# Algebra II Individual Test 

February 18,2006 Middleton Invitational

## NO CALCULATOR

E is NOTA, meaning "None Of These Answers."
For this entire test, let $i \equiv \sqrt{-1}$.

1. Give the equation in standard form of the line through the points $(2,-4)$ and $(3,1)$.
A) $5 x-y=14$
B) $5 x-y=2$
C) $5 x+y=8$
D) $5 x+y=16$
E) NOTA
2. Express $(13)_{5}+(13)_{9}$ in base 7 .
A) $(22)_{7}$
B) $(26)_{7}$
C) $(202)_{7}$
D) $(206)_{7}$
E) NOTA
3. How many distinct integers satisfy the inequality $\quad|2 x-3|<|4-x| \quad ?$
A) 1
B) 2
C) 3
D) 4
E) NOTA
4. $f(x)=a x^{4}+b x$, where a and b are nonzero constants. The remainder of the polynomial division $f(x) \div(x+1)$ is $4 a$. Find $|a+b|$, if the result of the polynomial division $\frac{f(x)-30}{x-2}$ is itself a polynomial.
A) 3
B) 4
C) 5
D) 6
E) NOTA
5. Let $G(x)$ be the greatest integer less than or equal to $x$. Find $G(0!)-G(-1.3)+G\left(\left(\frac{5}{7}\right)^{-\frac{1}{2}}\right)$.
A) 3
B) 4
C) 5
D) 6
E) NOTA
6. Consider the rational function $r(x)=\frac{P(x)}{Q(x)}$, where P and Q are polynomial functions of at least degree one. How many of the following four statements must be true?

- If $r(x)$ has the asymptote with equation $y=2$, then $r(x) \neq 2$ for all $x$.
- If $Q(k)=0$, then $r(x)$ has the asymptote with equation $x=k$.
- If $P(k)<Q(k)<0$, then $r(k)<0$.
- If $r(k)=0$, then $P(k)=0$.
A) 1
B) 2
C) 3
D) 4
E) NOTA

7. What is the slope of a line perpendicular to the line given by $2 x-3 y=10$ ?
A) $\frac{2}{3}$
B) $\frac{3}{2}$
C) $-\frac{2}{3}$
D) $-\frac{3}{2}$
E) NOTA
8. The distance between the foci of an ellipse is twice the length of its minor axis. If the eccentricity of the ellipse is E , then find $E^{-2}$.
A) $\frac{5}{4}$
B) $\frac{5}{2}$
C) 5
D) 17
E) NOTA
9. Find $|x|$, if $\quad \log _{81}(9)-\log _{5}\left(\frac{1}{25}\right)=\log _{4}\left(2^{x}\right)$.
A) 3
B) 4
C) 5
D) 6
E) NOTA
10. A parabola's vertex is in quadrant II and its directrix is the line $y=0$. Which way does the parabola open?
A) Left
B) Right
C) Up
D) Down
E) NOTA
11. Which pair of words completes a true statement when respectively placed in the blanks below?
" $f(x)=(0.4)^{x}$ is an exponential $\qquad$ function with a range of all $\qquad$ numbers."
A) growth; real
B) growth; positive
$\begin{array}{lll}\text { C) decay; real } & \text { D) decay; positive } & \text { E) NOTA }\end{array}$
12. Only five contestants remain on a game show. Of them, my favorites are Keith and Caitlin. On each of the next 3 episodes, a contestant chosen completely at random will be removed from the show. What is the probability that my favorites will be the last two contestants left?
A) $\frac{2}{5}$
B) $\frac{1}{5}$
C) $\frac{1}{10}$
D) $\frac{1}{12}$
E) NOTA
13. Express $\frac{2+3 i}{1+2 i}$ as a simplified fraction with a real denominator:
A) $\frac{8-i}{3}$
B) $\frac{-8+i}{3}$
C) $\frac{8-i}{5}$
D) $\frac{-8+i}{5}$
E) NOTA
14. By definition, which of the following numbers could be used as the base of a logarithm?
A) -3
B) 0
C) 1
D) $\sqrt{2}$
E) NOTA
15. Solve for $p: 3 p+t=r p-5$
A) $\frac{t-5}{3-r}$
B) $\frac{t+5}{-3 r}$
C) $\frac{t-5}{r+3}$
D) $\frac{t+5}{r-3}$
E) NOTA

