Calculus Individual Test January 2003

For each of the questions below, NOTA means none of the other answers provided are correct.

1. Given $f(x)=\sin^2 x$, for what value of x (in radian measure) is $f'(2x)=0$?

A) 0 B) $\frac{\pi}{6}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{3}$ E) NOTA

2. The region bounded by the line $y=4$ and the curve $y=x^2$ is cut into two regions of equal area by the line $y=k$. What is the value of k?

A) 2 B) $2\sqrt{2}$ C) $2\sqrt[3]{2}$ D) $2\sqrt[4]{2}$ E) NOTA

3. Given $f(x)=x^n$, where n is a natural number, What is the n^{th} derivative of $f(x)$?

A) $n!x$ B) $(n-1)!x^{n-1}$ C) $n!$ D) 0 E) NOTA

4. Given an odd continuous function f and the fact $\int_{-a}^{0} f(x)\,dx = N$, what is the value of $\int_{0}^{2a} f(x)\,dx$?

A) $2N$ B) 0 C) Cannot be determined D) $-2N$ E) NOTA

5. At what x value is the line tangent to $f(x)=\sqrt{3x+1}$ parallel to the line $2x-3y=6$?

A) $\frac{1}{6}$ B) $\frac{9}{4}$ C) $\frac{65}{48}$ D) $\frac{81}{64}$ E) NOTA

6. Evaluate $\int_{0}^{\pi}\left(\sin^2 x+\cos^2 x\right)dx$

A) 0 B) $\frac{\pi}{2}$ C) π D) $\frac{5\pi}{4}$ E) NOTA

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7. Find the value c guaranteed by the mean value theorem for $f(x)=4\sqrt{x}+3x$ on the interval [1,4].

A) There are none B) $\frac{3}{2}$ C) $\frac{9}{4}$ D) $\frac{27}{8}$ E) NOTA

8. Evaluate $\lim_{h\to 0}\frac{\cot\left(\frac{\pi}{4}+h\right)-1}{h}$

A) -1 B) -2 C) 1 D) 2 E) NOTA

9. Find the equation of the line normal to $y=\frac{3}{x^2}$ at $x=2$.

A) $y-\frac{3}{4}=\frac{3}{4}(x-2)$ B) $y+\frac{3}{4}=\frac{-3}{4}(x-2)$

C) $y-\frac{3}{4}=\frac{-4}{3}(x-2)$ D) $y-\frac{3}{4}=\frac{4}{3}(x-2)$

E) NOTA

10. Evaluate $\lim_{h\to 0}\frac{\sin^2(5x)}{2x^2}$

A) 0 B) $\frac{2}{5}$ C) 1 D) $\frac{5}{2}$ E) NOTA

11. How many asymptotes does the graph of the function $f(x)=\frac{x^3-2x}{x^2-4}$ have?

A) 0 B) 1 C) 2 D) 3 E) NOTA

12. A spherical balloon is being inflated at a constant rate of $3\,cm^3/\text{min}$. What is the rate of change of the surface area at the instant that the volume of the balloon is $36\pi cm^3$?

A) $\frac{3}{4}\,cm^2/\text{min}$ B) $\frac{4}{3}\,cm^2/\text{min}$ C) $2\,cm^2/\text{min}$

D) $2\pi\,cm^2/\text{min}$ E) NOTA

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13. Simplify $\frac{f(x+h)-f(x)}{h}$ for $f(x)=\frac{1}{x+1}$

A) $\frac{1}{(x+1)(x+h+1)}$ B) $\frac{-1}{(x+1)(x+h+1)}$

C) $\frac{-x}{(x+1)(x+h+1)}$ D) $\frac{x}{(x+1)(x+h+1)}$

E) NOTA

14. Evaluate $\lim_{h\to 0}\frac{1-\cos^2 5h}{h}$

A) undefined B) 5 C) 1 D) 0 E) NOTA

15. Find the value of k which makes the function $f(x)=\begin{cases}0.4x+k^2 & x<1\\ kx+2.4 & 1\le x\end{cases}$ continuous.

