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The abbreviation "**NOTA**" denotes "**None of the Above Answers is Correct**"
The abbreviation "**DNE**" denotes "**Does Not Exist**"

1. Find the range of $f(x)=\frac{|x|}{x}$.

a) $\{1\}$ b) $\{-1,1\}$ c) $(0,\infty)$ d) $(-\infty,0)\cup(0,\infty)$ e) NOTA

2 The number of cars produced at a factory is given by $200\sqrt{xy}$, where x and y are the amount of capital and labor used, respectively. At a particular point in time:

I. The factory has 2 units of capital
II. The factory has 3 units of labor
III. Capital is increasing at the rate of 1 unit per month
IV Labor is decreasing at the rate of 0.5 units per month

At this point in time, calculate (to the nearest car) the rate of change in the number of cars produced per month.

a) 82 b) 122 c) 164 d) 200 e) NOTA

3. Find $f^{3}(x)$ given $f(x)=\frac{x^2-1}{2-2x}$.

a) $\frac{-1}{2}$ b) $-x$ c) $\frac{-x^2+2x}{(1-x)}$ d) $\frac{(x^2+1)}{2(x-1)}$ e) NOTA

4. Evaluate: $\lim_{x\to 0}\frac{\sqrt[3]{8+x}-\sqrt[3]{8}}{x}$.

a) 0 b) $\frac{1}{12}$ c) $\frac{4}{3}$ d) 2 e) NOTA

5. Find the positive difference between the average values of $y=2x^3+2$ on the intervals $3\ge x\ge 0$ and $-3\le x\le 0$.

a) 4 b) 12 c) 27 d) 81 e) NOTA

6. Let $y(x)=x^{\sum_{n=1}^{11}(n+x)}$ Find $y'(1)$.

a) 1 b) 67 c) 77 d) e^{67} e) NOTA

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7. An expanding ring with inner radius $r = 3\,cm$ and outer radius R has area $A = 72\pi\ cm^2$ at time $t = 0$ seconds. If the rate of change of the outer circumference is a constant 10π **cm/sec** and $\frac{dr}{dt}$ is 50% greater than $\frac{dR}{dt}$, at what time t (in seconds) will the ring have no area?

a) 1.98 b) 2.4 c) 8.4 d) 14.4 e) NOTA

8. Evaluate: $\lim_{x\to 0} \frac{2xe^x - e^x - x + 1}{4x^2}$.

a) $\frac{-1}{8}$ b) $\frac{19}{50}$ c) ∞ d) DNE e) NOTA

9. The price of a share of stock in Cham.com [NASDAQ: CHAM] is modeled by the function $P(t) = t - \sin(\pi t) + P(t_0)$, where t is the number of hours after noon and $P(t_0)$ is the price of the stock at noon. Find the sum of the value(s) that satisfy the Mean Value Theorem for derivatives for CHAM's stock price from 1:00PM to 2:00PM.

a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $\frac{\pi}{2}$ d) π e) NOTA

10. Determine the second derivative of $f'(x) = 200 + 20x + .2x^2$ at $x = -48$.

a) .4 b) .5 c) .6 d) .8 e) NOTA

11. Find the obtuse angle of intersection (in degrees) between the curves $f(x) = 2x^2$ and $g(x) = 20 - 3x^2$ at their point of intersection in the first quadrant. What is the one's digit? (e.g. the "2" in 52.3°)

a) 4 b) 5 c) 6 d) 8 e) NOTA

12. What is the sum of the coefficients of the highest and lowest order terms in the derivative of $f(x)$ given $f(x) = (x^2 - 6x)(x^7 + 3)(x^3 - 2)$?

a) 7 b) 8 c) 24 d) 48 e) NOTA

13. Find $f^{100}(1)$ for $f(x) = 3^{2x} + \cos(\pi x) + x^{99}$.

a) $2^{100}(\ln 3)^{100} - \pi^{100}$ b) $9(\ln 9)^{100} - \pi^{100}$

c) $9(\ln 9)^{100}$ d) $9(\ln 9)^{100} + \pi^{100}$ e) NOTA

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14. A hot apple pie at 85° is placed on a rack to cool in a room with an ambient temperature of 25°. At 3:00PM the temperature of the pie was 75° and 50° at 3:10PM. If the pie obeys Newton's Law of Cooling, when was it placed on the rack (to the nearest minute)?

