Precalculus – January 2003 Regional Competition

For all questions, answer E. NOTA means none of the above answers are correct.
For all questions, let $i=\sqrt{-1}$.

1. Let $f(x-1)=3x+8$. If $f(A)=26$ then determine the value of A.

A. 5 B. 6
C. 7 D. 83 E. NOTA

2. For P>0, sin(2x)=P. Which of the following equals the expression $(\sin x+\cos x)^2$?

A. P B. 1-P
C. P-1 D. 1+P E. NOTA

3. If $\sqrt{56-\sqrt{56-\sqrt{56-\ldots}}}=x$ then x equals which of the following?

A. 7 B. 8
C. 56 D. 7 or 8 E. NOTA

4. Which angle is coterminal with -269°?

A. -91° B. 0°
C. 31° D. 451° E. NOTA

5. For $f(x)=\dfrac{(x-3)(x+4)(x-5)}{(x+3)(x-4)(x+5)}$, which is true of f at x=5?

A. f is continuous.
B. $f<0$.
C. f returns multiple values.
D. f's graph has a hole.
E. NOTA

6. Which of the following statements is true?

A. $Arc\sin\left(\frac{1}{2}\right)=\frac{\pi}{3}$ B. $Arc\tan\left(-\sqrt{3}\right)=-\frac{\pi}{3}$

C. $Arc\sec\left(\frac{\sqrt{2}}{2}\right)=\frac{\pi}{4}$ D. $Arc\cot(1)=\pi$

E. NOTA

7. Over a given period, a stock is a 'hit' if its cumulative percent price change is more than that of a benchmark figure, a 'miss' if it is less and a 'push' if it is equal. From the beginning of period Q to Monday's stock market close, stock XYZ's price change is 2.3% and the benchmark price change is 1.8%. The price changes for Tuesday, Wednesday, Thursday, and Friday, respectively, for XYZ are –0.8%, 1.2%, -3.0% and 2.8%. The respective price changes for the benchmark are 0.3%, 0.6%, -4.5% and 1.7%. If period Q ends Friday night, is XYZ a miss, hit, or push in period Q?

A. Miss B. Hit
C. Push D. Cannot be determined
E. NOTA

8. For $4\sin^2\theta-4\sin\theta=3$, determine the greatest solution for θ given $0<\theta<2\pi$.

A. $\frac{\pi}{3}$ B. $\frac{5\pi}{6}$
C. $\frac{5\pi}{3}$ D. $\frac{11\pi}{6}$ E. NOTA

9. Determine the sum: $(i+3)+1+\frac{1}{i+3}+\ldots$.

A. $\frac{-2}{5}+\frac{-1}{5}i$ B. $\frac{22}{5}+\frac{4}{5}i$

C. $\frac{7}{5}+\frac{-1}{5}i$ D. $\frac{-7}{5}+\frac{1}{5}i$ E. NOTA

10. Simplify the following expression for $0<\theta<90^\circ$:

$$\frac{\sin^2\theta+\tan^2\theta+\cos^2\theta}{\left(\frac{\sec\theta}{\cot^2\theta}\right)\left(\frac{\sin\theta}{\tan^2\theta}\right)}.$$

A. $\csc\theta$ B. $\sec\theta$
C. $\cot\theta$ D. $\csc\theta\sec\theta$
E. NOTA

11. Let f(x)=3cos(x)-2. If the range of g(x) is [-7,-1], then which of the following could be true for g(x)? (Assume the domains of f and g are all real numbers.)

A. $g(x)=|f(x)|$ B. $g(x)=2f(x)$
C. $g(x)=f(x)-2$ D. $g(x)=f(x+2)$
E. NOTA

12. B(x) is the graph of all co-planar points for whom the sum of the distances to the points (1,3) and (1,5) is 6. The graph of B(x) is a(n)…

A. Line B. Ellipse
C. Parabola D. Hyperbola
E. NOTA

13. If $|10x-4|=3+|4x+5|$ then determine the sum of all possible values of x.

A. $\frac{12}{7}$ B. $\frac{19}{7}$

C. $\frac{20}{7}$ D. 3 E. NOTA

14. If Q is randomly selected from the first 11 whole numbers, what is the probability that i^Q equals 1?

A. $\frac{1}{11}$ B. $\frac{2}{11}$

C. $\frac{3}{11}$ D. $\frac{4}{11}$ E. NOTA

15. Given $a_n=a_{n-1}+\log n$ for n>1. If $a_2=\log 2$ then $10^{a_{10}}$ equals which of the following?

A. 10 B. 10^{10}
C. 10! D. $10^{10!}$ E. NOTA

16. The ratio of green eggs to green eggs plus hams in a restaurant with 195 green eggs is 0.65. How many green eggs must the restaurant remove so that the ratio of green eggs to green eggs plus hams will be 0.50?

