January Regional *Precalculus*

Individual Test Solutions

1. D
2. C
3. B
4. A
5. D
6. A
7. E
8. C
9. A
10. B
11. A
12. D
13. C
14. B
15. D
16. D
17. C
18. A
19. C
20. D
21. A
22. D
23. D
24. A
25. B
26. C
27. D
28. E
29. B
30. A

Team Solutions

1. 1

2. $\sqrt{2}$

3. 38°

4. 106

5. 242

6. 9

7. 13

8. 3.44

9. $\frac{101}{201}$

10. 3

11. 78

12. −4

13. 58.75

14. $\frac{13}{6}$

15. $\frac{32}{81}$

The abbreviation NOTA denotes "None of these answers."

1. Given $f(t)=\sqrt[3]{t}$, $g(t)=\cos t$, $h(t)=(fg)(t)$, and $0\le t\le \pi$ find $h(\pi)$.

A) $\cos(-1)$ *B)* -1 *C)* $\cos\sqrt[3]{\pi}$ *D)* $-\sqrt[3]{\pi}$ *E)* NOTA

2. Solve $\frac{x+3}{x}-\frac{2}{x+3}=\frac{6}{x^2+3x}$ over the reals.

A) $\{-3\}$ *B)* $\{-3,-1\}$ *C)* $\{-1\}$ *D)* $\{1,3\}$ *E)* NOTA

3. What is the y-intercept of the graph of $f(x)=b^x$ for real numbers b and x, for positive constant b?

A) 0 *B)* 1 *C)* b *D)* $|x|$ *E)* NOTA

4. Suppose $f(x)=Q+R\ln x$ where Q and R are real constants. If $f(1)=5$ and $f(e)=4$, find $\frac{Q}{R}$.

A) -5 *B)* $\frac{1}{5}$ *C)* 4 *D)* $\frac{1}{4}$ *E)* NOTA

5. Let (a, b) and (c, d) represent the endpoints of the latus rectum of $x^2-6x-4y+17=0$. Find the sum of the ordinates of these endpoints.

A) -4 *B)* -2 *C)* 4 *D)* 6 *E)* NOTA

6. Solve the inequality $\frac{2}{x+3}\le\frac{1}{x-1}$ $\{x : x\in\mathbb{R}\}$

A) $(-\infty,-3)\cup(1,5]$ *B)* $(-\infty,5]$ *C)* $[5,\infty)$ *D)* $(-3,1)\cup[5,\infty)$ *E)* NOTA

7. In right triangle ABC with right angle B, angle $C = 39°$ and side $a = 23$, find side b to the nearest tenth.

A) 14.5 *B)* 17.9 *C)* 28.4 *D)* 36.5 *E)* NOTA

8. What is the period of the function $f(t)=\sin^2 t-\cos^2 t$.

A) $\frac{\pi}{2}$ *B)* 2 *C)* π *D)* 2π *E)* NOTA

9. Find an equation in terms of x and y whose graph includes the graph of: $x = t^2$, $y = 2t + 1$ for any t.

A) $4x = (y-1)^2$ *B)* $4x = (y+1)^2$ *C)* $4y = (x-1)^2$ *D)* $4y = (x+1)^2$ *E)* NOTA

10. 24 students are on tour of the United Nations Building. 12 students speak Russian, 6 speak German, and 15 speak Spanish. Only one student speaks all three languages. 2 students speak Russian and German, but not Spanish. One student does not speak Russian or Spanish and 6 do not speak Spanish or German. All 24 students speak at least one of the three languages. How many students speak both Russian and Spanish, but not German?

A) 2 *B)* 3 *C)* 6 *D)* 9 *E)* NOTA

11. Find the inverse of the coefficient matrix for the system of equations: $\begin{array}{c} x + 2y = 6 \\ 3x + 4y = 12 \end{array}$.

