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"NOTA" in each question denotes "None of the Above"

1)  $\int dx =$

- A)  $1 + C$                       B)  $C$                       C)  $\frac{dx^2}{2} + C$                       D)  $x + C$                       E) NOTA

2) A rectangle of perimeter 20 inches is rotated about one of its sides to generate a right circular cylinder. The rectangle which generates the cylinder with the largest volume has an area, in square inches, of: (round to nearest unit)

- A) 21                      B) 22                      C) 24                      D) 25                      E) NOTA

3) What is  $\frac{dy}{dx}$  if  $y = x^{x+1}$ ?

- A)  $(x \ln(x) + x + 1)(x^x)$     B)  $(x + 1)(x^x)$     C)  $x^{x+1} \ln x$                       D)  $\ln x$                       E) NOTA

4) Find the coefficient of the term containing  $x^{14}$  from the MacLaurin expansion of  $\sin(x^2)$ .

- A)  $-\frac{1}{7!}$                       B)  $\frac{1}{7!}$                       C) 0                      D)  $\frac{1}{14!}$                       E) NOTA

5) Find the area contained in the polar graph of  $r=3 \sin \Theta \cos \Theta$ .

- A)  $\frac{\pi}{4}$                       B)  $\frac{\pi}{16}$                       C)  $\frac{9\pi}{4}$                       D)  $\frac{9\pi}{8}$                       E) NOTA

6) The exact average value of  $f(x) = \frac{1}{\sqrt{x^2 + 1}}$  on  $[0,3]$  is

- A)  $\frac{10 + \sqrt{10}}{20}$                       B)  $\frac{1}{3} \ln(3 + \sqrt{10})$                       C)  $\frac{3}{2}$                       D)  $\frac{\pi - 2 \tan^{-1} \frac{1}{3}}{6}$                       E) NOTA

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7) Approximate the area under  $y = \frac{1}{x}$  on  $[7,9]$  using 4 subdivisions and Simpson's Method, to the ten-thousandths place.

- A) .2410      B) .2513      C) .2516      D) .2622      E) NOTA

8) A bowl is shaped like the graph of  $z = x^2 + y^2$ . Water is flowing into the bowl at the rate of  $1 \frac{\text{unit}^3}{\text{sec}}$ . How fast is the water level rising when it is 1 unit deep (unit/sec)?

- A)  $\frac{1}{\pi}$       B)  $\frac{2}{\pi}$       C)  $\pi$       D)  $\frac{1}{2\pi}$       E) NOTA

9) Approximate  $\sqrt[3]{29}$  using differentials to the nearest thousandths place:

- A) 3.071      B) 3.072      C) 3.073      D) 3.074      E) NOTA

10) At any point  $(x,y)$  on a certain curve, the slope is equal to  $xy^2$ . If the curve contains the point  $(0,4)$ , its equation is:

- A)  $y = \frac{4x^2}{x^2 - 2}$       B)  $y = \frac{1 - 4x^2}{2}$       C)  $y = \frac{4}{1 - 2x^2}$       D)  $y = \frac{4}{1 - 4x^2}$       E) NOTA

11) Find  $\frac{d^2y}{dx^2}$  if  $X(t) = \sqrt{t^2 + 3}$  and  $y(t) = \sin(t^2)$ .

- A)  $2x \cos(x^2 - 3)$       B)  $(2x - 3) \sin(x^2 - 3)$       C)  $2x \sin(x^2 - 3)$       D)  $2x \cos x^2 - 3$       E) NOTA

12) The volume of the solid generated by revolving about  $y = x - 7$  the region bounded by  $y = \frac{1}{\sqrt{x}}$  and the x-axis between  $x=1$  and  $x=4$ . The tenths digit of the exact volume is:

- A) 4      B) 5      C) 6      D) 7      E) NOTA

13) An ice cream cone contains 2 perfectly symmetrical, spherical scoops of chocolate ice cream, each with a radius of 4.3 cm. The ice cream melts out the bottom of the cone and drips to the ground at  $.05 \frac{\text{cm}^3}{\text{sec}}$  per scoop. Assuming that the scoops of ice cream stay perfectly symmetrical and spherical throughout the melting process and each melts at the same rate, exactly how fast is the total surface area of the scoops changing when the radius of each scoop is 3 cm?

- A)  $\frac{1}{60}$                       B)  $\frac{1}{30}$                       C)  $\frac{1}{15}$                       D)  $\frac{\pi^2}{3}$                       E) NOTA

14) Find the approximate value for C that satisfies the mean value theorem on  $[0,1]$  for  $f(x) = \frac{\tan^{-1} x}{x^2 + 1}$ . (Round to the nearest thousandth).

- A) .308                      B) .393                      C) .471                      D) .500                      E) NOTA

15) Approximate the value for  $\int_0^6 (x^3 - 2x^2 + 4) dx$  using trapezoidal rule and 3 subdivisions of equal width.

