

School Bowl - Mu Level
2000 Mu Alpha Theta National Convention

1. Let $A = \frac{dy}{dx}$ when $y = \sqrt{0.25}$

Let $B = \lim_{x \rightarrow \infty} \frac{8x^2 - 5x + 2}{3 - 7x + 2x^2}$

Find $\lim_{h \rightarrow A} \frac{\sqrt{x+h} - \sqrt{x}}{h}$ when $x = B$

(Use the values of A and B found above.)

2. Let $R =$ the region bounded by $y = x^2$ and the x -axis
from $x = 1$ to $x = 4$

Let $A =$ the exact area of R

Let $B =$ the approximate area of R obtained by using the Trapezoidal
Rule with $n = 3$

Let $C =$ the average value of $f(x) = x^2$ on $[1,4]$

Find $\frac{2AB}{C}$

3. Given $f(x) = \frac{g(x)}{x^2}$, $h(x) = x^2 g(x)$, and $k(x) = x^2 + g(x)$
and $g(3) = -2$ and $g'(3) = 5$

Let $A = f'(3)$

Let $B = h'(3)$

Let $C = k'(3)$

Find $A \left(\frac{B}{C} \right)^3$

4. Let $A = \int_2^3 \frac{dx}{x^2 - x}$

Let $B = \int_{-1}^2 |x^2 - 1| dx$

Let $C = \int_1^3 x^{-2} dx$

Find $\frac{e^A C}{B}$

5. Let $A =$ the y-intercept of the line which is tangent to the curve $3x^2 + 4x^2y + xy^2 = 8$ at the point $(1,1)$

Let $B =$ the x-coordinate of the point on the graph of $y = \sqrt{3x}$ that is closest to the point $(5,0)$

Find $A + B$

6. Let $A = \lim_{x \rightarrow \frac{\pi}{2}} [(1 - \sin x) \tan x]$

Let $B =$ the maximum value of $g(x) = \int_0^x \sin t dt$, where $x \geq 0$

Find $\int_A^B x^2 e^{x/2} dx$

7. A cube of ice melts without changing shape at the uniform rate of $4 \text{ cm}^3/\text{min}$. Let $A =$ the rate of change of the surface area of the cube, in cm^2/min , when the volume of the cube is 125 cm^3 .

There are two asymptotes to the graph of $xy + y = (x - 2)^2$. Let their equations be $x = B$ and $y = Cx + D$.

Find $|A|BCD$

8. Let A = the volume of the solid generated by rotating the region bounded by the graph of $x^2 + y^2 + 8x - 6y + 21 = 0$ about the line $x = 2$.

Let B = the volume of the solid generated by rotating the first quadrant region bounded between the graphs of $y = x^2$ and $y^2 = 8x$ about the x -axis

Find $\frac{A}{B}$

9. Let $f(x) = \int_1^x \sin(\ln 2t) dt$ for all $x > 0$

Find $f(1) + f'(.5e^{\pi}) + f''(.5) + f'''(.5)$

10. The number of bacteria in a culture is growing at a rate of $2,000e^{5t/6}$ per unit of time t . At $t = 0$, the number of bacteria present was 2,400. Find the number present at $t = 12$. Give the exact answer.

11. Let A = the slope of the curve $y = x^3 - 3x^2 - 9x + 20$ at its point of inflection

Let B = the length of a side of an expanding square when the rate of change of its area is three times the rate of change of the length of a side.

Let C = the minimum value of the function $y = x^2 - 5x + 4$ on the interval $1 \leq x \leq 2$

Let D = the total distance traveled by a particle moving on the x -axis so that its distance from the origin at time t is given by $s(t) = 10t - 4t^2$ between $t = 1$ and $t = 2$

Find $AB + CD$

12. The graphs of $f(x) = x$ and $g(x) = \frac{1}{c} x^2$ intersect at the point $(0,0)$ and at the first quadrant point (c,c) . Find this positive value of c so that the area of the region bounded by the graphs of these two functions is 54 square units.

13. If $\frac{d^2f}{dx^2} = x - 1$, $f'(0) = 1$ and $f(1) = 0$, then find $f(-1)$.

14. Let $A =$ the set of all values of k for which $\int_1^{\infty} x^{-k} dx$ converges

Let $B =$ the set of all values of k for which $\sum_{n=1}^{\infty} \frac{(k-1)^n}{n 2^n}$ converges

Find $A \cap B$

15. Let $A = \int_0^{\pi/4} \tan^4 x dx$

Let $B = \int_0^{\pi/2} \cos^3 x dx$

Find $A + B$