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16. Let $m(x)=f(g(h(x)))$, where $f(x)=|3-x|$,
$g(x)=(x-2)^{2}$, and $h(x)=\sqrt{x}+\sqrt{x-5}$.
Which of the following inequalities is true?
A) $m(1)>m(4)>m(9)$
B) $m(1)>m(9)>m(4)$
C) $m(4)>m(1)>m(9)$
D) $m(4)>m(9)>m(1)$
E) NOTA
17. Let $f(x)=a x^{5}+b x^{2}-3 x-2$, where $a$ and $b$ are positive integers. Which of the following statements are true?
I. As $x$ approaches $-\infty, f(x)$ approaches $\infty$.
II. $f(x)$ has at least one real root.
III. $f(x)$ has at least two nonreal roots.
A) I \& II only
B) II \& III only
C) I \& III only D) I, II, \& III
E) NOTA
18. Solve for $x: \quad 9^{x}=4$
A) $\log _{2} 3$
B) $\log _{3} 2$
C) $\frac{2}{\log _{2} 3}$
D) $\frac{2}{\log _{3} 2}$
E) NOTA
19. If $a^{2}+b^{2}=4 a b$, and $a+b=4$, then find the product $a b$, where $a b \neq 0$.
A) -1
B) 1
C) $\frac{8}{3}$
D) $\frac{4 \sqrt{5}}{5}$
E) NOTA
20. If $(5 x+A)(2 x+B)=10 x^{2}+29 x-21$, then evaluate $(A+B)$, if A and B are integers.
A) 4
B) -4
C) 7
D) -7
E) NOTA
21. Solve for $k: \quad 2^{k}=\sqrt[5]{\left(\frac{1}{8}\right)^{3}}$
A) $\frac{1}{5}$
B) $-\frac{9}{5}$
C) $-\frac{5}{9}$
D) -9
E) NOTA
22. My tax refund (R) increases by $5 \%$ every year. The first year $(t=0)$ it was only $\$ 500$. If $t$ is an integer measured in years, then how can I properly model this situation algebraically?
A) $R(t)=500 e^{1.05 t}$
B) $R(t)=500+25 t$
C) $R(t)=500(1.05)^{t}$
D) $R(t)=\left(500+\frac{.05}{t}\right)^{5 t}$

## E) NOTA

23. Find the sum of the solutions to the equation

$$
(x-5)^{2}-(2 x-5)^{2}=x-1 .
$$

A) -4.6
B) 1.5
C) 3
D) 5.5
E) NOTA
24. Let $F(x)=(x-2)$ !. What odd value of $n$ satisfies the inequality $2^{n} \leq \frac{F(9)\left({ }_{14} C_{6}\right)}{F(12)}<2^{n+2}$ ?
A) 1
B) 3
C) 5
D) 7
E) NOTA
25. $|M|+|N|=M^{2}+N^{2}=12$, where $M$ and $N$ are complex conjugates. Find $\left|\frac{M+N}{M-N}\right|^{2}$.
A) $\frac{1}{5}$
B) $\frac{7}{5}$
C) $\frac{7}{6}$
D) 2
E) NOTA

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26. Find the constant $k$ such that the vertex of the parabola defined by the equation $y=x^{2}+2 k x+\left(k^{2}-9 k+9\right)$ lies on the $x$-axis.
A) 3
B) 2
C) 1
D) $-\frac{2}{3}$
E) NOTA
27. Which point is a focus of the graph defined by the equation $5 y^{2}-4 x^{2}+20=0 \quad$ ?
A) $(1,0)$
B) $(0,1)$
C) $(3,0)$
D) $(0,3)$
E) NOTA
28. Your initial bankroll of $N$ dollar experiences a $30 \%$ decrease. It is then increased by $40 \%$, and subsequently adjusted to $75 \%$ of its new value. Find the ratio of the final amount to $N$.
A) $\frac{33}{40}$
B) $\frac{22}{15}$
C) $\frac{200}{147}$
D) $\frac{147}{200}$
E) NOTA
29. Two perpendicular lines intersect at $(6,6)$. Their $y$-intercepts are a distance of 13 apart. Find the distance between their $x$-intercepts.
A) 7
B) 12
C) 13
D) 25
E) NOTA
30. $f(x)= \begin{cases}2^{x}, & \text { if }|x| \text { is odd } \\ f\left(-\frac{x}{2}\right), & \text { if }|x| \text { is even }\end{cases}$

Evaluate $|f(32) \cdot f(9) \cdot f(14) \cdot f(2) \cdot f(40)|$.
A) $\frac{1}{32}$
B) $\frac{1}{8}$
C) 1
D) 32
E) NOTA

1. (A) The slope is $\frac{1-(-4)}{3-2}=5$. Manipulating $y+4=5(x-2)$ gives the answer.

