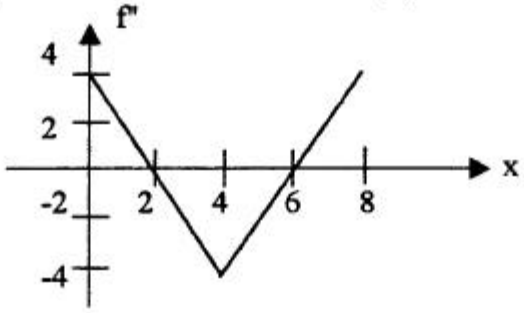
Limits & Derivatives – Mu Level
2000 Mu Alpha Theta National Convention

Note: For each of the following questions, answer E, NOTA, means "None of th

1. Suppose f(3) = 2, f '(3) = 5 and f "(3) = -2. Then $\frac{d^2}{dx^2}\left(f^2(x)\right)$ at x = 3 is equa

   A. -20 B. 20 C. 38 D. 42 E. NOTA

2. The graph of f " is shown below. If f '(1) = 0, then f '(x) = 0 at x = 1 and x =



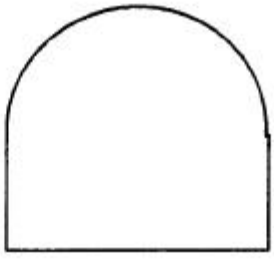
   A. 0 B. 2 C. 3 D. 4 E. NOTA

3. If f(x) is continuous at the point where x = a, which of the following statement false?

   A. $\lim_{x \to a} f(x)$ exists B. $\lim_{x \to a} f(x)$ = f(a) C. f '(a) exists

   D. $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$ E. NOTA

4. The figure shown consists of a rectangle capped by a semicircle. Its area is 10 minimum perimeter of the figure is approximately



   A. 10.584 yd B. 28.284 yd C. 37.793 yd

   D. 38.721 yd E. NOTA

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For questions 5 – 11, use the table below showing the values of differentiable functions f and g.

x	f	f '	g	g '
0	2	1	5	-4
1	3	2	3	-3
2	5	3	1	-2
3	10	4	0	-1

5. If $B = f \bullet g$, then $B'(2) =$

A. -20 B. -7 C. -6 D. -1 E. NOTA

6. If $K(x) = \left(\frac{f}{g}\right)(x)$, then $K'(0) =$

A. $-\frac{13}{25}$ B. $-\frac{1}{4}$ C. $\frac{13}{25}$ D. $\frac{13}{16}$ E. NOTA

7. If $P(x) = f(x^3)$, then $P'(1) =$

A. 2 B. 6 C. 8 D. 12 E. NOTA

8. If $H(x) = \sqrt{f(x)}$, then $H'(3) =$

A. $\frac{1}{4}$ B. $\frac{1}{2\sqrt{10}}$ C. $\frac{2}{\sqrt{10}}$ D. $4\sqrt{10}$ E. NOTA

9. If $S(x) = f^{-1}(x)$, then $S'(3) =$

A. –2 B. $\frac{1}{25}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E. NOTA

10. If $M(x) = f(g(x))$, then $M'(1) =$

A. –12 B. –6 C. 6 D. 12 E. NOTA

11. If $A = f + 2g$, then $A'(3) =$

A. –2 B. 2 C. 7 D. 8 E. NOTA

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12. If $f(x)=\begin{cases} x^2 & \text{for } x \le 1 \\ 2x-1 & \text{for } x > 1 \end{cases}$, then

A. f(x) is not continuous at x = 1
B. f(x) is continuous at x = 1 but f'(1) does not exist
C. f'(1) exists and equals 1
D. f'(1) = 2
E. NOTA

13. The table below shows values of f "(x) for various values of x.

x	-1	0	1	2	3
f "(x)	-4	-1	2	5	8

The function f could be

A. a linear function B. a quadratic function C. a cubic function

D. A fourth-degree function E. NOTA

14. Based on the values of f shown in the table below, estimate f'(2).

x	1.92	1.94	1.96	1.98	2.00
f(x)	6.00	5.00	4.40	4.10	4.00

A. –0.10 B. –0.20 C. –5 D. –10 E. NOTA

15. If $x = t^2 - 1$ and $y = t^4 - 2t^3$, then when t = 1, $\frac{d^2y}{dx^2}$ is

A. 1 B -1 C. 0 D. 3 E. NOTA

16. If $y = x^2 + x$, then the derivative of y with respect to $\frac{1}{1-x}$ is

A. $(2x+1)(x-1)^2$ B. $\frac{2x+1}{(1-x)^2}$ C. $\frac{3-x}{(1-x)^3}$ D. 2x +1 E. NOTA

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17. Suppose $\lim_{x\to 0}\frac{g(x)-g(0)}{x}=1$. It follows necessarily that