A) $\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$ *B)* $\begin{bmatrix} -\frac{1}{2} & -\frac{3}{2} \\ -1 & -\frac{1}{2} \end{bmatrix}$ *C)* $\begin{bmatrix} 1 & -\frac{3}{2} \\ -1 & 2 \end{bmatrix}$ *D)* $\begin{bmatrix} -2 & 1 \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$ *E)* NOTA

12. Which of the following is equivalent to the expression $\log_f (g+h)$, where f, g, and h are all real numbers greater than 1?

A) $f \ln(h+g)$ *B)* $f \ln h \ln g$ *C)* $\ln \frac{h+g}{f}$ *D)* $\frac{\ln(h+g)}{\ln f}$ *E)* NOTA

13. Solve for x: $y = 1 + \cfrac{1}{1 + \cfrac{1}{x}}$

A) $x = 1 - \frac{1}{y}$ *B)* $x = \frac{y-1}{y}$ *C)* $x = \frac{1-y}{y-2}$ *D)* $x = \frac{y+1}{2+y}$ *E)* NOTA

14. Find the perimeter (to the nearest tenth) of a 45° slice of large circular cheese pizza if the slice is a sector and the pizza has a 14-inch-diameter.

A) 14.1" *B)* 19.5" *C)* 20.3" *D)* 36.0" *E)* NOTA

15. The expression $\frac{5x^2 + 7x - 4}{x^3 + 4x^2}$ is equal to $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+4}$. Find C.

A) -1 *B)* 1 *C)* 2 *D)* 3 *E)* NOTA

16. How many real numbers between 0 and 2π solve the equation $4\cos^2 x - 3 = -\cos x$?

A) 0 *B)* 1 *C)* 2 *D)* 3 *E)* NOTA

17. Find all horizontal and vertical asymptotes of $f(x) = \dfrac{2x-4}{x^2-4}$.

A) $x=0,\ x=2,\ y=-2$
B) $x=-2,\ x=2,\ y=0$
C) $x=-2,\ y=0$
D) $x=0,\ y=2$
E) NOTA

18. Which ordered pair is **not** in the *inverse* of the relation given by $x^2y + y^2 = 10$?

A) $(1,\sqrt{3})$ *B)* $(2,-\sqrt{3})$ *C)* $(1,-3)$ *D)* $(2,\sqrt{3})$ *E)* NOTA

19. Given $\sin\theta = 0.4$ and $\cot\theta > 0$, find $\sin(-\theta)+\tan(-\theta)+\csc(\theta)$ to the nearest hundredth.

A) −3.29 *B)* −1.76 *C)* 1.66 *D)* 2.51 *E)* NOTA

20. Find twice the product of all the real zeros of the function $g(x) = x^3 + 3x^2 - 16x - 48$.

A) −64 *B)* −24 *C)* 48 *D)* 96 *E)* NOTA

21. How many positive real x-coordinates of the solutions exist for the system of equations: $\begin{array}{l} y = 2e^{2x} - 3 \\ e^{x} = -\frac{1}{5}y \end{array}$?

A) 0 *B)* 1 *C)* 2 *D)* 3 *E)* NOTA

22. If $2x^2(x-4)^{-\frac{1}{2}} + \frac{x}{2}(x-4)^{\frac{1}{2}}$ is rewritten in the form $\dfrac{Ax^2-4x}{2\sqrt{x-4}}$, find the value of A.

A) −15 *B)* −4 *C)* 2 *D)* 5 *E)* NOTA

23. For oblique triangle DEF where E is an obtuse angle, $\sin(D+E)$ is equal to:

A) $\sin(D+F)$ *B)* $\sin(E)$ *C)* $\sin(E+F)$ *D)* $\sin(F)$ *E)* NOTA

24. What is the domain of the function $f(x) = \dfrac{\sqrt{4-x^2}}{x+4}$?

A) $[-2,2]$ *B)* $(-2,2)$ *C)* $(-\infty,-2)\cup(2,\infty)$ *D)* $[0,2]$ *E)* NOTA

25. Which is an equation of a sinusoid that passes through the point $(2,0)$, has period $\frac{\pi}{3}$ and amplitude 3?