2. (B) $(13)_{5}=3\left(5^{0}\right)+1\left(5^{1}\right)=8$. $(13)_{9}=3\left(9^{0}\right)+1\left(9^{1}\right)=12$. Do stair-step division and read the remainders down.
3. (C) Solving for the points of equality using the equations $2 x-3=4-x$ and $-(2 x-3)=4-x$ yields critical values at $\frac{7}{3}$ and -1 . Testing intervals revels that the solutions lie between these numbers, and the integers on the interval are 0,1 , and 2 .
4. (D) Using the Remainder Theorem, $f(-1)=a-b=4 a$, so $b=-3 a$. The definition of a polynomial dictates that $f(2)-30=0$. Solve $16 a+2(-3 a)=30$ to find that $a=3$ and $b=-9$.
5. (B) Note that 0 ! $=1$, and that $\sqrt{1}<\sqrt{\frac{7}{5}}<\sqrt{4} . \quad 1-(-2)+1=4$.
6. (A) Only the last statement is true.
7. (D) For any line $A x+B y=C$, the slope of a perpendicular line is $\frac{B}{A}$.
8. (A) Eccentricity $=\frac{c}{a}$ from the ellipse's standard form, so $E^{-2}=\frac{a^{2}}{c^{2}} . b=\frac{c}{2}$, and $c=\sqrt{a^{2}-b^{2}} \rightarrow c^{2}=a^{2}-\frac{c^{2}}{4} \rightarrow 5 c^{2}=4 a^{2} \rightarrow \frac{a^{2}}{c^{2}}=\frac{5}{4}$.
9. (C) Solving $\frac{1}{2}-(-2)=\frac{1}{2} x, x=5$.
10. (C) The parabola opens away from the $x$-axis, its directrix.
11. (D) True for $f(x)=a^{x}$ whenever $0<a<1$.
12. (C) Only one combination of two contestants will satisfy me, and there are ${ }_{5} C_{2}=\frac{5!}{2!3!}=10$ possible combinations.
13. (C) $\left(\frac{2+3 i}{1+2 i}\right)\left(\frac{1-2 i}{1-2 i}\right)=\frac{2+3 i-4 i-6 i^{2}}{1-4 i^{2}}=\frac{8-i}{5}$
14. (D) Any positive real number except 1 may be used.
15. (D) $3 p-r p=-5-t \rightarrow p(3-r)=-(t+5) \rightarrow p=\frac{t+5}{r-3}$
16. (B) $f(g(h(1)))=f(g(1+2 i))=f\left((1+2 i-2)^{2}\right)=f(-3-4 i)=|6+4 i|=2 \sqrt{13}$.

$$
\begin{aligned}
& f(g(h(4)))=f(g(2+i))=f\left((2+i-2)^{2}\right)=f(-1)=|3+1|=4 . \\
& f(g(h(9)))=f(g(5))=f\left((5-2)^{2}\right)=f(3)=|3-9|=6 .
\end{aligned}
$$

17. (B) I is not true, because $f$ has a positive leading coefficient and is an odd degree, so $f(x)$ approaches $-\infty$ as $x$ approaches $\infty$. II and III are both true by Des Cartes' Rule of Signs.
18. (B) $9^{x}=4 \rightarrow \log _{9} 4=x \rightarrow x=\frac{\log 2^{2}}{\log 3^{2}} \rightarrow x=\frac{2 \log 2}{2 \log 3}=\frac{\log 2}{\log 3}=\log _{3} 2$
19. (C $(a+b)^{2}=a^{2}+b^{2}+2 a b=4 a b+2 a b=6 a b ;(a+b)^{2}=4^{2}=16 ; 6 a b=16 \rightarrow a b=\frac{8}{3}$
20. (A) $10 x^{2}+29 x-21=(5 x-3)(2 x+7)$ so $-3+7=4$.
21. (B)

$$
\sqrt[5]{\left(\frac{1}{8}\right)^{3}}=\sqrt[5]{\left(2^{-3}\right)^{3}}=\sqrt[5]{2^{-9}}=\left(2^{-9}\right)^{\frac{1}{5}}=2^{-\frac{9}{5}}
$$

