Calculus Team **FAMAT Regional** **January**

1. What are the coordinates of the point on the graph of $y = -2x^2 - 9x - 5$ at which the tangent line is perpendicular to the line $y = x - 10$?

2. The following is a table of values for a continuous differentiable function f.
Let A be the y-intercept of the normal to f at x = 4.
Let B be the average rate of change of f on [2,5].
Let C be the x-intercept of the tangent line at x = 2.
Let D be the slope to the line parallel to the normal at x = 2.
Find A + B + C + D.

x	1	2	3	4	5
f(x)	1	3	5	7	10
f '(x)	0.25	0.5	1	2	3

3. A particle is moving along the curve $y = \ln(5x + 2)$. At the instant when the particle crosses the x-axis, the ordinate is changing at the rate of 10 units per second. Find the rate of change in units per second of the abscissa at this time.

4. List the letters of the statements that are true.

A) $\dfrac{d^2y}{dx^2} = \left(\dfrac{dy}{dx}\right)^2$

B) You can always divide by e^x.

C) If P is a polynomial, then $\lim\limits_{x \to a} P(x) = P(a)$.

D) If $f(x) = 4e^3$, then $f'(x) = 12e^2$.

E) $f(x) = \sin(2x)$, $f^{99} = 2^{99}\cos(2x)$

F) $\lim\limits_{x \to -\infty} \dfrac{2x}{\sqrt{x^2 + 1}} = -2$

G) If $f''(x) = (x - a)^2 (x - b)^2$, then f has no points of inflection.

5. Let A(x) be the area of a rectangle whose vertices are (x. ln(x)), (8, ln(x)), (8,0) and (x, 0), $1 \le x \le 8$. To the nearest thousandth what is the greatest value of A(x).

6. If $f(x) = e^{\cos(x)}$, let A be the number of zeroes f ' has on $[0, 3\pi]$.
If $f'(x) = \cos(x^x)$, let B be the number of critical points that f has on (0.2, 2.6).
If $f'(x) = 0.5 + \sin(x) + 0.1\ln(x)$, let C be the number of points of inflection of f on [0, 15].
If $f(x) = (x^2 - 1)(x^2 + 2)$, let D be the number of values that satisfy the mean value theorem in the interval $-3 \le x \le 4$.
Find A + B + C + D.

7. A function g is defined for all real values of a and b such that $g(a + b) - g(a) = 10ab - 4b^2$.
Find g '(x).

8. $f'(x) = (x - 0.1)^2 \sqrt{1.08 - 0.9x^2}$. Graph f ' on [-1, 1]. Find the x-coordinate of each point of inflection of f to the nearest thousandth. Then find their sum.

9. $h(x) = e^{g(x)}$, $g(x) > 0$, g is concave up, and h is always increasing. Both h and g are twice differentiable. List the letters of the statements that must be true.

A) $h(0) = 1$ B) g is always increasing. C) $h(x) > 0$

D) h is always concave up. E) h has one horizontal tangent line.

10. If $(x - y)^2 = y^2 - xy$, find the slope of the tangent line at $x = 1$.

11. $f(x) = e^{\tan^{-1}(3x)} - 2x$. Find each of the answers to the nearest thousandth then find their sum.

Let A be the x-coordinate of the point where the rate of change of f is the greatest.

Let B be the slope of the tangent at the x-intercept of f.

Let C be the x-coordinate of the point where the normal is first parallel to $4x - 3y = 1$.

Let D be the value of c guaranteed by the Mean Value Theorem over [0,3].

12. Find $f'(x)$ if it is known that $\frac{d}{dx}[f(2x)] = x^2$.

13. A particle moves on a vertical line so that is coordinates at t, $t \geq 0$, are given by $y(t) = t^3 - 12t + 5$. On what interval(s) is the speed increasing?

14. A conical tank has radius 3 ft and depth of 10 ft. If water is poured into the tank at the rate of $2ft^3$/min, to the nearest thousandth how fast in ft/min is the water level rising when the water in the tank is 6 ft deep?

15. $f(x) = x^5 + x$. Find the value of $\frac{d}{dx}f^{-1}(x)$ at $x = 2$.