A) $y=3\sin(6x-2)$ *B)* $y=3\sin(6x-12)$ *C)* $y=6\sin\left(\frac{2}{3}\pi x-\frac{4}{3}\right)$ *D)* $y=6\sin\left(\frac{2}{3}x-\frac{2}{3}\right)$ *E)* NOTA

26. The population of Las Vegas , Nevada in January 2000 was 478,000 and has been increasing at the rate of 6.28% each year. At that rate, in what year will the population be 1 million?

A) 2002 *B)* 2007 *C)* 2012 *D)* 2034 *E)* NOTA

27. For triangle *JKL* with angle $K = 57°$, side $j = 11$ and side $k = 10$, find all possible measures for angle *J*, to the nearest tenth.

A) no triangle possible *B)* 92.3° *C)* 124.3° and 55.7° *D)* 112.7° and 67.3° *E)* NOTA

28. State the range of $h(\theta) = \tan\theta\cos\theta$.

A) $(-\infty,\infty)$ *B)* $[-\pi,\pi]$ *C)* $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ *D)* $(-1,1)$ *E)* NOTA

29. If the area of a triangle with sides 7, 8, and 9 equals $a\sqrt{b}$ in simplified radical form, find ab.

A) 30 *B)* 60 *C)* 84 *D)* 504 *E)* NOTA

30. Find the acute angle θ that satisfies $\cot\theta = \frac{4}{\sqrt{48}}$

A) $\frac{\pi}{3}$ *B)* $\frac{\pi}{4}$ *C)* $\frac{\pi}{5}$ *D)* $\frac{\pi}{6}$ *E)* NOTA

1. D

$h(t) = (fg)(t) = f(t)\cdot g(t)$

$= \sqrt[3]{t}\cos t$

$h(\pi) = \sqrt[3]{\pi}\cos\pi = \sqrt[3]{\pi}(-1)$

$= -\sqrt[3]{\pi}$

2. C

$\frac{x+3}{x} - \frac{2}{x+3} = \frac{6}{x(x+3)}$

$(x+3)^2 - 2x = 6 \Rightarrow x^2+4x+3=0$

$(x+3)(x+1)=0$, x **is undefined at** -3, $\therefore x=-1$

3. B

The y-intercept occurs when x = 0. Any real number greater than zero raised to the zero power is equal to 1.

4. A

$f(1) = 5 : Q + R\ln 1 = 5$

$Q + R(0) = 5 \Rightarrow Q = 5$

$f(e) = 4 : Q + R\ln e = 4$

$Q + R(1) = 4 \Rightarrow Q + R = 4 \Rightarrow$

$R = -1, \therefore \frac{Q}{R} = -5$

5. D

$x^2 - 6x - 4y + 17 = 0$ is a parabola with vertex (3, 2), focus (3, 3), and directrix $y = 1$. The latus rectum passes through the focus ⊥ to the axis of symmetry with endpoints on the parabola at (1, 3) and (5, 3). Sum of the ordinates = 3 + 3 = 6

6. A

$\frac{2}{x+3} \le \frac{1}{x-1} \Rightarrow \frac{2}{x+3} - \frac{1}{x-1} \le 0 \Rightarrow$

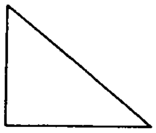
$\frac{x-5}{(x+3)(x-1)} \le 0$

critical numbers are -3, 1, 5 & values that satisfy the inequality are $(-\infty,-3)\cup(1,5]$

7. E

$\cos 39° = \frac{23}{b}$,

$b \approx 29.6$



8. C

$\sin^2 t - \cos^2 t = -(\cos^2 t - \sin^2 t)$

$= -\cos 2t$

period: $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

9. A

$x = t^2 \Rightarrow \sqrt{x} = t \Rightarrow y = 2t+1$

$\Rightarrow \frac{y-1}{2} = t$

substituting:

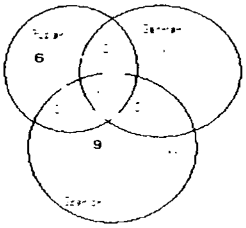
$\sqrt{x} = \frac{y-1}{2} \Rightarrow \left(2\sqrt{x}\right)^2 = (y-1)^2$

$4x = (y-1)^2$

10. B Given :

Russian=12, German=6, Spanish=15; $R\cap G\cap S = 1; R\cap G = 2$; 1 speaks strictly German & 6 speak strictly Russian.

