Collcute High School

1. Which of the following is equal to e?

I. $\sum_{n=1}^{\infty}\frac{1}{n!}$ II. 2.718281828 III $\lim_{x\to\infty}(\cos x + x)^{\frac{1}{x}}$

A) I B) II C) III
D) I and II E) I, II, and III

2. What is the *nth* derivative, $f^{[n]}(x)$, of the function $y = x^n$?

A) n B) n^2 C) $(n-1)!$ D) $n!$ E) NOTA

3. If $y = x^4 + 2x^3 + 3x^2$, then the derivative of y with respect to $x^2 + 1$ is

A) $4x + 3$ B) $2x^2 + 6x + 3$ C) $2x^2 + 3x + 3$
D) $4x^3 + 2x^2 + 6x$ E) NOTA

4. The graph of $f(x) = \frac{2x^2 - x - 3}{3x^2 - x - 2}$ crosses its horizontal asymptote when

A) x = 5 B) x = 1 C) x = -1 D) x = -5 E) NOTA

5. The number of bacteria in a culture is growing at a rate of $1500e^{5t/3}$ per unit of time. At $t = 0$, the number of bacteria present was 2500. The number of bacteria present at $t = 6$ is

A) $2500e^6$ B) $1500e^{10}$ C) $2500e^{10}$ D) $900e^{10}$ E) NOTA

6. Choose (a, b) such that $0 \le a \le \pi$ and $0 \le b \le 1$. What is the probability that (a,b) lies outside the region defined by $\int_0^{\pi} \sin x dx$?

A) $\frac{1}{2}$ B) $\frac{\pi - 2}{\pi}$ C) $\frac{\pi - 2}{\pi^2}$ D) $\frac{\pi^2 - 2}{\pi}$ E) NOTA

7. $\lim_{n\to\infty}\left(\sqrt{n^2 + n} - \sqrt{n^2 + 1}\right)$

A) 0 B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) ∞ E) NOTA

8. $\lim_{n\to\infty}\sum_{x=1}^{n}\frac{2x^3+x^2+x}{n^4}=$

A) 0 B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) 1 E) NOTA

9. Given the set of (Roman) numbered statements:

I. If f is continuous on $[a,b]$, then it is integrable on $[a,b]$

II. If f is continuous on $[a,b]$, then it is differentiable on $[a,b]$

III. If f is integrable on $[a,b]$, then it is continuous on $[a,b]$

IV. If f is differentiable on $[a,b]$, then it is continuous on $[a,b]$

V. If f is differentiable on $[a,b]$, then it is integrable on $[a,b]$

VI. If f is integrable on $[a,b]$, then it is differentiable on $[a,b]$

Which of the following subsets indicate all true statements?

A) { I, II, IV } B) {I, III, IV} C) {III, IV, VI}
D) { I, IV, V} E) NOTA

10. If water weighs 62 lb / ft^3, find the work required to pump all the water from a full cylindrical container to its top. The dimensions of the cylinder are: diameter 3 ft. and height 4 ft. [The answer will be in ft-lbs]

A. 18π B) 576π C) 1116π D) 4464π E) NOTA

11. $\lim_{n\to\infty}\frac{\sqrt{1}+\sqrt{2}+\cdots+\sqrt{n}}{n^{\frac{3}{2}}}$ is equal to the definite integral

A) $\int_0^1\sqrt{x}dx$ B) $\int_0^1\frac{1}{\sqrt{x}}dx$ C) $\int_1^2\sqrt{x}dx$ D) $\int_0^1\frac{1}{x}dx$ E) NOTA

12. Which of the following improper integrals diverge?

A) $\int_0^\infty e^{-x^2}dx$ B) $\int_{-\infty}^0 e^x dx$ C) $\int_0^1\frac{dx}{x}$ D) $\int_0^\infty e^{-x}dx$ E) NOTA

13. The figure shown has an area of 10 and its center of mass (centroid) is at the point P(2,4). Find the volume of the solid which is formed by rotating the figure about the x-axis.

A) 20π B) 40π C) 80π D) 160π E) NOTA

14. If $f(x)$ is continuous at the point where $x=a$, which of the following statements may be false?

A) $\lim_{x\to a}f(x)$ exists B) $\lim_{x\to a}f(x)=f(a)$ C) $f'(a)$ exists
D) $f(a)$ is defined E) $\lim_{x\to a^-}f(x)=\lim_{x\to a^+}f(x)$

15. Find: $\lim_{x\to\infty}\dfrac{(4+3x)(12x-1)(20+8x)}{(4x+3)(12x+1)(2-6x)}$

A) -1 B) 1 C) 0 D) ∞ E) NOTA

16. Find all assymptotes of the following function:

$y=\dfrac{4x^4-8x^3-15x^2+6x+9}{x^3-7x-6}$

A) $x=-2$ B) $x=-2, x=-1, x=3, y=4x-8$ C) $x=-2, x=-1, x=3$

D) $x=-3, x=1, x=2$ E) NOTA

17. Let $f(x)=\dfrac{1}{3^{1/x}+1}$ and $g(x)=\dfrac{x}{3^{1/x}+1}$. At $x=0$, which of the following is true?

A) Both f and g have essential discontinuities.
B) Both f and g have removable discontinuities.
C) f has an essential discontinuity and g has a removable discontinuity.
D) f has a removable discontinuity and g has an essential discontinuity.
E) NOTA