Newton's law of cooling states that the rate of change of the temperature T of an object is proportional to the difference between T and the constant temperature of the surrounding medium, called the *ambient temperature*.

a) 2:26PM b) 2:49PM c) 2:57PM d) 2:58PM e) NOTA

15. Let the Quotient Derivative function, $QD(f(x), g(x))$, equal the derivative of $f(x)$ divided by $g(x)$ or $\frac{f'(x)}{g(x)}$. Find: $QD\left[QD(x^5, x), 5x^2\right]$ if $x \neq 0$.

a) $\frac{4}{5x}$ b) x c) $4x$ d) $\frac{x^2}{5}$ e) NOTA

16. Let $L = \lim_{x\to\infty} \sqrt{(x^2+6x)} - x$ Let $K = \lim_{x\to\infty} \sqrt{(x^2+8x-3)} - x$

Find: L^K.

a) 8 b) 81 c) 256 d) ∞ e) NOTA

17. Let $T(t) = 13\sin(t+5) + 80$ model the temperature in the Amazon rain forest at time t on the interval (0, 7). Find all values of t for which $T(t)$ is increasing.

a) $(0, \frac{5\pi}{2} - 5) \cup (\frac{7\pi}{2} - 5, 7)$ b) $(\frac{5\pi}{2} - 5, \frac{7\pi}{2} - 5)$
c) $(0, \frac{3\pi}{2} - 3) \cup (\frac{5\pi}{2} - 3, 7)$ d) $(\frac{3\pi}{2} - 3, \frac{5\pi}{2} - 3)$ e) NOTA

18. If $f(x) = ax^3 + bx^2 + c$ does not have a relative maximum, what **must** be true of b?

a) $b < 0$ b) $b = 0$ c) $b > 0$ d) $b = a$ e) NOTA

19. An equation of the line tangent to the graph of a differentiable function $f(x)$ at x = 0 is $g(x) = 5x + 4$.

Evaluate: $\lim_{x\to 0} \frac{xf(x)}{\sin(2x)}$.

a) 0 b) 1 c) 2 d) 4 e) NOTA

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20. Find the y-intercept of the tangent line to the graph of $y = \arcsin\sqrt{x}$ at the point $(\frac{1}{2}, \frac{\pi}{4})$

a) $\frac{\pi}{4} - \frac{\sqrt{2}}{2}$ b) $\frac{\pi}{4} - \frac{1}{2}$ c) $\frac{\pi}{4} - \frac{\sqrt{3}}{6}$ d) $\frac{\pi}{4} - \frac{\sqrt{6}}{6}$ e) NOTA

21. Evaluate: $\int_{-\pi}^{\pi} \frac{\sin x}{\cos x}\,dx.$

a) 0 b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) π e) NOTA

22. Find equations for the linearization of $y = x^2 - x + 3$ at $x = 1$ and at $x = 2$. What is the x-value of the intersection of the two equations?

a) -3.5 b) 0 c) 1 d) 1.5 e) NOTA

23. Find the point on the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$ that is closest to $(2, \sqrt{3})$. Round to two decimal places.

a) (1.64, 1.82) b) (1.97, 1.74)
c) (2.03, 1.72) d) (2.34, 1.62) e) NOTA

24. Use two iterations of Newton's Method to approximate the x-value at the intersection of $y = 3 - \ln x$ and $y = 4x$ with $x_0 = 1$ as the initial guess. Round to three decimal places.

a) .795 b) .796 c) .804 d) .805 e) NOTA

25. Find a Riemann Sum formulation for the following integral $\int_{10}^{20} x^3\,dx$

a) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 \left(\frac{10i}{n}\right)$ b) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{10i}{n}\right)^3 \left(\frac{10}{n}\right)$

c) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(10 + \frac{10i}{n}\right)^3 \left(\frac{10}{n}\right)$ d) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{20i}{n}\right)^3 \left(\frac{20}{n}\right)$ e) NOTA

26. Use a differential to estimate $\sqrt{69}$ given that $\sqrt{64} = 8$.

a) $\frac{70}{9}$ b) $\frac{108}{13}$ c) $\frac{133}{16}$ d) $\frac{151}{18}$ e) NOTA

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27. Given: $f(a,b,c)=a^1b^2c^3$. Find: $\frac{\partial f}{\partial b}(a,b,c)$.

a) $2b$ b) ac^3 c) $6bc^2$ d) $2abc^3$ e) NOTA

28. Let $f(x)=x^3-x^2-3$ for $\frac{3}{4}\le x\le\frac{5}{2}$ Find $g'(1)$ if $g(x)$ is the inverse of $f(x)$.

a) -3 b) $\frac{-1}{3}$ c) $\frac{1}{8}$ d) 1 e) NOTA

29. Let R be a solid in 3-space and f a function defined on R such that:

$$\iiint_R f(x,y,z)\,dV=3$$
$$\iiint_R (5f(x,y,z)+2)\,dV=53$$

Determine the volume of R.

a) 19 b) $\frac{58}{3}$ c) 34 d) 56 e) NOTA

30. Find the equation of the tangent line to the graph of $y=\sqrt{4-x^2}$ at the point (0, 2).

a) $y=2$ b) $y=\frac{-1}{2}x+2$
c) $y=x+2$ d) DNE e) NOTA