A) -2 B) -1 C) 1 D) 2 E) NOTA

16. Given $f'(x)=\sin(x^2)$, then the graph of $y=f(x)$ is increasing on which of the following intervals?

A) $(0,\pi)$ B) $(\pi,2\pi)$ C) $(-\sqrt{\pi},\sqrt{\pi})$

D) $(\sqrt{\pi},\sqrt{2\pi})$ E) NOTA

17. For $f(x)=x^2-\frac{1}{\sqrt{x}}$ what is the x-coordinate of the point of inflection?

A) $\left(\frac{3}{4}\right)^{2/5}$ B) $\left(\frac{3}{8}\right)^{2/5}$ C) $\left(\frac{3}{8}\right)^{5/2}$ D) 1 E) NOTA

18. The location function of a particle in motion is $s(t)=2t^3-9t^2+5$. What is the value of $a(t)$, the acceleration function, when velocity is first zero? Assume that $t>0$.

A) -18 B) 0 C) 3 D) 18 E) NOTA

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19. The base of a solid in quadrant I is bounded by the curve $y = 4 - x^{2/3}$. If each cross section perpendicular to the x-axis is a square with one edge in the xy-plane, the volume of the resulting solid is

A) $\frac{1024}{35}$ B) $\frac{2896}{35}$ C) $\frac{328}{3}$ D) $\frac{2560}{3}$ E) NOTA

20. Given the following facts $f(2) = 6$, $f'(2) = 4$, $f''(2) = 2$, evaluate $\frac{d^2\left(f^3(x)\right)}{dx^2}$ at $x = 2$

A) -360 B) 36 C) 612 D) 792 E) NOTA

21. Let $f(x) = ax^4 + bx^2$ where $ab < 0$. Which of the following is true?

A) f has three critical values
B) f has two points of inflection
C) There is a point of inflection when $x = 0$
D) f has two roots
E) NOTA

22. $f(x) = -x^3 + Ax^2 + Bx + 30$. Given the facts that $f'(x) > 0$ on $(-2,10)$ and $f''(4) = 0$, find A and B

A) A = -12, B = -60
B) A = -12 , B = -36
C) A = 12, B = -60
D) A =12, B = 60
E) NOTA

23. Given the graph of a continuous function $y = f(x)$ such that $f(a) = c$ and $f(b) = d$. Geometrically, the average rate of change of $f(x)$ on [a,b] is represented by

A) The length of a horizontal line segment
B) The length of a vertical line segment
C) An area
D) Slope of the segment joining (a,c) and (b,d)
E) NOTA

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24. Let $f(x)$ be a function such that $f(2)=3$. Which of the following statements MUST be true?

I) $\lim_{x\to 2} f(x)=3$

II) $f(x)$ is continuous at $x=2$

III) $f(x)$ is differentiable at $x=2$

A) I only B) II only C) III only D) all must be true E) NOTA

25. $f(x)=\begin{cases}\dfrac{x^3-8}{x-2} & x\neq 2\\ k & x=2\end{cases}$. If $f(x)$ is continuous, then $k=$

A) 0 B) 2 C) 4 D) 12 E) NOTA

26. Find the equation of the line tangent to $x^2+3y^2=4$ at (1,1)

A) $y+1=\dfrac{-1}{3}(x+1)$ B) $y-1=\dfrac{-x}{3y}(x-1)$

C) $x+3y=2$ D) $y-1=\dfrac{-1}{3}(x-1)$

E) NOTA

27. A farmer wishes to fence in a yard of 20,000 square feet using a large barn as one of the sides. Find the minimum amount of fencing to be used for the job.

A) 40 ft B) 200 ft C) 500 ft D) 20,000 ft E) NOTA

28. If the half life of a radioactive substance is 8 years, how long will it take, in years, for two thirds of the substance to decay?

A) 7.69 B) 12 C) 12.21 D) 12.68 E) NOTA

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29. The height of fluid in a cylinder with a radius of 4 cm is increasing at the rate of $2\,{}^{cm}/_{\min}$. Find the rate of change of the volume of fluid in the cylinder with respect to time when the height of the fluid is 10 cm.

A) $16\pi\,{}^{cm^3}/_{\min}$ B) $\frac{1}{16\pi}\,{}^{cm^3}/_{\min}$ C) $\frac{5}{8\pi}\,{}^{cm^3}/_{\min}$

D) $160\pi\,{}^{cm^3}/_{\min}$ E) NOTA

30. Use the trapezoidal rule with two equal subintervals to approximate, correct to three decimal places, $\int_1^2 \sin\sqrt{x}\,dx$

A) 0.042 B) 0.692 C) 0.915 D) 0.928 E) NOTA