

Calculus Team · JANUARY REGIONAL Question 1

$$f(x) = 11x^3 + 10x^2 + 9x^3 + 8x^2 + 7x + 6$$

$$h(x) = \int_1^2 x^{\cos x} dx$$

$$A = f^{(3)}(3)$$

$$B = g'(\sqrt{\pi})$$

$$C = h(1)$$

$$D = x'(0)$$

Find $A + B + C + D$

$$g(x) = \cos(x^2)$$

$$x(0) = t^2 + e^t$$

Calculus Team · JANUARY REGIONAL Question 2

$$A = \lim_{z \rightarrow 1} \left(\frac{1}{z} - \frac{1}{z-1} \right) \left(\frac{1}{z-5} \right)$$

$$B = \lim_{y \rightarrow 0} \frac{1 - \sec^2 2y}{y^2}$$

$$C = \lim_{h \rightarrow 0} \frac{\cos(\pi+h) + 1}{h}$$

$$D = \lim_{x \rightarrow 0} \frac{\sin 5x}{2x}$$

Find $A * B * C * D$

Calculus Team · JANUARY REGIONAL Question 3

$$s(x) = \frac{1}{2x}$$

$$t(x) = (x-6)^3 - 3(x-6) + 2$$

$$u(x) = x^3 - 4x^3 + 1$$

$$v(x) = -3 \ln(x^2)$$

- A = The intervals where $s(x)$ is decreasing
- B = The intervals where $t(x)$ is increasing
- C = The intervals where $u(x)$ is concave down
- D = The intervals where $v(x)$ is concave up

Find $A \cup B \cup C \cup D$

Calculus Team · JANUARY REGIONAL Question 4

A = The number of points of inflection of $\cos(2x)$ on $[0, 4\pi]$

B = The average value of $\frac{4x^4 + 8x^2 + 12}{\sin x}$ on $(-\infty, \infty)$

C = The number of values of c on $[0, 2\pi]$ that satisfy the Mean Value Theorem for derivatives for $f(x) = \cos x$.

Find: $A^B + C$.

Calculus Team · JANUARY REGIONAL Question 5

Let C be the curve in \mathbb{R}^3 defined by $x = t^2$, $y = 4t^3$, $z = 9t$, for $t \geq 0$. Calculate the distance along C from $(1, 4, 9)$ to $(16, 32, 36)$

Calculus Team · JANUARY REGIONAL Question 6

Find the area of the ellipse $(x - \sqrt{3})^2 + \frac{81(y - \pi)^2}{100} = 81$.

Calculus Team · JANUARY REGIONAL Question 7

If the function $f(x)$ is continuous and differentiable for all x and $f'(2) = 3$, $f''(x) = 0$ and $f(3) = 12$, find the equation for the function $f(x)$.

Calculus Team · JANUARY REGIONAL Question 8

$$\int_0^A 2t dt = 16$$

$$\int_2^{3M} \frac{dt}{t} = 9$$

$$\int_0^A \int_{\frac{1}{x}}^{12} \tan^2 x dx = 12 \quad C \in [0, 4]$$

Find: $|A * B * C|$

Calculus Team - JANUARY REGIONAL Question 9

$$f(x) = \frac{x^2}{(x-1)}$$

$$g(x) = x \sin^2 x$$

$$h(x) = xe^x + 2^x$$

Find: $f'(2) + g'(\pi) + h'(1)$.

Calculus Team - JANUARY REGIONAL Question 10

Given $v(x) = g(h(x))$

$$u(x) = h(g(x))$$

$$s(x) = \int_{h(x)}^{v(x)} f(t) dt$$

$$h(-3) = -3$$

$$h(1) = -2$$

$$h'(-3) = 1$$

$$h'(1) = 6$$

$$g(-3) = 1$$

$$g(1) = -3$$

$$g'(-3) = 3$$

$$g'(-2) = \frac{1}{2}$$

$$g'(1) = 1$$

$$f(-3) = \frac{1}{2}$$

$$f(-2) = \frac{1}{2}$$

$$f(1) = \frac{1}{2}$$

$$f'(-3) = 3$$

$$f'(-2) = 2$$

$$f'(1) = 2$$

Find: $s'(1)$.

Calculus Team - JANUARY REGIONAL Question 11

Find all asymptotes of $g(x) = \frac{x^2 + 2x - 15}{x^2 - 5x + 6}$.

Calculus Team - JANUARY REGIONAL Question 12

The base of a solid is the region between the parabolas $x=y^2$ and $x=3-2y^2$. Find the volume of the solid given that the cross sections perpendicular to the x-axis are rectangles of height 2.