$\therefore$ Russian: $12 - 9 = 3$ in $R\cap S$.



Only 3 can speak both Russian & Spanish.

11. A The coefficient matrix is $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$. It's inverse is :

$\frac{1}{1(4)-2(3)}\begin{bmatrix}4 & -2\\ -3 & 1\end{bmatrix} = \begin{bmatrix}-2 & 1\\ \frac{3}{2} & -\frac{1}{2}\end{bmatrix}$

12. D

Using the change of base formula

$\log_f(g+h) = \frac{\ln(g+h)}{\ln f}$

$= \frac{\ln(h+g)}{\ln f}$

13. C

$1 + \frac{1}{1+\frac{1}{x}} = 1 + \frac{x}{x+1} = \frac{2x+1}{x+1} = y$

$y(x+1) = 2x+1 \Rightarrow xy - 2x = 1 - y$

$x = \frac{1-y}{y-2}$

14. B

Perimeter = 2sides + arc length

$= 7 + 7 + \frac{\pi(7)(45°)}{180°} = 19.5"$

15. D

$\frac{5x^2+7x-4}{x^3+4x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+4}$

$5x^2+7x-4 = Ax(x+4) + B(x+4) + Cx^2$

$5x^2+7x-4 = (A+C)x^2 + (4A+B)x + 4B$

solving the system of equations:

$A+C=5,\ 4A+B=7,\ 4B=-4$

yields A=2, B=-1, **C=3**

16. D

$4\cos^2 x + \cos x - 3 = 0$

$(4\cos x - 3)(\cos x + 1) = 0$

$4\cos x = 3$ $\quad$ $\cos x = -1$

$\cos x = \frac{3}{4}$ $\quad$ $x = \cos^{-1}(-1)$

$x = 41.41,\ 318.59$ $\quad$ $x = \pi$

17. C

$f(x) = \frac{2x-4}{x^2-4} = \frac{2(x-2)}{(x+2)(x-2)}$

$x = 2$ is a hole
$x = -2$ is a vertical asymptote
$y = 0$ is a horizontal asymptote

18. A

$(\sqrt{3})^2(1) + 1^2 \neq 10,\ \therefore (1, \sqrt{3})$ is not

$(-\sqrt{3})^2(2) + (2)^2 = 10,\ \therefore (1, \sqrt{3})$ is

$(-3)^2(1) + (1)^2 = 10,\ \therefore (1, \sqrt{3})$ is

$(\sqrt{3})^2(2) + (2)^2 = 10,\ \therefore (1, \sqrt{3})$ is

19. C

$\sin\theta = \frac{4}{10} = \frac{2}{5}\ \&\ \csc\theta = \frac{5}{2}$

20. D

$x^3 + 3x^2 - 16x - 48 = 0$

$x^2(x+3) - 16(x+3) = 0$

$(x+4)(x-4)(x+3) = 0$

$x = -4, -3, 4$

twice (-4)(-3)(4) is 96

21. A

By substitution $\begin{matrix} y = 2e^{2x} - 3 \\ y = -5e^x \end{matrix}$ becomes

$2e^{2x} - 3 = -5e^x \Rightarrow 2e^{2x} + 5e^x - 3 = 0$

$(2e^x - 1)(e^x + 3) = 0$

$2e^x = 1$ $\quad$ $e^x = -3$

$e^x = 1/2$ $\quad$ $x = \ln(-3)$

$x = \ln(1/2)$ $\quad$ $\varnothing$

$x = -.693$

no positive solutions

22. D

$2x^2(x-4)^{-\frac{1}{2}} + \frac{x}{2}(x-4)^{\frac{1}{2}}$

$= \frac{1}{2}x(x-4)^{-\frac{1}{2}}[4x + (x-4)]$

$= \frac{x(5x-4)}{2\sqrt{x-4}} = \frac{5x^2-4x}{2\sqrt{x-4}} = \frac{Ax^2-4x}{2\sqrt{x-4}}$