A. 45 B. 90
C. 105 D. 300 E. NOTA

17. If $\cos(75^\circ)=R(\cos 30^\circ-\sin 30^\circ)$ where R is a constant, then R could be which of the following?

A. $\sin 45^\circ$ B. $1-\cos 45^\circ$
C. $\cos 105^\circ$ D. $1+\sin 135^\circ$
E. NOTA

18. In square units, what's the area of a triangle with sides of length 5, 9, and 10 units?

A. $\sqrt{42}$ B. $6\sqrt{14}$
C. $60\sqrt{3}$ D. $12\sqrt{665}$ E. NOTA

19. Ten people play 5-on-5 basketball. If two of the players insist upon playing on the same team, in how many ways can the group be divided into two teams of five?

A. 120 B. 252
C. 336 D. 1680 E. NOTA

20. A number X is added to the sine of thirty degrees and the sum is multiplied by the natural log of e^6. This product is subtracted from two raised to the second power and the difference is divided by the cotangent of $\frac{3\pi}{4}$, yielding a result of 17. Determine the Xth root of 64.

A. 1 B. 2
C. 4 D. 8 E. NOTA

21. Determine the dot product of $5j-2k$ and $-3j-4k$.

A. –7 B. -23
C. $26k$ D. $-26k$ E. NOTA

22. Let $f(x)=x^5+4x^3-6x^2+9$. Determine the remainder when f(x) is divided by the factor (x-1).

A. –2 B. 1
C. 8 D. 9 E. NOTA

23. In triangle ABC, AB=10, BC=12, and $m<C=50°$. Give the sum of all possible values of $\overline{AC}$ to the nearest tenth.

A. 11.7 B. 12.2
C. 15.4 D. 30.6 E. NOTA

24. Determine the value of -6k if

$$\begin{bmatrix} 1 & -2 & 5 \\ 3 & k & 8 \\ -4 & 3 & -6 \end{bmatrix} = 8k+3.$$

A. 21 B. 23
C. 46 D. 106 E. NOTA

25. What is the distance between the points $(8,60°)$ and $(4\sqrt{2},45°)$, written in polar coordinates?

A. $\sqrt{3}$ B. $2\sqrt{3}$
C. $4\sqrt{3}+4$ D. $4\sqrt{3}-4$ E. NOTA

26. The number 224_5, written in base 5, is equal to KAT_n, written in base n. If the product $K \bullet A \bullet T = 0$, then n could be which of the following? (K, A, and T are whole numbers <10 and not necessarily distinct.)

A. 3 B. 6
C. 7 D. 8 E. NOTA

27. Starting from point A, Mark walks 10ft in a straight line, turns P degrees, and walks 12ft in a straight line. He then turns Q degrees such that he is facing point A and notes that point A is 13ft in front of him. To the nearest whole number, what is P+Q? (Note: P and Q are positive numbers less than 180.)

A. 61 B. 108
C. 119 D. 133 E. NOTA

28. If a=sin(x) and b=cos(x) then which of the following equals the expression $a^3 - b^3$?

A. (a-b) B. (a+b)
C. (a-b)(1+ab) D. (a-b)(1-ab)
E. NOTA

29. The radius of a cylinder decreases 10% for each unit of time after time=0 but its height remains constant. To the nearest percent, by how much does the cylinder's volume decrease from time=0 to time=2 (assume the cylinder has a positive volume at time=0)?

A. 19 B. 34
C. 66 D. 81 E. NOTA

30. A circle with center at the origin has radius 10. Rays B and C, are drawn from the origin with slopes of $\sqrt{3}$ and -1, respectively. An ant (of negligible length) crawls counterclock-wise along the circle from its intersection with B. If the ant crawls at a rate of $\frac{\pi}{2}$ units per minute, in how many minutes will it reach the intersection of the circle and C?

A. $\frac{25}{3}$ B. 10
C. $\frac{125}{3}$ D. 50 E. NOTA