22. (C) Each year, the amount is multiplied by 1.05. This is normal exponential growth, not continuous growth (modeled by choice A), linear growth (modeled by choice B), or pure nonsense (modeled by choice D).
23. (C)

$$
(x-5)^{2}-(2 x-5)^{2}-x+1=0 \rightarrow\left(x^{2}-10 x+25\right)-\left(4 x^{2}-20 x+25\right)-x+1=0
$$

$$
-3 x^{2}+9 x+1=0 \text {, and the sum of the solutions is }-\frac{b}{a}=-\frac{9}{-3}=3 .
$$

24. (A)

$$
\frac{(7)!\left(\frac{14!}{6!8!}\right)}{(10)!}=\frac{7!14!}{6!8!10!}=\frac{[(6!)(7)][(10!)(11)(12)(13)(14)]}{(6!)(8!)(10!)}=\frac{(7)(11)(12)(13)(14)}{(8)(7)(6)(5)(4)(3)(2)}=\frac{(13)(11)(7)}{(8)(5)(3)(2)}=\frac{1001}{240} \approx 4.2
$$

25. (B) Let $M=a+b i$, so $N=a-b i$. $|a+b i|=|a-b i|=\sqrt{a^{2}+b^{2}}$, so $2 \sqrt{a^{2}+b^{2}}=12 \rightarrow a^{2}+b^{2}=36$. $12=M^{2}+N^{2}=(a+b i)^{2}+(a-b i)^{2}=a^{2}+2 a b i-b^{2}+a^{2}-2 a b i-b^{2}=2 a^{2}-2 b^{2}$, so $a^{2}-b^{2}=6$. $a^{2}-b^{2}+a^{2}+b^{2}=42 \rightarrow a^{2}=21 \rightarrow a=\sqrt{21}$, and by substitution $b=\sqrt{15}$. $\left|\frac{M+N}{M-N}\right|^{2}=\left|\frac{a+b i+a-b i}{a+b i-(a-b i)}\right|^{2}=\left|\frac{2 a}{2 b i}\right|^{2}=\left|\frac{a}{b i}\right|^{2}=\left|\frac{\sqrt{21}}{i \sqrt{15}}\right|^{2}=\frac{7}{5}$.
26. (C) $b^{2}-4 a c=0$, so $(2 k)^{2}-4(1)\left(k^{2}-9 k+9\right)=0 \rightarrow 4 k^{2}-4 k^{2}+36 k-36=0 \rightarrow 36 k=36 \rightarrow k=1$.
27. (C) $5 y^{2}-4 x^{2}=-20 \rightarrow \frac{x^{2}}{5}-\frac{y^{2}}{4}=1$ The hyperbola has a horizontal transverse axis, so the foci are on the $x$-axis. $c=\sqrt{a^{2}+b^{2}} \rightarrow c=3$.
28. (D) $N\left(\frac{70}{100}\right)\left(\frac{140}{100}\right)\left(\frac{75}{100}\right)=\frac{147}{200} N$
29. (C) The equations of the lines are $y=m x+b+13$ and $y=-\frac{x}{m}+b$. Substituting $(6,6)$ yields $-6 m-7=b$ and $6+\frac{6}{m}=b . \quad-6 m-7=6+\frac{6}{m} \rightarrow 6 m^{2}+13 m+6=0 \rightarrow(2 m+3)(3 m+2)=0 \rightarrow m=-\frac{2}{3},-\frac{3}{2}$.

$$
\begin{array}{|l|l|}
\hline \text { Case I: } m=-\frac{2}{3} & \text { Case II: } m=-\frac{3}{2} \\
\hline
\end{array}
$$

$$
-6 m-7=b \rightarrow-6\left(-\frac{2}{3}\right)-7=b \rightarrow b=-3 \quad-6 m-7=b \rightarrow-6\left(-\frac{3}{2}\right)-7=b \rightarrow b=2
$$

$$
y=m x+b+13
$$

$$
y=-\frac{2}{3} x+10 \quad y=\frac{3}{2} x-3
$$

$x$-intercept: $15 \quad \bar{j} \quad x$-intercept: 2
distance between $=15-2=13$

$$
\begin{array}{lll}
y=m x+b+13 & y=-\frac{x}{m}+b \\
y=-\frac{3}{2} x+15 & y=\frac{2}{3} x+2
\end{array}
$$

