Question #1
National Alpha Bowl 2000

Given that P and Q are the complex roots of the equation

$$5x^2 + 4x + 25 = 0,$$

find the numerical value of $(P + 5)(Q + 5)$.

Question #2
National Alpha Bowl 2000

Let P = the sum of the x and y coordinates of the point of intersection of the lines with equations $2x - 3y = 6$ and $3x - 2y = 10$

Let Q = the y-intercept of the line that is the perpendicular bisector of the segment joining the points (1, 5) and (−3, 2)

Let R = the sum of m and b, where m and b are the slope and y-intercept of the line whose parametric equations are $x = 2t - 5$ and $y = 3t + 1$

Let S = the sum of the coordinates of point C, where C lies on the line containing (1, 8) and (4, 13) and the ordinate of point C is twice its abscissa

Find: $S(Q + \frac{R}{P})$

Question #3
National Alpha Bowl 2000

Let A = the sum of all values of y such that (0, y) will lie on the circle with diameter 8 and center (−3, 1)

Let B = the length of the latus rectum of the parabola with equation $y^2 + 12x + 6y - 15 = 0$

Let C = $h + k + e\sqrt{2}$, where (h, k) is the center and e is the eccentricity of the ellipse with equation $x^2 + 2y^2 + 6x - 4y - 5 = 0$

Let D = the distance between the foci of the hyperbola with equation $(x + 2)^2 - 2(y - 3)^2 = 10$

Find: ACD^2

Question #4
National Alpha Bowl 2000

Let A = $\cos 2x$, given that $\sin x + \cos x = -\frac{1}{5}$ and $\frac{3\pi}{4} < x < \pi$

Let B = the radian measure of $\text{Arctan}\left(\frac{1}{2}\right) + \text{Arctan}\left(\frac{1}{3}\right)$

Let C = the smallest solution of the equation $1 + \sin x + \sin^2 x = \cos^2 x$, given that $0 < x < 2\pi$

Let D = $\cos(\text{Arccsc } 3)$

Find: $\frac{ABD^2}{C}$

Question #5
National Alpha Bowl 2000

Let A = $\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - 4}$

Let B = $\lim_{x \to \infty} \frac{3x + 4}{2x - 7}$

Let C = $\lim_{x \to \infty} \frac{x^2 \cos\left(\frac{1}{x}\right)}{2x^2 - 1}$

Let D = $\lim_{x \to 0} \frac{x}{1 - \sqrt{1 - x}}$

Find: $\frac{B}{A} + \frac{D}{C}$

Question #6
National Alpha Bowl 2000

Given: $f(x) = \frac{x^3 - 7x + 6}{x^2 - 4x - 21}$

If x = A is a vertical asymptote of f(x), f(9) = B, the sum of the real zeros of f(x) is C, and y = Dx + 4 is a slant asymptote of f(x), find the numerical value of BD – AC.

Question #7
National Alpha Bowl 2000

Let A = $\log_{0.75} (\sin \frac{\pi}{3})$

Let B = $\log_3 m + \log_3 n$, given that m and n are the roots of $9x^2 - 8x + 1 = 0$

Let C = $\lim_{x \to \infty} [\log_3 (18x - 5) - \log_3 (2x + 1)]$

Let D = $\log_4 8 + (\log_4 6)(\log_6 2)$

Find: ABCD

Question #8
National Alpha Bowl 2000

The line $y = Ax + B$ is tangent to the graph of $f(x) = 2x^2 - 5x + 9$ at the point (1, 6).

Find: AB

Question #9
National Alpha Bowl 2000

Given: $f(x) = 3 \sin (2x - \frac{\pi}{2})$

Let A = the amplitude of f(x)

Let B = the period of f(x)

Let C = the phase shift (horizontal displacement) of the graph of f(x) if f(x) is considered to be a translation of the graph of $y = 3 \sin 2x$

Let D = $f(\frac{2\pi}{3})$

Find: $\frac{AB}{CD}$

Question #10
National Alpha Bowl 2000

Let $P = xy^2$, where x and y are the rectangular coordinates of the point with polar coordinates $(4, \frac{2\pi}{3})$

Let $Q = ab$, if the complex number 2 cis $(\frac{3\pi}{4})$ is written in rectangular form a + bi

Find: PQ

Question #11
National Alpha Bowl 2000

Let A = the value of k if the vectors $\langle 2, k, -3 \rangle$ and $\langle -4, 1, 2 \rangle$ are perpendicular

Let B = $\cos\theta$, where θ is the angle between the vectors $\langle 2, 2, 1 \rangle$ and $\langle 3, 6, -2 \rangle$

Let C = $|u \times v|$, where u and v are vectors, $u = \langle -2, -1, 2 \rangle$ and $v = \langle 1, 0, 1 \rangle$

Find: $7ABC^2$

Question #12
National Alpha Bowl 2000

The sum to infinity of a certain geometric series is 8,

whereas the sum of the second and third terms is 3.

Find all possible values of the common ratio of this series.

Question #13
National Alpha Bowl 2000

Let A = the absolute value of the tangent of the angle between the lines with equations $2x - y + 7 = 0$ and $3x - 4y - 1 = 0$

Let B = the tangent of the inclination of the line with equation $3x - 5y = 17$

Find: AB

Question #14
National Alpha Bowl 2000

Evaluate: $\sum_{n=0}^{\infty} \frac{1}{n^2 + 5n + 6}$

Question #15
National Alpha Bowl 2000

Given: $f(x) = x^4 - Ax^3 + Bx^2 + 9x - 16$ (A and B are positive integers)

Find: the product of all possible rational roots of f(x)