Question #1
National Theta Bowl 2000

The lines with equation $Ax + 2y = B$ and $4x + 3y = C$ are perpendicular and both lines contain the point $(-2, 3)$.

Find: $A + B + C$

Question #2
National Theta Bowl 2000

Let A = the 50th term of the arithmetic sequence $-3, -7, -11, -15, \ldots$

Let B = the 20th term of the geometric sequence $8192, -4096, 2048, -1024, \ldots$

Let C = the sum of the first 50 terms of the arithmetic series $-3 + 3 + 9 + 15 + \ldots$

Let D = $\sum_{n=1}^{\infty} 3(\frac{4}{5})^n$

Find: $A + BCD$

Question #3
National Theta Bowl 2000

Given: $z = 1 + 2i$ $\quad (i = \sqrt{-1})$

Let A = the conjugate of z

Let B = the reciprocal of z

Let C = the opposite of z

Let D = the absolute value of z

If the product $ABCD^2$ is written in the form $x + yi$ $(x, y \in \Re)$,

Question #4
National Theta Bowl 2000

Given: $x^2 + y^2 + 12x - 2y + 21 = 0$

$x^2 - y^2 + 4x + 8y - 21 = 0$

$x^2 + 4y^2 + 10x + 24y + 45 = 0$

Line 1 contains the center of the circle.

Line 2 contains the centers of the hyperbola and the ellipse.

Line 1 is perpendicular to Line 2.

If the equation for Line 1 is written in the form $Ax + By = C$, where A, B, and C are relatively prime integers and $A > 0$, find the sum of A, B, and C.

Question #5
National Theta Bowl 2000

There are 120 red marbles and 80 blue marbles in a bag that contains 200 marbles. If only blue marbles are added to the bag so that the probability of randomly drawing a blue marble from the bag becomes $\frac{2}{3}$, how many blue marbles must be added to the bag?

Question #6
National Theta Bowl 2000

The cubic equation $x^3 - 3x^2 + Ax + B = 0$ has rational coefficients and one of the roots is $-1 + i\sqrt{3}$.

Find: AB

Question #7
National Theta Bowl 2000

Let A = the number of degrees in the measure of the largest angle in a triangle whose angles are in the ratio 2:3:4

Let B = the number of units in the distance from the center of a circle with diameter 26 to a chord of the circle with length 10

Let C = the number of cubic units in the volume of a cube with a diagonal of length $\sqrt{12}$

Let D = the number of square units in the area of a right triangle with hypotenuse of length 10 and legs with lengths in the ratio 1:2

Find: $\frac{AB}{CD}$

Question #8
National Theta Bowl 2000

Given: $A = \begin{bmatrix} 3 & -2 \\ 5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 4 \\ 6 & -8 \end{bmatrix}$

Let V = the sum of the entries in A^{-1}

Let W = the sum of the entries in AB

Let X = the sum of the entries in B^2

Let Y = the value of the determinant of A

Let Z = the value of the determinant of B

Find: $W - X + VYZ$

Question #9
National Theta Bowl 2000

Let A = the value of $\log_{0.008}(25\sqrt[3]{25})$

Let B = the sum $d + e + f$ (where d, e, and f are integers), if $\log_{10} 2 = x$, $\log_{10} 3 = y$ and $\log_{10} 150 = dx + ey + f$

Let C = the value of x if $(2x)^{\log_{10}2} = (4x)^{\log_{10}4}$ $(x \neq 0)$

Let D = the sum $\log_3 p + \log_3 q$, if p and q are the roots of the quadratic equation $3x^2 - 7x + 1 = 0$

Find: $\frac{BD}{AC}$

Question #10
National Theta Bowl 2000

The parabola with equation $y = x^2 - 8x + 10$ intersects the line with equation $y = 2x - 11$ in the points (A, B) and (C, D).

Find: $A + B + C + D$

Question #11
National Theta Bowl 2000

Let A = the value of the discriminant of the quadratic equation $3x^2 + 4x = 1$

Let B = the minimum value of the quadratic expression $x^2 - 4x + 8$ $(x \in \Re)$

Let C = the sum of the x and y coordinates of the vertex of the graph of the quadratic function $f(x) = -x^2 - 6x + 2$

Let D = the sum of the x and y coordinates of the focus of the graph of the quadratic function $f(x) = x^2 + 2x - 3$

Find: $AB + CD$

Question #12
National Theta Bowl 2000

Given the following recursively defined function:

$f(1) = 100$

$$f(n) = \begin{cases} \frac{1}{2} f(n-1), \text{ if } f(n-1) \text{ is even} \\ \frac{1}{2} f(n-1) + \frac{1}{2}, \text{ if } f(n-1) \text{ is odd.} \end{cases} \quad (\text{for } n \geq 2)$$

Find: $f(5)$

Question #13
National Theta Bowl 2000

Given: a and b are distinct positive integers and $5a + b = 32$

Find the sum of all possible values of a.

Question #14
National Theta Bowl 2000

Given: $f(a,b) = (a^b)(b^a)$

Find: $f(4, f(1,2))$

Queston #15
National Theta Bowl 2000

Given: $f(x) = x^4 - Ax^3 + Bx^2 + 9x - 12$ (A and B are positive integers)

Find: the product of all possible rational roots of $f(x)$