18. For $f(x)=x^{n-1}\ln x$, the n^{th} derivative, $f^{[n]}(x)=$

A) $\dfrac{(n-2)!}{x}$ B) $\dfrac{(n-1)!}{x}$ C) $\dfrac{n!}{x}$ D) $\dfrac{(n+1)!}{x}$ E) NOTA

19. Find the value(s) of x_0 guaranteed by Rolle's Theorem for $f(x)=x^3-12x$ on the interval $0\le x\le 2\sqrt{3}$.

A) 2 B) -2, 2 C) 1 D) -3, 1 E) NOTA

20. What is the derivative of $|x|$?

A) +1, -1 B) $\dfrac{\sqrt{x^2}}{x}$ C) $-|x|^{-2}$ D) undefined E) NOTA

21. To approximate the value of $\ln(2.1)$, John uses linear interpolation, while Humberto uses differentials. If their calculations are correct, what is the absolute value of the difference between their answers? (Assume ln 2 = .693 and ln 3 = 1.099)

A) 0.0094 B) 0.0187 C) 0.0356 D) 0.0406 E) NOTA

22. If $g(x)=\dfrac{x^2-|x|-6}{x^2-9}$, then find $\lim_{x\to 0^+} g'(x)$

A) 1/9 B) 1/6 C) 1/3 D) 2/3 E) NOTA

23. Calculate $\int_1^3 x^2 dx$ using the trapezoid rule with 6 subdivisions of the interval $[1,3]$.

A) 26/3 B) 235/27 C) 461/54 D) 13/2 E) NOTA

24. What is the coefficient of x^2 in the Maclaurin series of $f(x)=\frac{1}{\sqrt{1+\sin 2x}}$?

A) 6 B) 3 C) 2 D) 1 E) NOTA

25. Find the equation of the line tangent to the curve $e^x+\cos(y)-\frac{3}{2}=0$, if the line passes throught the point $\left(0,\frac{\pi}{3}\right)$.

A) $y=\frac{2\sqrt{3}}{3}x+\frac{\pi}{3}$ B) $y=\frac{2\sqrt{3}}{3}x-\frac{\pi}{3}$ C) $y=\frac{1}{2}x+\frac{\pi}{3}$

D) $y=\frac{1}{2}x-\frac{\pi}{3}$ E) NOTA

26. Use the mean value theorem to find c if $y=\frac{\sin(x)}{x}$ and $a=-\frac{\pi}{4}$, $b=\frac{\pi}{4}$.

A) 0 B) 1 C) $-\frac{\pi}{4}$ D) $\frac{\pi}{4}$ E) NOTA

27. Find the curvature of $y^2=2x$ when $x=2$.

A) $\frac{\sqrt{5}}{25}$ B) $\sqrt{5}$ C) $\frac{\sqrt{5}}{5}$ D) $5\sqrt{5}$ E) NOTA

28. A hotel chain has asked the R&R Railroad Company to run a special train to accomodate its customers. The railroad company agrees if at least 200 people will use the service. The fare is to be \$8 per person if 200 go and will decrease by one cent for everybody for each person over 200 that goes. If k is the number of passengers that will give the raidroad maximum revenue, find $2k-200$.

A) 230 B) 400 C) 640 D) 800 E) NOTA

29. Two cars leave an intersection at the same time, one heading 30^0 from due north and 50 mph and the other heading 90^0 from due north at 40 mph. After one hour, how fast is the distance between the two cars changing?

A) $5\sqrt{21}$ B) $10\sqrt{21}$ C) $\frac{41\sqrt{21}}{21}$ D) $\frac{30\sqrt{21}}{7}$ E) NOTA

30 $\lim_{r\to 0}\frac{1}{2}r^{-1}\left((r+e)^{r+e}-e^e\right)$

A) e B) $\frac{1}{2}e^e$ C) e^e D) $2e^e$ E) NOTA

1. C (I = e -1, II not exact!, III =e)
2. D
3. C
4. D
5. E $900e^{5t/3}+1600$
6. B
7. C
8. C
9. D
10. C
11. A
12. C
13. C
14. C
15. A
16. E (x=3, x=-1 are removable so not assymptotes. **x=-2 vertical, y=4x-8, oblique**)
17. C
18. B
19. A
20. B
21. A
22. A
23. B
24. E $\frac{3}{2}$
25. A
26. E MVT does NOT apply because function is not continuous.
27. A $\frac{|y''|}{\left(1+(y')^2\right)^{3/2}}$
28. D
29. B
30. C