$x$-intercept: 10
$x$-intercept: -3
30. (A) $f(32)=f(-16)=f(8)=f(-4)=f(2)=f(-1)=2^{-1} . \quad f(9)=2^{9} . \quad f(14)=f(-7)=2^{-7}$.

$$
f(2)=f(-1)=2^{-1} \cdot \quad f(40)=f(-20)=f(10)=f(-5)=2^{-5} \cdot \quad 2^{-1} \cdot 2^{9} \cdot 2^{-7} \cdot 2^{-1} \cdot 2^{-5}=2^{-5}=\frac{1}{32} .
$$

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Question \# 1
$\mathbf{A}=|4-4 i|$
$\mathbf{B}=f(-5)$ if $f(x-7)=2 x^{2}-31 x+104$
$\mathbf{C}!=$ The number of possible distinct arrangements of 23 people around a circular table

Find: $\frac{2|\mathrm{C}-\mathrm{B}|}{\mathrm{A}}$

## Middleton Invitational Algebra II Team (no calculator)

$\mathbf{A}=$ the sum of the coefficients in the expansion of $(39 x-41 y)^{10}$
$\mathbf{B}=\frac{17545 i^{2}-3829 i}{17545-3829 i^{3}}$
$\mathbf{C}=$ the area of the conic section with equation $\frac{(x-9)^{2}}{1024}+\frac{(y+18)^{2}}{1024}=1$
$\mathbf{D}=$ the slope of the line through the points $(68,73) ;(62,55)$

Find: $\left(\frac{\mathrm{BCD}}{\mathrm{A} \pi}\right)^{\mathrm{B}}$

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$$
\begin{aligned}
& \mathbf{A}=3+\frac{2}{1+\frac{5}{3+\frac{2}{1+\frac{5}{3+\ldots}}}} \\
& {\left[\begin{array}{cc}
3 & \mathrm{E} \\
-7 & \mathrm{P}
\end{array}\right]+\left[\begin{array}{cc}
5 & -1 \\
2 \mathrm{P} & \mathrm{E}
\end{array}\right]=\left[\begin{array}{cc}
8 & 6 \\
-35 & \mathbf{C}
\end{array}\right]}
\end{aligned}
$$

Find: $\quad-\mathrm{C}+\mathrm{A}$

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February 18, 2006
Question \# 4

A 60-degree sector of a circle has an area of $24 \pi$. Let $\mathbf{A}=$ the surface area of the figure created when the complete circle is revolved around its diameter.
$\mathbf{B}=$ the volume of a frustum of a right circular cone with radii of 6 units and 12 units, and height of 6 units.
$\mathbf{C}=(p-g)$, if $p$ and $g$ are relatively prime positive integers, $f(2)=-\frac{p}{g}, \quad$ and

$$
f(x)=(3)^{\frac{1}{2}} \sqrt{x^{-3}+x^{-4}}-x^{2}
$$

$\mathbf{D}=|x|$, if $x^{24}-8^{4}=0$
Find: $\quad D\left(\frac{A-B}{C \pi}\right)$

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Question \# 5
$\mathbf{A}=$ the sum of the digits of the result when, $\sum_{x=0}^{6} x^{2}$ is evaluated
$\mathbf{B}=$ the number of real values of x that satisfy $P^{-1}(\mathrm{x})=P(\mathrm{x})$, given that $P(x)=3+\frac{1}{x}$
$\mathbf{C}=x-y+z$, given the system $\left\{\begin{aligned} 3 x+6 y-4 z= & 14 \\ -7 x+2 y+8 z= & 16 \\ -x-3 y-9 z= & -10\end{aligned}\right.$
$\mathbf{D}=$ the value of $x$ for which $4\left(\log _{x} 5+\log _{x} 2\right)+\frac{3 \log 100}{\log x}=10$
Find: $(C)\left(B-\frac{A}{D}\right)$

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February 18, 2006
Question \# 6
$\mathbf{A}=$ the number of trailing zeros at the end of $26!$
$\mathbf{B}=$ the area bounded by the graph of $|x|+|y|=5$ in the Cartesian plane.
$\mathbf{C}=\frac{y}{x}$, given that $3, x, y, 375, \ldots$ is a geometric sequence.
$\mathbf{D}=$ the sum of the roots of $-3 x^{2}+2 x^{3}-7 x^{2}+12=0$.

