

# ALPHA INDIVIDUAL TEST

SAN DIEGO 2000

(NOTA means none of the above)

1. What is the third term in the expansion of  $(x+y)^9$ ?

a)  $84x^7y^2$

b)  $84x^6y^3$

c)  $36x^7y^2$

d)  $36x^6y^3$

e) NOTA

2. Evaluate  $\sum_{n=1}^{100} (2n+1)$ .

a) 10,000

b) 12,500

c) 10,500

d) 10,200

e) NOTA

3. Find the center of the ellipse:  $4x^2 + 9y^2 - 32x + 18y + 37 = 0$ .

a) (16, -9)

b) (-8, 2)

c) (4, -1)

d) (2, 3)

e) NOTA

4. Evaluate:  $\cos(\sin^{-1} 2x)$

a)  $\sqrt{4x^2 - 1}$

b)  $\sqrt{4x^2 + 1}$

c)  $\sqrt{1 - 4x^2}$

d)  $1 - 2x$

e) NOTA

5. Find the sum of the first thirty terms of the following sequence: -2, 3, 8, ...

a) 5211

b) 2511

c) 2119

d) 2115

e) NOTA

# ALPHA INDIVIDUAL TEST (PAGE 2)

SAN DIEGO 2000

6. In an arithmetic sequence,  $t_4 = -2$  and  $t_{28} = 70$ . Find the value of  $t_{77}$ .

- a) 217                      b) 218                      c) 224                      d) 235                      e) NOTA

7. Find the largest angle in a triangle with sides 16, 26 and 36. Round to the nearest degree.

- a)  $111^\circ$                       b)  $116^\circ$                       c)  $74^\circ$                       d)  $79^\circ$                       e) NOTA

8. An airplane flies 400 miles with a heading of 70 degrees and then due east for 300 miles. What is the distance of the plane (to the nearest mile) from its starting point?

- a) 576 mi.                      b) 690 mi.                      c) 500 mi.                      d) 161 mi.                      e) NOTA

9. Solve the system for  $x$  and  $y$  and then evaluate  $x^2y$ :  $27^{2x-y} = \frac{1}{81}$  and  $\log_x \frac{729}{64} = 6$ .

- a)  $\frac{12}{5}$                       b)  $\frac{13}{2}$                       c)  $\frac{39}{4}$                       d)  $\frac{39}{8}$                       e) NOTA

10. Given a geometric sequence with the first term and common ratio both not 0, and an arithmetic sequence with first term 0, a third sequence 1, 1, 2, ... is formed by adding corresponding terms of the 2 given sequences. Find the sum of the first 10 terms of the third sequence.

- a) 1024                      b) 1023                      c) 978                      d) 45                      e) NOTA

ALPHA INDIVIDUAL TEST (PAGE 3)  
SAN DIEGO 2000

11. Given  $g(x) = 2x + 8$  and  $f(x) = \frac{1}{x+2}$ , find  $g \circ f^{-1}(-2)$ .

a)  $\frac{-5}{2}$

b) 3

c) 6

d) impossible

e) NOTA

12. If, for all  $x$ ,  $f(x) = f(2a)^x$  and  $f(x+2) = 27f(x)$ , then find  $a$ .

a)  $\frac{1}{2}$

b) 2

c)  $\frac{1}{4}$

d) 4

e) NOTA

13. Matrix A has 2 rows, 3 columns. Matrix B has 4 rows, 2 columns. The existing product of these 2 matrices consists of how many elements?

a) 4

b) 6

c) 8

d) 12

e) NOTA

14. Find the sum  $\sqrt[3]{5+2\sqrt{13}} + \sqrt[3]{5-2\sqrt{13}}$ .

a) -1

b) 0

c) 1

d) 2

e) NOTA

15. Find the coefficient of  $abc^2$  in the expansion of  $(a+b+c+d)^4$ .

a) 8

b) 12

c) 16

d) 24

e) NOTA

ALPHA INDIVIDUAL TEST (PAGE 4)  
SAN DIEGO 2000

16. Find the sum:  $\binom{100}{1} + 2\binom{100}{2} + 3\binom{100}{3} + \dots + 100\binom{100}{100}$ .

a)  $100 \cdot 2^{100}$

b)  $100 \cdot 2^{99}$

c)  $99 \cdot 2^{100}$

d)  $99 \cdot 2^{99}$

e) NOTA

17. Simplify the product:  $\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) \dots \left(1 - \frac{1}{n}\right)$ .

a)  $1/n$

b)  $2/n$

c)  $3/n$

d)  $4/n$

e) NOTA

18. In how many ways can 3 Americans, 4 Germans, 2 Frenchmen, and 3 Russians sit around a circular table if those of the same country sit together?

a) 1728

b) 2592

c) 5184

d) 10368

e) NOTA

19. If each of 2 intersecting lines intersects a hyperbola and neither line is tangent to the hyperbola, then what are the possible number of places where the lines can intersect the hyperbola?

a) 0, 1 or 2

b) 2 or 3

c) 3 or 4

d) 4 only

e) NOTA

20. A parabola  $y = ax^2 + bx + c$  has vertex (4, 2). If (2, 0) is on the parabola, then find  $abc$ .

a) -2

b) 3

c) 12

d) -24

e) NOTA

ALPHA INDIVIDUAL TEST (PAGE 5)  
SAN DIEGO 2000

21. The equation  $x^4 - 16x^3 + 94x^2 + px + q = 0$  has 2 double roots. Find  $p + q$ .

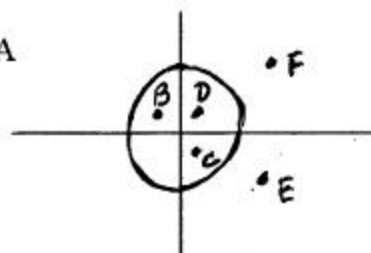
- a) -15                      b) 15                      c) 30                      d) -30                      e) NOTA

22. If  $f(2x) = \frac{2}{2+x}$ , for all  $x > 0$ , then find  $2f(x)$ .

- a)  $\frac{4}{4+x}$                       b)  $\frac{4}{2+x}$                       c)  $\frac{8}{4+x}$                       d)  $\frac{8}{2+x}$                       e) NOTA

23. The diagram below shows several numbers in the complex plane. The circle is the unit circle centered at the origin. Which of the shown points could be the reciprocal of F?

- a) B                      b) C                      c) D                      d) E                      e) NOTA



24. Find the exact value of the cosine of the angle between the vectors  $(3 \ 4 \ 5)$  and  $(-1 \ 4 \ 3)$ .

- a)  $\frac{\sqrt{13}}{7}$                       b)  $\frac{5\sqrt{13}}{7}$                       c)  $\frac{\sqrt{13}}{14}$                       d)  $\frac{5\sqrt{13}}{14}$                       e) NOTA

25. How many factors of  $2^{95}$  are there which are greater than 1 million?

- a) 120                      b) 96                      c) 76                      d) 52                      e) NOTA

