Advanced Calculus – Mu Level
2000 Mu Alpha Theta National Convention

"NOTA" in each question denotes "None of these answers."

1. The shape of the surface defined by $2x^2 + 2y^2 + 3z^2 = 6$ is a(n):

   A) oblate spheroid
   B) prolate spheroid
   C) elliptic hyperboloid of one sheet
   D) elliptic hyperboloid of two sheets
   E) NOTA

2. Rewrite the equation
   $x^2 + y^2 + z^2 - 8x = 0$
   in spherical coordinate form.

   A) $\rho = \sqrt{1 - 8\cos\phi\cos\theta}$ B) $\rho = \sqrt{1 - 8\sin\phi\cos\theta}$

   C) $\rho = 8\sin\phi\cos\theta$ D) $\rho = 8\cos\phi\cos\theta$ E) NOTA

3. Which of the following is/are true of a conservative vector field $\mathbf{F}$ continuous on an open disk B? Let C be a sectionally smooth curve lying in B.

   I. There exists some scalar function ϕ for which $\mathbf{F} = \nabla\phi$
   II. The line integral $\int_C \mathbf{F} \bullet dR$ is dependent on the path C
   III. If C is a closed path on B, $\int_C \mathbf{F} \bullet dR = 0$

   A) I only
   B) II only
   C) I and II only
   D) I, II, and III
   E) NOTA

4. If $f(x,y) = x^2 e^{2xy} + \frac{x}{y}$
   find $\frac{\partial^2 f(x,y)}{\partial x \partial y}$

   A) 8 B) $4x^4 e^{2xy} + \frac{2x}{y^3}$ C) $2x^2 e^{2xy}(3 + 2xy) - \frac{1}{y^2}$ D) $2e^{2xy}(1 + 4xy + 2x^2y^2)$ E) NOTA

5. The dimensions of a rectangular box are measured to be 8 cm., 12 cm., and 15 cm., and ea measurement is correct to within .02 cm. of the actual length. Using differentials, approxima the greatest possible error in the volume of a box calculated from these measurements. Ro your answer to the nearest hundredth.

A) 7.91 B) 7.92 C) 7.93 D) 7.94 E) NOTA

6. If $u = x^2 - y^2$
$x = 3r - s$
$y = r + 2s$

Find $\frac{\partial u}{\partial r}$

A) 0 B) $4r - 6s$ C) $32r - 48s$ D) $16r - 10s$ E) NOTA

7. The temperature at any point (x,y) of a rectangular plate lying in the xy plane is determined b the equation:
$T(x, y) = x^2 + xy$
Find the direction, measured from the positive x, (in degrees rounded to the nearest hundredth) for which the rate of change of the temperature at the point (-3,1) is a maximum.
A) $-18.43°$ B) $30.96°$ C) $161.57°$ D) $210.96°$ E) NOTA

8. Find the equation for the plane tangent to the surface
$x^2 + y^2 - 3z = 2$
at the point (-2, -4, 6)

A) $-7x + 3y - 5z + 28 = 0$
B) $4x + 8y + 3z + 22 = 0$
C) $9x - 8y + z - 20 = 0$
D) $2x + 7y - z + 38 = 0$
E) NOTA

9. The point (3, -1, 11) on the surface
$z = 6x - 4y - x^2 - 2y^2$

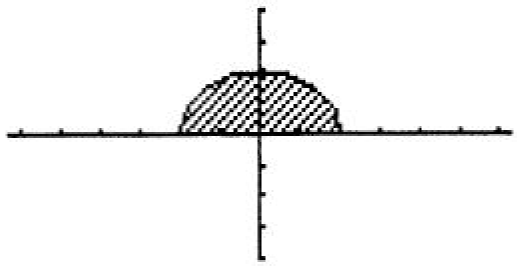
A. is a relative, but not absolute, maximum
B. is a relative, but not absolute, minimum
C. is an absolute maximum
D. is an absolute minimum
E. NOTA

10. Given $\nabla f(x, y) = (6x - 5y)\mathbf{i} - (5x - 6y^2)\mathbf{j}$
find $f(x, y)$

A) $6\mathbf{i} + 12y\mathbf{j}$ B) $\left(3x^2 - \frac{5y^2}{2} + C\right)\mathbf{i} - \left(\frac{5x^2}{2} - 2y^3 + C\right)\mathbf{j}$

C) $x - 5y + 6y^2 + C$ D) $3x^2 - 5xy + 2y^3 + C$ E) NOTA

11. A flat, semicircular sheet of metal with a radius of 2 units is placed on the coordinate (xy) axe: as shown:



If the mass per unit area of the metal is directly proportional to distance from the origin, then which of the following will provide the total mass of the sheet? Let k be a constant.