- A) 190                      B) 204                      C) 232                      D) 464                      E) NOTA

16) Which of the following is a line normal to the curve  $y = \frac{1}{x^2}$  at  $(1,1)$

- A)  $x - 2y = 1$                       B)  $y = \frac{x^4}{2}$                       C)  $y = \frac{x^4}{8}$                       D)  $2y - x = 1$                       E) NOTA

17)  $\lim_{h \rightarrow 0} \frac{\ln(\ln(6+h)) - \ln(\ln 6)}{h}$

- A) .2986                      B)  $\frac{1}{6}$                       C)  $\frac{1}{6 \ln 6}$                       D)  $\frac{1}{\ln 6}$                       E) NOTA

18) The maximum value of  $f(x) = xe^{-x^2}$  is

- A) 0                      B)  $\frac{1}{\sqrt{2}}$                       C)  $\frac{1}{\sqrt{2e}}$                       D)  $\infty$                       E) NOTA

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19) The base of a solid is the graph that is defined by the area bounded by  $y = \sin x$  and  $y = \cos x$  on  $[0, \frac{\pi}{4}]$ . Each cross section perpendicular to the x-axis is a semicircle with diameter lying in the xy plane. Approximate the volume of the solid to the ten-thousandths place.

- A) .1121                      B) .4481                      C) .4483                      D) .8966                      E) NOTA

20) If  $\int_0^{x^3} (t^4 \sin^2 t^2 + \frac{1}{2} t^3) dt = h(x)$ , find  $h'(x)$ .

- A)  $3x^{14} \sin^2 x^6 + \frac{3}{2} x^{11}$     B)  $9x^{16} \sin^2 x^6 + \frac{3}{2} x^{11}$     C)  $x^{12} \sin^2 x^6 + \frac{1}{2} x^9$     D)  $x^4 \sin^2 x^2 + \frac{1}{2} x^3$     E) NOT

21) What is  $\frac{dy}{dx}$  if  $x^2 + y^2 = \sin(xy)$ ?

- A)  $\frac{-\cos(xy) - 2x}{2y - x}$     B)  $\frac{y - 2x}{2y - x \cos(xy)}$     C)  $\frac{-y \cos(xy) + 2x}{x \cos(xy) + 2y}$     D)  $\frac{y \cos(xy) - 2x}{2y - x \cos(xy)}$     E) NOTA

22) Calculate exactly  $\int_{e^{e^e}}^{e^{e^{e^2}}} \frac{dx}{x(\ln x)(\ln(\ln x))(\ln(\ln(\ln x)))}$

- A) 0                      B)  $\ln 2$                       C)  $2 \ln 2$                       D)  $(\frac{1}{2})(e^{e^{e^2}} - e^{e^e})$     E) NOTA

23) A 20-foot ladder is placed against a flat, vertical wall. The weather changes from bright and sunny to dark and rainy. Since it is rainy, the ladder begins to slip down the wall at  $.1 \frac{\text{feet}}{\text{sec}}$ . Exactly how fast will the bottom of the ladder be moving away from the wall when the top of the ladder is 5 feet from the ground? ( $\frac{\text{feet}}{\text{sec}}$ )

- A)  $\frac{\sqrt{15}}{10}$                       B) .3873                      C)  $\frac{\sqrt{15}}{150}$                       D) Not enough info                      E) NOTA

24) Find the domain of  $y = \frac{1}{\sqrt{\ln(\sqrt{x^2 - 1})}}$ .

- A)  $[-1,1]$       B)  $(-\infty, \sqrt{2}] \cup (\sqrt{2}, \infty)$       C)  $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$       D)  $(-\sqrt{2}, \sqrt{2})$       E) NOTA

25) The half-life of Floridium is 1800 years. If you have 6 grams of it now, approximately how many years ago did you have 180 grams? (round to nearest hundredth)

- A) 30.00      B) 366.83      C) 8832.00      D) 8832.40      E) NOTA

26) The first three terms of the MacLaurin series of the function,  $f(x) = \int_0^x \frac{\sin t}{t} dt$  are:

- A)  $1 - \frac{1}{6}x^2 + \frac{1}{120}x^4$       B)  $x - \frac{1}{18}x^3 + \frac{1}{600}x^5$       C)  $x^2 - \frac{1}{6}x^4 + \frac{1}{120}x^6$       D)  $\frac{1}{3}x^3 - \frac{1}{30}x^5 + \frac{1}{720}x^7$       E) NOTA

27) The volume of a cube is decreasing at a rate of  $40 \frac{\text{cm}^3}{\text{sec}}$ . How fast, in  $\frac{\text{cm}^2}{\text{sec}}$ , is the surface area of the cube decreasing at the instant when each edge of the cube is 5 cm?

- A) -32      B)  $-\frac{8}{15}$       C)  $\frac{8}{15}$       D) 32      E) NOTA

28) The motion of a particle has the jerk motion equal to  $-8t$ , and is stationary and not accelerating at  $t=3$ . Which of the following gives the particle's velocity at any time (t)?

- A)  $-\frac{4t^3}{3} + 36t - 72$       B)  $-\frac{4t^3}{3} + 36t$       C)  $-4t^2$       D)  $-4t^2 + 36$       E) NOTA

29) A baseball player smashes a home run over the right field wall, 400 feet from home plate. The ball travels in a parabolic path of  $x(t) = 150t$  and  $y(t) = 3 + 53t - 16t^2$ , in feet. Exactly how far above the top of the 9 foot tall wall did the ball cross before it landed in the stands?

- A)  $\frac{8}{3}$       B)  $\frac{194}{9}$       C)  $\frac{275}{9}$       D)  $\frac{356}{9}$       E) NOTA

30) If  $\lim_{x \rightarrow 3} \frac{g(3) - g(x)}{x - 3} = -1.743$ , then at the point at  $x=3$ , the graph of  $g(x)$  must be

- A) decreasing    B) increasing    C) concave up    D) concave down    E) NOTA