$\therefore A = 5$

23. D

$D + E = 180 - F$

$\sin(D+E) = \sin(180 - F)$

$= \sin(F)$

24. A The domain of f must satisfy

$4 - x^2 \geq 0$

$4 \geq x^2$

$\pm 2 \geq x$

$[-2, 2]$

25. B

$y = a\sin(bx + c),\ amplitude: a = 3$

$period: \frac{2\pi}{b} = \frac{\pi}{3},\ b = 6$

$y = 3\sin(6x + c)$

$0 = 3\sin(6(2) + c)$

$\sin^{-1} 0 = 12 + c$

$-12 = c$

$\therefore y = 3\sin(6x - 12)$

26. C

$P(t) = P_0(1 + r)^t$

$1{,}000{,}000 = 478{,}000(1.0628)^t$

$2.0921 = 1.0628^t$

$\ln 2.0921 = t \ln 1.0628$

$\frac{\ln 2.0921}{\ln 1.0628} = t$

12.12 *years from* 2000

2012

27. D This is a case of SSA

$\frac{\sin J}{11} = \frac{\sin 57°}{10}$

$\sin J = .9225$

$J = \sin^{-1} .9225$

$J = 67.294°$ or

$J = 180° - 67.299° = 112.706°$

28. E

Where cosine is zero, the function h is undefined.

29. B

Using Heron's Formula:

$Area = \sqrt{12(5)(4)(3)} = \sqrt{720}$

$= 12\sqrt{5} = a\sqrt{b}$

$a = 12 \quad b = 5 \quad ab = 12(5) = 60$

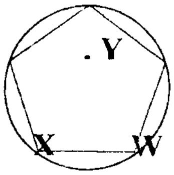
30. A $\cot\theta = \frac{4}{\sqrt{48}} = \frac{4}{4\sqrt{3}}$

$\tan\theta = \frac{\sqrt{3}}{1}$

$\theta = \frac{\pi}{3}$

January Regional **Precalculus Team: Question 8**

In a regular pentagon, line segment xy is drawn from the center y of the polygon to a vertex x such that angle θ is formed by xy and side xw. If distance $xy = 2.5\sec\theta$, find the length of the apothem to the nearest hundredth.



January Regional **Precalculus Team: Question 9**

Write $\dfrac{M}{L+N}$ as a fraction in simplest form if: $L = \sum_{k=1}^{100} k$, M = the sum of the positive even integers from 2 to 200 inclusive, $N = \sum_{j=101}^{200} j$

January Regional **Precalculus Team: Question 10**

$f(\theta) = a\sin(b\theta + c)$ is the function whose phase shift equals $-\dfrac{5}{2}$, amplitude equals 3, and period equals π. $g(\theta) = d\cos(h\theta + k) + m$ is the function whose vertical shift equals -1, phase shift equals $\dfrac{4}{3}$, amplitude equals 2, and period equals $\dfrac{2\pi}{3}$. Find the value of $g(\pi) - f(\pi)$ to the nearest whole number.

January Regional **Precalculus Team: Question 11**

$Ax^2 + By^2 + Cx + Dy + E = 0$ is the equation of the ellipse with center $(-2,3)$, major axis with length 10, eccentricity $\dfrac{3}{5}$, and passing through $(3,3)$. Find $E \cdot \dfrac{A+B+C}{D}$ (with A and B relatively prime to the nearest whole number.

January Regional **Precalculus Team: Question 12**

What is the coefficient of the "x" term in the 4th degree polynomial function with real coefficients, zeros at $x = -1,\ 3,\ 2-i$, and $f(0) = 30$.