A) $k\int_0^{\sqrt{4-x^2}} \int_{-2}^{2} \sqrt{x^2 + y^2}\, dxdy$ B) $k\int_0^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \sqrt{x^2 + y^2}\, dxdy$

C) $k\int_0^{\pi} \int_0^{2} r\, dr d\theta$ D) $k\int_{-2}^{2} \sqrt{x^2 + y^2}\, dx$ E) NOTA

12. Let R be the closed three-dimensional region in xyz space defined by a sphere of radius 3 centered on the origin. Consider this sphere to be a solid object whose density at any point varies directly with the square of the cartesian x coordinate of that point. If k is a constant, wh of the following is a spherical coordinate expression of the mass of the sphere?

A) $k\iiint_R \rho \sin\phi \cos^3\theta\, d\rho\, d\theta\, d\phi$ B) $k\iiint_R \rho^4 \sin^3\phi \cos^2\theta\, d\rho\, d\theta\, d\phi$

C) $k\iiint_R \rho^2 \sin^2\phi \cos^2\theta\, d\rho\, d\theta\, d\phi$ D) $k\iiint \rho^2 \sin^5\phi\, d\rho\, d\theta\, d\phi$ E) NOTA

13. If **F** is a vector field with all vectors lying in the xy plane, then which of the following is a possibl evaluation of $(\nabla \bullet \mathbf{F})$?

A) $(5x^2+2xy)\mathbf{i}+(x^2+e^y)\mathbf{j}$ B) $(\ln(xy)+3x)\mathbf{i}+(\ln(xy))\mathbf{j}$

C) $2e^{2x}+3x^2z+2y^2$ D) $4ye^{xy}-2x^2y$ E) NOTA

14. If $\mathbf{F}(x,y,z)=z^2\mathbf{i}+e^{-\cos(xy)}\mathbf{j}+\cos x^2\mathbf{k}$ and $\text{curl}\,\mathbf{F}=\mu\,\mathbf{i}+\alpha\,\mathbf{j}+\theta\,\mathbf{k}$ find α

A) 0 B) $y\sin(xy)e^{-\cos(xy)}$ C) $2(z+x\sin x^2)$ D) $x\sin(xy)e^{-\cos(xy)}$ E) NOTA

15. If R is the region enclosed by a simple closed positively oriented curve C, then the area of R is given by:

A) $\int_C (ydx+2xdy)$ B) $\int_C (xdy+ydx)$ C) $\frac{1}{2}\int_C (ydx+xdy)$ D) $\frac{1}{2}\int_C (ydx-xdy)$ E) NOTA

16. Let R be a closed cubic region in xyz space having side length $\sqrt{.5}$, one vertex on the origin, an its interior lying entirely in the first octant. Let f(x,y,z) be defined as:

$f(x,y,z)=x^2+y^2+\ln(2.2z^2+.1)$

and let P be the point in R at which f(x,y,z) is maximized. To the nearest thousandth, find the magnitude of the vector which begins at the origin and terminates at P.

A) 1.000 B) 1.087 C) 1.182 D) 1.225 E) NOTA

17. The two surfaces $z=\sqrt{x^2+y^2}$ and $x^2+4y^2+4z^2=173$ intersect in a space curve containing the point $P_0(3,4,5)$. Which of the following vectors is tangent to this curve at this point?

A) $<-190,-490,776>$ B) $<590,970,-926>$

C) $<630,-100,-130>$ D) $<640,-300,144>$ E) NOTA

18. At the point (5,0,10) what is the directional derivative of the function

$f(x,y,z)=x^2+\dfrac{y}{x}+z^3$ along the vector <2,5,-4>?

A) $\dfrac{-3982}{\sqrt{63}}$ B) $\dfrac{-5893}{\sqrt{5}}$ C) $\dfrac{-1179}{\sqrt{45}}$ D) $\dfrac{-427}{10\sqrt{901}}$ E) NOTA

19. A cylindrical rod of length 10 units and diameter 2 units has a density which varies with the square of the perpendicular distance from the base of the rod. What is the rod's moment of inertia about its central axis? Let k be a constant.

A) $\dfrac{20k\pi}{3}$ B) $\dfrac{1000k\pi}{3}$ C) $\dfrac{100k\pi}{9}$ D) $\dfrac{2000k\pi}{9}$ E) NOTA

20. Which integral is $\int_0^{16}\int_0^{\sqrt{x}} f(x,y)\,dy\,dx$ with the order of integration reversed?

A) $\int_0^{4}\int_0^{y^2} f(x,y)\,dx\,dy$ B) $\int_0^{16}\int_0^{\sqrt{x}} f(x,y)\,dx\,dy$ C) $\int_0^{4}\int_{y^2}^{16} f(x,y)\,dx\,dy$ D) $\int_0^{\sqrt{x}}\int_0^{16} f(x,y)\,dx\,dy$ E) NOTA