Find: $A^{\left(\frac{B}{C D}\right)}$

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## February 18, 2006 Question \# 7

$\mathbf{A}=$ the absolute value of $(729 @ 23)$, given that $(\mathrm{d} @ \mathrm{~m})=\mathrm{d}^{\left(\frac{1}{\mathrm{m-17}}\right)}$.
$\mathbf{B}=$ the ratio of an angle to its complement, measured in degrees, if the ratio of the angle to its supplement is $3: 7$

The directrix and the axis-of-symmetry of the parabola described by $x^{2}-6 x-8 y=-25$ intersect at $\left(x_{1}, y_{1}\right)$. Let $\mathbf{C}=x_{1} \bullet y_{1}$.

Find: $A^{-1} B+C$

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$$
R(x)=i \sqrt{x-2}, \text { where } i=\sqrt{-1} . \quad G(x)=2+x^{2}
$$

The interval for which $R=$ a real number, is expressed as $(-\infty, \mathbf{A}]$
Let $\mathbf{B}$ and $\mathbf{C}$ be the roots of $G$.

The domain of the inverse of $G$ is expressed as $[\mathbf{D}, \infty)$.
$\mathbf{E}=$ the non-negative value of x which makes $[R(x)]^{2}=G(x)$ true.
Find: $\frac{B}{C}+A D-E$

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Question \# 9
$\mathbf{A}=$ the area of a right triangle with integral leg lengths, given that the length of the median from the right angle to the hypotenuse is 25 , and one leg's length is less than 15 .
$\mathbf{B}=x^{2}+\frac{36}{x^{2}}$, when $x+\frac{6}{x}=5$.

Find: A+B.

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Algebra II Team (no calculator)
$\mathbf{A}=f(f(f(1)))$, when $f(x)=\left\{\begin{array}{cl}3 x-5 & , x=1 \\ 1+x^{2} & , x<1 \\ x^{3}-1 & , x>1\end{array}\right.$
$\mathbf{B}=$ the eccentricity of the conic section given by the equation below.

$$
\frac{(x+2)^{2}}{4}+\frac{(y-3)^{2}}{225}=1
$$

$\mathbf{C} \pi=$ The area of the conic section given by the equation above.

Find: $\quad B C \sqrt{A+101}$

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$\mathbf{A}=$ the sum of the first 100 natural numbers
$\mathbf{B}=16+8+4+2+\ldots$
$\mathbf{C}=$ the distance from P to the midpoint of the segment $\overline{P E}$, given that $\mathrm{P}:(1,-3), \mathrm{E}:(7,5)$

Find: $\frac{A C^{-1}}{B}$

## Middleton Invitational <br> Algebra II Team (no calculator)

## February 18, 2006 <br> Question \# 12

$\mathbf{A}=i^{20123565644897933213132006}$
$\mathbf{B}=$ the area bounded by the line $y=-x+10$, the x -axis and the y -axis.
$\mathbf{C}=$ the number of points where the graphs of $y=2^{x}$ and $y=x^{2}$ intersect.
$\mathbf{D}=$ the sum of the values of x that make both equations true, if $3 x y=21$ and $2 x^{2}-y^{2}=12$.

Find. $\sqrt{C}\left(\frac{A+C}{B-D}\right)^{D}$

Let each true statement have a value of 2 , and each false statement a value of -3 .

1. $0!=1$.
2. The quotient of the eccentricity of a hyperbola and the eccentricity of a circle is negative.
3. $\pi$ is a real number.
4. $\sqrt{\frac{4}{9}}$ is an irrational number.
5. For any real number $\mathrm{p}, p \bullet \frac{1}{p}=1$ by the identity property of equality.

Let A be the sum of the values of the true statements. Let B be the sum of the values of the false statements. Give the value of $|A-B|$.