January Regional **Precalculus Team: Question 13**

Mrs. Ana Log gave a challenging Ch.5 Precalculus test to her class. The lowest score was a 50 (out of 100), which Mrs. Log scaled to a 62 (out of 100). The highest score was an 85 (out of 100), which she scaled to a 90 (out of 100). At the end of the semester, Mrs. Log increased all test grades by 3%. At the end of the semester, Polly's chapter 5 Test grade was recorded as a 72%. What was Polly's original score on the chapter 5 Test if Mrs. Log always uses a linear scale?

January Regional **Precalculus Team: Question 14**

The function $f(x) = Px^7 + Qx^6 + Rx^5 + Sx^4 + Tx^3 + Ux^2 + Vx + W$ has real integral coefficients with A = the maximum possible number of y-intercepts of f, B = the maximum possible number of x-intercepts of f, C = the maximum possible number of local extrema. Find the exact value of $A + \dfrac{B}{C}$.

January Regional **Precalculus Team: Question 15**

If $\cos x = \dfrac{-1}{3}$, $\pi < x < \dfrac{3\pi}{2}$ find the value of $\sin 2x \cos 2x \tan 2x$ as a fraction whose numerator and denominator are integers with no common factors.

January Regional — ***Precalculus Team: Question 1***

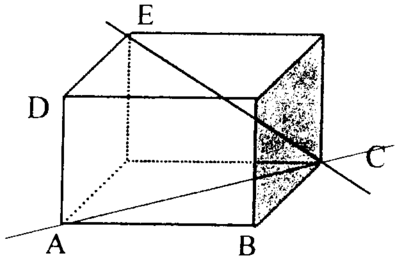
The vertex of $f(x)=x^2+Ax+B$ is $(2,4)$. The vertex of $g(x)=x^2+8x+C$ lies on the x-axis. The vertex of $h(x)=x^2+Dx+E$ lies on the y-axis. Find $\left(\left(C^A\right)^B\right)^D$.

January Regional — ***Precalculus Team: Question 2***

The lateral areas of two similar solids are 196π in.^{2} and 324π in.^{2}. The volume of the smaller cylinder is 686π in.^{3}. Let F = the volume of the larger cylinder in cubic inches. Let K = the geometric mean of 4 and 18. Find the value of $\frac{4F}{486K\pi}$.

January Regional — ***Precalculus Team: Question 3***

$\angle ACE$ is inscribed in a right rectangular prism with $AB=6,\ BC=4,\ \&\ AD=2$. Find the measure of $\angle ACE$ to the nearest degree.



January Regional — ***Precalculus Team: Question 4***

For the line $\frac{2}{3}x+\frac{1}{4}y=-\frac{5}{6}$, let: I = the product of the x-intercept and the y-intercept. J = the distance from the x-intercept to the y-intercept. K = the sum of the coordinates of the midpoint between the x-intercept and the y-intercept. Written in simplest form $\frac{I+J}{K}=\frac{a+b\sqrt{c}}{d}$. Find the sum of the absolute values of *a, b, c,* and *d.*

January Regional — ***Precalculus Team: Question 5***

Find the sum of the coefficients of the x terms with a non-zero exponent in the expansion of $(2x-3)^5$.

January Regional — ***Precalculus Team: Question 6***

For $y=mx+b$, find the value of y if: b is the value of

$\ln(\sin\theta)+\ln(\cos\theta)+\ln(\tan\theta)+\ln(\csc\theta)+\ln(\sec\theta)+\ln(\cot\theta)$, for $0<\theta<\frac{\pi}{2}$. m is the maximum number of real zeros possible for $f(x)=Ax^3+Bx^2-Cx+D$. x is the remainder of $\frac{g(x)}{h(x)}$ for $g(x)=10x^{75}-8x^{65}+6x^{45}+4x^{32}-2x^{15}+5$ and $h(x)=x+1$.

