January Regional
Calculus Individual Test

For all questions, "E. NOTA" means none of the above answers is correct. Unless otherwise stated, assume all numbers are real.

1. Let $f(x)=3x^2-\cos(x)$. Find $f'(\pi)$.

A. $6\pi-1$
B. 6π
C. $3\pi^2+1$
D. π^3
E. NOTA

2. Find the slope of the line tangent to the graph of $xy^3+3y^2-2x^2-18=0$ at the point $(3,2)$.

A. -12
B. $-\frac{1}{12}$
C. $\frac{1}{12}$
D. 12
E. NOTA

3. Evaluate: $\lim_{x\to 2^+} e^{\sqrt{x^2-4}}$

A. 0
B. e^{-1}
C. e
D. Limit does not exist
E. NOTA

4. Find the maximum value of $y=xe^{-x}$ on $[-\ln 2, \ln 2]$.

A. $\ln\frac{1}{4}$
B. $\frac{1}{e}$
C. $\ln\sqrt{2}$
D. f attains no maximum value
E. NOTA

5. Suppose $f(x)$ is continuous at $x=a$. Which of the following are true?

I. $f(a)$ exists
II. $\lim_{x\to a} f(x)$ exists
III. $f'(a)$ exists
IV. For every $\varepsilon>0$ there exists $\delta>0$ such that if $|x-a|<\delta$, then $|f(x)-f(a)|<\varepsilon$

A. I only
B. I and II only
C. I, II, and IV only
D. I, II, III, and IV
E. NOTA

6. Find the radius of the largest neighborhood about $x=3$ such that $|3x-9|<.01$. Round your answer to the nearest thousandth.

A. 0.001
B. 0.002
C. 0.003
D. 0.004
E. NOTA

7. Evaluate: $\lim_{x\to-\infty} \frac{\sqrt{4x^2+5x+1}}{x+1}$

A. -4
B. 0
C. 2
D. ∞
E. NOTA

8. How many asymptotes does the graph $f(x)=\frac{x^3+3x^2-4x}{x^3+x^2-x-1}$ have?

A. 1
B. 2
C. 3
D. 4
E. NOTA

9. Find the equation of the line tangent to the graph $f(x)=\sin(x)\tan(x)$ when $x=\frac{\pi}{4}$.

A. $2x+3y\sqrt{2}=\sqrt{2}\left(\frac{3\pi}{4}-1\right)$

B. $2x-3y\sqrt{2}=\sqrt{2}\left(\frac{3\pi}{4}-1\right)$

C. $3x\sqrt{2}+2y=\sqrt{2}\left(\frac{3\pi}{4}-1\right)$

D. $3x\sqrt{2}-2y=\sqrt{2}\left(\frac{3\pi}{4}-1\right)$

E. NOTA

10. Lauren wishes to enclose a garden with a rectangular wooden fence. To conserve wood, she decides to use one side of his house as a side of the rectangle. Lauren only has enough wood to build 24 feet of fence. Find the maximum area enclosed by the fence. Express your answer in square feet.

A. 36
B. 48
C. 64
D. 72
E. NOTA

11. Which of the following statements is **false** concerning a function $f(x)$ defined on $[a,b]$?

A. If $f(x)$ is convex on (a,b), then $f(x)$ is continuous on (a,b)

B. $f(x)$ attains its maximum value on $[a,b]$

C. If $f(x)$ is differentiable on (a,b), then $f(x)$ is continuous on (a,b)

D. If $f(x)$ is differentiable on $[a,b]$ and $f(a)=f(b)$, then $f(x)$ has a critical point in (a,b)

E. NOTA

12. A ball is dropped from the top of a 100 foot building. If the acceleration due to gravity is $-32\,{}^{\text{ft}}/_{\text{sec}^2}$, how long in seconds will it take for the ball to hit the ground?

A. 1
B. 2
C. 3
D. 4
E. NOTA

13. For what values of x is $y=\frac{x^2+3x+1}{x^2+1}$ increasing?

A. $x=-1$ and $x=1$

B. $\left(\frac{-3-\sqrt{5}}{2},\frac{-3+\sqrt{5}}{2}\right)$

C. $(-1,1)$

D. $(-\infty,-1)\cup(1,\infty)$

E. NOTA

14. At what values of x does the graph of $f(x)=x^5-10x^3+20x^2-15x+3$ change concavity?

A. $x=-3$ and $x=1$
B. $x=-2$ and $x=1$
C. $x=-3$
D. $x=-2$
E. NOTA

15. Suppose $f(x)$ and $g(x)$ are continuous at $x=2$, $\lim_{x\to 2} f(x)=2$, and $\lim_{x\to 2} g(x)=0$. Evaluate $\lim_{x\to 2}\left(f(x)+g(x)\right)^{g(f(x))}$.

