

Sequences

Identify the following sequences and then evaluate.

1. $\lim_{n \rightarrow \infty} \frac{1}{n} \left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \dots + \left(\frac{n}{n}\right)^2 \right]$
2. a) $\lim_{n \rightarrow \infty} \frac{1}{n} \left[\sin\left(\frac{\pi}{n}\right) + \sin\left(\frac{2\pi}{n}\right) + \dots + \sin\left(\frac{n\pi}{n}\right) \right]$
 b) $\lim_{n \rightarrow \infty} \frac{\pi}{n} \left[\sin\left(\frac{\pi}{n}\right) + \sin\left(\frac{2\pi}{n}\right) + \dots + \sin\left(\frac{n\pi}{n}\right) \right]$
3. $\lim_{n \rightarrow \infty} \frac{1}{n} \left[e^{\left(\frac{3}{n}\right)} + e^{\left(\frac{6}{n}\right)} + \dots + e^{\left(\frac{3n}{n}\right)} \right]$
4. $\lim_{n \rightarrow \infty} \frac{\sqrt{1} + \sqrt{2} + \dots + \sqrt{n}}{n^{\frac{3}{2}}}$
5. $\lim_{n \rightarrow \infty} \ln \left[\left(e + \frac{e^2 - e}{n} \right) \left(e + \frac{2(e^2 - e)}{n} \right) \left(e + \frac{3(e^2 - e)}{n} \right) \dots (e^2) \right] \left(\frac{e^2 - e}{n} \right)$
6. Given that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$, simplify the sum $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$
7. $\lim_{n \rightarrow 0} \frac{1}{n} \left(n - \frac{n^3}{3!} + \frac{n^5}{5!} - \dots \right)$
8. $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81} + \dots$
9. $\int_0^2 \left(\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{n(n+1)} + \dots \right) dx$
10. $\lim_{n \rightarrow 0} \left(1 + n + \frac{n^2}{2} + \frac{n^3}{3!} + \frac{n^4}{4!} + \dots \right)$
11. $\lim_{n \rightarrow \infty} \left(\frac{1}{n^2} + \frac{4}{n^2} + \frac{9}{n^2} + \dots + 1 \right) \frac{1}{n}$
12. $\lim_{n \rightarrow \infty} \left(\frac{(n+1)^2}{n^3} + \frac{(n+2)^2}{n^3} + \frac{(n+3)^2}{n^3} + \dots + \frac{1}{n} \right)$
 Hint: Rewrite as: $\lim_{n \rightarrow \infty} \frac{1}{n} \left(\frac{(n+1)^2}{n^2} + \frac{(n+2)^2}{n^2} + \frac{(n+3)^2}{n^2} + \dots + 1 \right)$