January Regional — ***Precalculus Team: Question 7***

In the blanks that correspond to the statement *i* through *iv*, write 0 if the statement is true and 1 if the statement is false. *i)* If $f(x)=\sqrt{x^2+0.0001}-0.01$ **and** $g(x)=|x|$, $f(x)=g(x)$. *ii)* If $y=\left(x^{200}\right)^{\frac{1}{200}}$, $y=x$ for $\{x : x\in\Re\}$. *iii)* $h(x)=\frac{x}{x^2-1}$ intersects its horizontal asymptote. *iv)* The greatest integer function has an inverse function.

____ ____ ____ ____
i *ii* *iii* *iv*

If the 4-digit number written in the blanks is a binary number, write the number in base 10.

1.

For $f(x)$: $\left\{\begin{array}{l}\frac{-A}{2}=2 \Rightarrow A=-4 \\ f(2)=2^2-8+B=4 \Rightarrow B=8\end{array}\right\}$

for $g(x)$: $\left\{\begin{array}{l}\frac{-8}{2}=-4 \\ f(-4)=16-32+C=0 \Rightarrow C=16\end{array}\right\}$

for $h(x)$: $\left\{\frac{-D}{2}=0 \Rightarrow D=0\right\}$.

$\therefore\ C^{A^{B^D}}=\left(\left(16^{-4}\right)^8\right)^0=1$

2.

The similarity ratio of the two cylinders is $\sqrt{\frac{196\pi}{324\pi}} \Rightarrow \sqrt{\frac{49}{81}} \Rightarrow \frac{7}{9}$. The ratio of the volumes is $\frac{7^3}{9^3}$.

Using proportions to solve for the larger volume:

$\frac{286\pi}{F}=\frac{7^3}{9^3} \Rightarrow F=1458\pi$. To find K:

$\frac{4}{K}=\frac{K}{18} \Rightarrow K^2=72 \Rightarrow K=6\sqrt{2}$.

Finally $\frac{4F}{486K\pi}=\frac{4\cdot 1458\pi}{486\left(6\sqrt{2}\right)\pi}=\frac{2}{\sqrt{2}}=\sqrt{2}$

3.

By the Pythagorean Theorem,

$AC=2\sqrt{13},\ AE=2\sqrt{5},\ \&\ CE=2\sqrt{10}$.

Using the law of cosines:

$\cos C=\frac{10+13-5}{2\sqrt{10}\left(\sqrt{13}\right)}$

$\&\ C\approx 38°$

4.

The x-intercept is $x=\frac{-5}{4}$, the y-intercept is $y=\frac{-10}{3}$.

$I=\frac{-5}{4}\cdot\frac{-10}{3}=\frac{25}{6}$, $J=\sqrt{\left(\frac{-5}{4}-0\right)^2+\left(0-\frac{-10}{3}\right)^2}=\frac{5\sqrt{73}}{12}$;

and the midpoint between the x- & y-intercepts is $\left(\frac{-5}{8},\frac{-10}{6}\right)$, whose sum $\frac{-55}{24}=K$.

$\frac{I+J}{K}=\frac{a+b\sqrt{c}}{d}=\frac{20+2\sqrt{73}}{11}$. Sum = 106

5.

$(2x-3)^5={}_5C_0(2x)^5+{}_5C_1(2x)^4(-3)+$
${}_5C_2(2x)^3(-3)^2+{}_5C_3(2x)^2(-3)^3$
$+{}_5C_4(2x)(-3)^4+{}_5C_2(-3)^5$

The sum of the coefficients of powers of x is

$32-240+720-1080+810=242$

6.

$b=\ln(\sin\theta)+\ln(\cos\theta)+\ln(\tan\theta)+\ln(\csc\theta)+$
$\ln(\sec\theta)+\ln(\cot\theta)$

$=\ln(\sin\theta\cos\theta\tan\theta\csc\theta\sec\theta\cot\theta)$

$=\ln\left(\sin\theta\cos\theta\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}\cdot\frac{\cos\theta}{\sin\theta}\right)$

$=\ln 1=0$

There are, at most, n real zeros for any nth degree polynomial. For a 3rd degree function there are, at most, 3 real zeros. Therefore, $m=3$.