21. $\mathbf{F}=\dfrac{2y}{e}\mathbf{i}+\left(\dfrac{2x}{e}+6y\right)\mathbf{j}$

and C is defined by the equation: $\mathbf{R}(t)=3e^t\mathbf{i}+\dfrac{4}{\pi}\tan^{-1}t\,\mathbf{j}\quad 0\le t\le 1$

evaluate $\int_C \mathbf{F}\bullet d\mathbf{R}$

A) 6 B) 7 C) 8 D) 9 E) NOTA

22. Find the critical point of $f(x,y,z)=x^2+y^2+z^2$ subject to the constraint $3x-2y+z-4=0$

A) $\left(\dfrac{6}{7},\dfrac{-4}{7},\dfrac{2}{7}\right)$ B) $\left(1,\dfrac{-2}{3},\dfrac{-1}{3}\right)$ C) $\left(\dfrac{3}{7},\dfrac{-4}{3},\dfrac{10}{21}\right)$ D) $\left(\dfrac{1}{4},\dfrac{1}{9},\dfrac{125}{36}\right)$ E) NOTA

23. Which of the following is not a conservative vector field over the disk centered at the origin w radius 5?

A) $\mathbf{F}(x,y)=\left(\dfrac{ye^{\tan^{-1}x}}{x^2+1}\right)\mathbf{i}+\left(e^{\tan^{-1}x}\right)\mathbf{j}$ B) $\mathbf{F}(x,y)=\left(\dfrac{-y}{x^2+y^2}\right)\mathbf{i}+\left(\dfrac{x}{x^2+y^2}\right)\mathbf{j}$

C) $\mathbf{F}(x,y)=\left(ye^x-\dfrac{y}{e^x}\right)\mathbf{i}+\left(e^x+\dfrac{1}{e^x}\right)\mathbf{j}$ D) $\mathbf{F}(x,y)=\left(\dfrac{-y^2\tan\frac{x}{6}}{6}\right)\mathbf{i}+\left(2y\ln\left(\cos\frac{x}{6}\right)\right)\mathbf{j}$ E) NOT.

24. Find $\dfrac{dF(x)}{dx}$ where $F(x)=\int_0^{3x}e^{-x^2t^2}\,dt$

A) $3e^{-9x^4}-2x\int_0^{3x}t^2e^{-x^2t^2}\,dt$ B) $-t^2\int_0^{3x}e^{-x^2t^2}\,dt$ C) $3e^{-9x^4}$ D) $e^{-9x^2t^2}$ E) NOTA

25. Find the surface area of the surface $z=xy$ inside the cylinder $x^2+y^2=1$

A) $\dfrac{2\pi}{3}(2\sqrt{2}-1)$ B) $\dfrac{\pi\sqrt{3}}{2}(\sqrt{2}-1)$ C) $(2\sqrt{2})\pi$ D) 2π E) NOTA

26. Find the minimum of $f(x,y)=x^2+y^2-xy+x-5y$

A) -14 B) -7 C) -3 D) 1 E) NOTA

27. An initially empty cone is formed by rotating the line y=x from x=0 to x=5 about the y-axis. A t=0, water begins filling this cone from above at the rate of 16 cubic units/second. In additior drain at the vertex of the cone begins opening at t=0 and continues to open in such a way th water drains out of the cone at the rate of t^2 cubic units/sec for any given t. To the nearest hundredth of a second, what is the first time that the water level in the cone will reach a heig 3.44 units?

A) .22 B) 3.90 C) 6.82 D) The water level never reaches 3.44 E) NOTA

28. To the nearest thousandth, find the area inside the circle $r = 2\cos\theta$ and outside the circle $r = 1$

A) .544 B) 1.307 C) 1.913 D) 1.977 E) NOTA

29. Which of the following is true for the function

$$f(x,y)=\begin{cases}\dfrac{x^2+xy}{x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 1 & \text{if } (x,y)=(0,0)\end{cases}\ ?$$

A. $\lim_{(x,y)\to(0,0)} f(x,y)$ exists, and $f(x,y)$ is continous at (0,0)

B. $\lim_{(x,y)\to(0,0)} f(x,y)$ exists, and $f(x,y)$ is discontinuous at (0,0)

C. $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist, and $f(x,y)$ is continuous at (0,0)

D. $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist, and $f(x,y)$ is discontinuous at (0,0)

E. NOTA

30. The region in the first quadrant enclosed by the graphs of $y = x^2$ and $y = 3x$ forms the base of solid. Cross-sections of this solid taken perpendicular to the y axis have the shape of isosceles right triangles with one leg in the xy plane. What is the volume of this solid?

A) $\frac{81}{20}$ B) $\frac{81}{40}$ C) $\frac{27}{20}$ D) $\frac{27}{40}$ E) NOTA