A. g is not defined at x = 0. B. g'(0) = 1 C. g'(1) = 0

D. g is not continuous at x = 0 E. NOTA

18. $\lim_{x\to\infty} \sin x$

A. is nonexistent B. is infinity C. is 1 or -1

D. oscillates between –1 and 1 E. NOTA

19. Suppose $\lim_{x\to -3^-} f(x)=-1$; $\lim_{x\to -3^+} f(x)=-1$; $f(-3)$ is not defined. Which, if any, of the following statements is false?

A. $\lim_{x\to -3} f(x) = -1$
B. f has a removable discontinuity at x = -3
C. f is continuous everywhere except at x = -3
D. If we redefine f(-3) to be equal to -1, then the new function will be continuous at x = -3.
E. NOTA

20. The value of $\lim_{h\to 0}\frac{e^{a+h}-e^{a}}{h}$ is

A. 0 B. $\frac{1}{a}$ C. 1 D. e^a E. NOTA

21. A particle moves along the x-axis so that at time t its position is given by $x(t) = (t + 1)(t - 3)^3$. For what values of t is the velocity of the particle increasing?

A. t > 3 B. 0 < t < 3 C. 1 < t < 3 D. t < 1 and t > 3 E. NOTA

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22. If $f(x)=\begin{cases} e^{-x}+2 & \text{for } x<0 \\ ax+b & \text{for } x\geq 0 \end{cases}$ is a differentiable function at 0, then a + b =

A. 0 B. 1 C. 2 D. 3 E. NOTA

23. Let f be a function defined for all real numbers and let a and b be real numbers. Which of the following statements is equivalent to "if $\varepsilon > 0$, then there exists $\delta > 0$ such that if $0 < |x - a| < \delta$, then $|f(x) - b| < \varepsilon$" ?

A. $\lim_{x\to 0}|f(x)-b|=a$ B. $\lim_{x\to a} f(x)=0$ C. $\lim_{x\to a} f(x)=b$

D. $\lim_{x\to b} f(x)=a$ E. NOTA

24. The equation of the line tangent to the curve $y=\frac{kx+8}{k+x}$ at x = -2 is y = x + 4. What is the value of k?

A. –3 B. –1 C. 1 D. 3 E. NOTA

25. Let f be a differentiable function for all x. Which of the following must be true?

I. $\frac{d}{dx}\int_0^3 f(x)dx = f(x)$ II. $\int_3^x f'(x)dx = f(x)$ III. $\frac{d}{dx}\int_3^x f(x)\,dx = f(x)$

A. II only B. III only C. I and II only D. II and III only E. NOTA

26. If f is a differentiable function such that for all $x > 0$, $f(x^2) = 2x^3$, then $f'(4)$ =

A. 4 B. 6 C. 12 D. 16 E. NOTA

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27. To prove that $\lim_{x \to 2}(4x + 1) = 9$, which 8 could be used?

I. $8 = \frac{16}{8}$ H. $8 = \frac{6}{6}$ III. $8 = \frac{46}{2}$

A. I only B. H only C. III only D. I and U only E. NOTA

28. If $f(X) = X3 - x - 6$ for all real numbers x, and if g is the inverse function of f, then f UP) g,(O) =

A. 0 B. I C. -I D. -6 E. NOTA

29. A square is inscribed in a circle. How fast is the area of the square changing when the area of the circle is increasing one square inch per minute?

A. I in'/min B. 7r in 2 /min C. I in 2 /minD. 2 in'/min E. NOTA

2 2)r

30. If f and g are twice differentiable functions such that g(x) = e' f(x) and g"(x) = ex h(x) + e' f(x), then h(x) =

A. f'(x) + f "(x) B. f (X) + (f ,(X))2 C. (f,,(X)+f,,(X))2

D. 2 f'(x) + f "(x) E NOTA