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Question \# 14

Let $A^{6}$ be the solution to $\log _{2} x+\log _{4} x=1$.
Let $\frac{\sqrt{2}+1}{\sqrt{3}-1}=\frac{\sqrt{B}+\sqrt{C}+\sqrt{D}+\sqrt{E}}{2}$.
Give the value of $A+B+C+D+E$.

## Middleton Invitational Algebra II Team (no calculator)

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Question \# 15

Let $\frac{4 x+1}{x^{2}-9}=\frac{A}{x-3}+\frac{B}{x+3}$.
Let $\sqrt{7+2 \sqrt{10}}=\sqrt{D}+\sqrt{E}$.
Give the value of $6 A+D+E$.

$$
\begin{aligned}
& \frac{2|\mathrm{C}-\mathrm{B}|}{\mathrm{A}}=\frac{28 * 2}{4 \sqrt{2}} \\
& =7 \sqrt{2}
\end{aligned}
$$

$$
\begin{aligned}
& \left(\frac{\mathrm{BCD}}{\mathrm{~A} \pi}\right)^{\mathrm{B}} \\
& =\left(\frac{-1 * 1024 \pi * 3}{1024 \pi}\right)^{-1} \\
& =-3^{-1} \text { or }-\frac{1}{3}
\end{aligned}
$$

$$
\begin{aligned}
& -\mathrm{C}+\mathrm{A} \\
& =7+\sqrt{15}
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{D}\left(\frac{\mathrm{~A}-\mathrm{B}}{\mathrm{C} \pi}\right) \\
& =\sqrt{2} *\left(\frac{72 \pi}{9 \pi}\right) \\
& =8 \sqrt{2}
\end{aligned}
$$

1. $\mathbf{A}=\sqrt{4^{2}+4^{2}}=4 \sqrt{2}$
$\mathbf{B}=\mathrm{f}(2-7)=\mathrm{f}(5)=2\left(2^{2}\right)-31(2)+104=50$
$\mathbf{C}!=(23-1)!=22!=22$
2. $\mathbf{A}=(39-41)^{10}=2^{10}$ or 1024
$\mathbf{B}=\frac{-17545-3829 \mathrm{i}}{17545+3829 \mathrm{i}}=-1$
C

$$
\begin{gathered}
=\frac{(x-9)^{2}}{1024}+\frac{(y+18)^{2}}{1024}=1, \text { cirle, } \mathrm{r}^{2}=1024, \text { area }=1024 \pi \\
\mathbf{D}=\frac{55-73}{62-68}=\frac{-18}{-6}=3
\end{gathered}
$$

3. $\mathbf{A}=x=3+\frac{2}{1+\frac{5}{x}}, x=3+\frac{2}{\frac{5+x}{x}}, x=3+\frac{2 x}{5+x}$

$$
(x-3)(x+5)=2 x, x^{2}-15=0, x=\sqrt{15}
$$

$\left[\begin{array}{cc}3 & \mathrm{E} \\ -7 & \mathrm{P}\end{array}\right]+\left[\begin{array}{cc}5 & -1 \\ 2 \mathrm{P} & \mathrm{E}\end{array}\right]=\left[\begin{array}{cc}8 & 6 \\ -35 & C\end{array}\right]$

$$
E-1=6, E=7
$$

$$
-7+2 P=-35, P=-14
$$

$$
P+E=7-14=-7=C
$$

4. $\mathbf{A}=\frac{60}{360} * \pi r^{2}=24 \pi, r^{2}=24 * 6, r^{2}=144$

$$
S A=4 \pi r^{2}=4(144)(\pi)=576 \pi
$$

$\mathbf{B}=\frac{1}{3}(6)(\pi)(144+36+12 * 6)=2 \pi(252)=504 \pi$
C
$=\sqrt{3} \sqrt{\frac{1}{8}+\frac{1}{16}}-4=\sqrt{3} * \frac{\sqrt{3}}{4}-4=\frac{3}{4}-\frac{16}{5}=-\frac{13}{4}$.