A. 0
B. $\frac{1}{2}$
C. 1
D. Cannot be determined
E. NOTA

16. Which of the following functions is continuous at $x=3$?

A. $f(x)=\begin{cases}1, & x\neq 3\\ 0, & x=3\end{cases}$

B. $f(x)=\begin{cases}\dfrac{|x-3|}{x-3}, & x\neq 3\\ 1, & x=3\end{cases}$

C. $f(x)=\begin{cases}\dfrac{x^3-3x^2-7x+21}{x-3}, & x\neq 3\\ 2, & x=3\end{cases}$

D. $f(x)=\begin{cases}\dfrac{x^2-9}{x-3}, & x\neq 3\\ 3, & x=3\end{cases}$

E. NOTA

17. Brianna, the owner of a widget company, has determined that the cost of producing x widgets is given by $C(x)=500+10x+\dfrac{1000}{x}$ if $x>0$. How many widgets should Brianna produce if she wants to minimize her costs?

A. 10
B. 240
C. 470
D. 700
E. NOTA

18. Suppose $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b). Then there exists $c\in(a,b)$ such that the line tangent to the graph $f(x)$ at $x=c$ is parallel to the secant line through $(a,f(a))$ and $(b,f(b))$. This is a geometric interpretation of which of the following theorems?

A. Mean Value Theorem
B. L'Hôpital's Rule
C. Taylor's Theorem
D. Fundamental Theorem of Calculus
E. NOTA

19. If $-x^2\le f(x)\le x^2$ for all x, what is $\lim_{x\to 0} f(x)$?

A. 0
B. 1
C. ∞
D. Cannot be determined
E. NOTA

20. Evaluate: $\dfrac{d}{dx}\left(\ln\left|\cos\left(x^2\right)\right|\right)$

A. $\sec\left(x^2\right)$
B. $-\tan\left(x^2\right)$
C. $-2x\tan\left(x^2\right)$
D. $2xe^{-\sin\left(x^2\right)}$
E. NOTA

21. Find the n$^{\text{th}}$ derivative of $f(x)=\dfrac{1}{x}$, where n is a whole number.

A. $\dfrac{(-1)^n n!}{x^{n+1}}$
B. $\dfrac{(-1)^n n!}{x^n}$
C. $\dfrac{(-1)^n (n+1)!}{x^n}$
D. $\dfrac{(-1)^{n+1} n!}{x^n}$
E. NOTA

22. Find the maximum value of $f(x)=\dfrac{-2x^3+6x^2-4x}{x-1}$.

A. 0
B. 1
C. 2
D. f attains no maximum value
E. NOTA

23. Let $f(x)=\begin{cases} x^2+ax+2, & [0,a] \\ 3-x, & (a,4] \end{cases}$ where $0<a<4$. For which value of a is $f(x)$ continuous on $[0,4]$?

A. $\frac{1}{2}$
B. $\frac{3}{2}$
C. $\frac{5}{2}$
D. $\frac{7}{2}$
E. NOTA

24. Evaluate: $\lim_{x\to 0}(1+x)^{1/x}$

A. 0
B. e^{-1}
C. 1
D. e
E. NOTA

25. Suppose $f(x)=0$ when x is rational and $f(x)=1$ when x is irrational. For what values of x is $f(x)$ continuous?

A. $x=0$
B. $\{x \mid x \text{ is rational}\}$
C. $\{x \mid x \text{ is irrational}\}$
D. $f(x)$ is nowhere continuous
E. NOTA

26. Given $\sqrt[7]{2187}=3$, use differentials to approximate $\sqrt[7]{2005}$. Round your answer to the nearest thousandth.

A. 2.962
B. 2.963
C. 2.964
D. 2.965
E. NOTA

27. Let $f(x)=x\cdot[x]$, where $[x]$ is the greatest integer less than or equal to x. What is $f'(2)$?

A. 0
B. 1
C. 2
D. 4
E. NOTA

28. Evaluate: $\lim_{x\to 0}\frac{\ln(4+x)-\ln 4}{x}$

A. 0
B. $\frac{1}{4}$
C. $\ln 4$
D. e^4
E. NOTA

29. Evaluate: $\lim_{n\to\infty}\frac{1^2+2^2+\cdots+n^2}{n^3}$

A. $\frac{1}{3}$
B. $\frac{1}{2}$
C. 1
D. Limit does not exist
E. NOTA

30. Find the maximum value of $f(x)=\frac{x}{\sqrt{x^3+1}}$.

A. $\frac{10\sqrt{189}}{189}$
B. $\frac{\sqrt[6]{108}}{3}$
C. $\sqrt[3]{2}$
D. $\sqrt[3]{2}\cdot\sqrt{3}$
E. NOTA