$x=$ The remainder of $\frac{g(x)}{h(x)}=g(-1)=3$. $y=3(3)+0=9$

7.

i) false $f(0)\neq g(0)$,

ii) false, $y=x$ for $\{x:x\geq 0\}$ only,

iii) true

iv) false, the greatest integer function is not one-to-one.

$1101_2=1(2)^3+1(2)^2+0(2)+1=13_{10}$

8.

Each interior angle $=\frac{3(180)}{5}=108°$.

The line segment xy bisects an interior angle so $\theta=54°$.

The hypotenuse of the small triangle is $h=2.5\sec\theta=2.5\sec 54°=4.2533$.

The apothem is $a=4.2553\sin 54°\approx 3.44$

9.

$L = \sum_{k=1}^{100} k = \frac{100(101)}{2} = 5050,$

$M = 2\sum_{k=1}^{100} k = 2(5050) = 10100,$

$N = \sum_{j=101}^{200} j = \frac{200(201)}{2} - \frac{100(101)}{2} = 15050,$

$\frac{M}{L+N} = \frac{10100}{5050+15050} = \frac{101}{201}$

10.

$f(\theta) = 3\sin(2\theta+5)$ and $g(\theta) = 2\cos(3\theta-4)-1$

$g(\pi)-f(\pi) \Rightarrow$

$(2\cos(3\pi-4)-1)-(3\sin(2\pi+5)) \approx 3.1841 \approx 3$

11.

Since the major axis is 10 units long

$3 = \sqrt{5^2-b^2} \therefore b = 4$.

Since the ellipse passes through (3, 3),

$\frac{(3+2)^2}{25} + \frac{(3-y_1)^2}{16} = 1$ & $y_1 = 3$.

The equation of the ellipse is $\frac{(x+2)^2}{25} + \frac{(y-3)^2}{16} = 1$

$\Rightarrow 16x^2 + 25y^2 + 64x - 150y - 111 = 0$.

$E \cdot \frac{A+B+C}{D} = -111 \frac{16+25+64}{-150} = 77.7 \approx 78$

12.

Since complex zeros occur in conjugate pairs, $2+i$ is also a zero of *f.*

$f(x) = a(x-3)(x+1)(x-2+i)(x-2-i)$

$= a(x^2-2x-3)(x^2-4x+5)$,

$= a(x^4-6x^3+5x^2+2x-15)$

$f(0) = 30$

$a(0^4-60^3+50^2+20-15) = 30$

$a = -2 \therefore$

$f(x) = -2x^4+12x^3-10x^2-4x+30$

The coefficient of the "x" term is -4.

13.

The points $(50,62)$, $(85,90)$ yield the equation:

$\textit{curved score} = .8(\textit{original score}) + 22$.

The semester score obtained by the equation:

$\textit{semester score} - 3 = .8(\textit{original score}) + 22$.

Polly's original score is found by

$72 - 3 = .8(\textit{original score}) + 22$

$\therefore (\textit{original score}) = 58.75$

14.

An nth degree polynomial function with real coefficients has at most one y-intercept $\therefore A = 1$;

at most, n x-intercepts $\therefore B = 7$

& at most, n-1 local extrema $\therefore C = 6$.

$A + \frac{B}{C} = 1 + \frac{7}{6} = \frac{13}{6}$

15.

$\cos x = \frac{-1}{3},\ \pi < x < \frac{3\pi}{2}, \sin x = \frac{-2\sqrt{2}}{3}, \tan x = 2\sqrt{2}d$

$\sin 2x \cos 2x \tan 2x \Rightarrow \sin 2x \cos 2x \frac{\sin 2x}{\cos 2x} \Rightarrow \sin^2 2x$

$\Rightarrow (2\sin x \cos x)^2 \Rightarrow \left(2 \cdot \frac{-2\sqrt{2}}{3} \cdot \frac{-1}{3}\right)^2 \Rightarrow \left(\frac{4\sqrt{2}}{9}\right)^2 \Rightarrow \frac{32}{81}$