$$
13-4=9
$$

$\mathbf{D}=x^{24}-8^{4}=0, \mathrm{x}^{24}=2^{12},|x|=\sqrt{2}$

$$
\begin{aligned}
& \text { (C) }\left(B-\frac{A}{D}\right) \\
& (-4)\left(2-\frac{10}{10}\right) \\
& =-4
\end{aligned}
$$

| $\mathrm{A}^{\left(\frac{\mathrm{B}}{\mathrm{CD}}\right)}$ |
| :--- |
| $=6^{\frac{50}{25}}$ |
| $=6^{2}$ or 36 |

$$
\begin{aligned}
& \mathrm{A}^{-1} \mathrm{~B}+\mathrm{C} \\
& \frac{1}{3} * \frac{3}{2} \\
& ={ }^{3}=\frac{1}{2}
\end{aligned}
$$

7. $\mathbf{A}=729 @ 23=729^{\frac{1}{23-17}}=\left(3^{6}\right)^{\frac{1}{6}}=3$.

$$
\mathbf{B}=\begin{aligned}
& \frac{a}{180-a}=\frac{3}{7}, 7 a+3 a=180 * 3 \\
& a=18 * 3=54 \\
& 90-54=36 \\
& \frac{54}{36}=\frac{3}{2}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\mathrm{B}}{\mathrm{C}}+\mathrm{AD}-\mathrm{E} \\
& =\frac{i \sqrt{2}}{-i \sqrt{2}}+4-0 \\
& =-1+4=3
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{A}+\mathrm{B} \\
& =336+13 \\
& =349
\end{aligned}
$$

$$
\begin{aligned}
& B C \sqrt{A+101} \\
& =30\left(\frac{\sqrt{221}}{15}\right)(\sqrt{124+101}) \\
& =30 \sqrt{221}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\mathrm{AC}^{-1}}{\mathrm{~B}} \\
& =\frac{5050}{5 * 32}=\frac{1010}{32} \\
& =\frac{505}{16}
\end{aligned}
$$

$\mathbf{C}=x^{2}-6 x-8 y=-25$. Axis of symmetry, $\mathrm{x}=3$.
Directrix, $y=0$.
$3 * 0=0$
8. $\mathbf{A}=2$
$\mathbf{B}=-i \sqrt{2}$
$\mathbf{C}=i \sqrt{2}$
D $=2$
$\mathbf{E}=0$
9. $\mathbf{A}=$ the median in a right triangle is half the length of the hypotenuse. Therefore, the hypotenuse is 50 , so the triangle must have sides $2(7-24-25)=14-48-50$. The area would be $A=\frac{1}{2}(14)(48)=(14)(24)=336$
$\mathbf{B}=\mathrm{x}+\frac{6}{\mathrm{x}}=5, \mathrm{x}^{2}-5 \mathrm{x}+6=0, \mathrm{x}=-3$ or $\mathrm{x}=-2$.
It turns out that both answer yield the same result. 13.

$$
3(1)-5=-2
$$

10. $\mathbf{A}=1+(-2)^{2}=5$

$$
5^{3}-1=124
$$

$\frac{(x+2)^{2}}{4}+\frac{(y-3)^{2}}{225}=1$
$a=15 \quad \mathrm{~b}=3 \quad \mathrm{c}=\sqrt{221}$
$\mathbf{B}=\frac{c}{a}=\frac{\sqrt{221}}{15}$
$\mathbf{C} \pi=a b \pi=15(2) \pi=30 \pi . \quad 30$

$$
\begin{aligned}
& \hline \hline \text { 11. } \mathbf{A}=\sum_{x=1}^{100} x=\frac{100(101)}{2}=5050 \\
& \mathbf{B}=\sum_{x=0}^{\infty} 16\left(\frac{1}{2}\right)^{x}=\frac{16}{1-\frac{1}{2}}=16 * 2=32 \\
& \mathbf{C}=\text { Midpoint, M: }(4,1) \\
& d=\sqrt{(1+3)^{2}+(4-1)^{2}}=\sqrt{16+9}=5
\end{aligned}
$$

$$
\begin{aligned}
& \sqrt{C}\left(\frac{A+C}{B-D}\right)^{D} \\
& =\sqrt{3}(1) \\
& =\sqrt{3}
\end{aligned}
$$

Find the absolute value of the difference between the true and false statements.
$|(2 * 2)-(3 *(-3))|$
$=|4+9|$
$=13$

$$
\begin{aligned}
& =\mathrm{A}+\mathrm{B}+\mathrm{C}+\mathrm{D}+\mathrm{E} \\
& =16+6+3+2+1 \\
& =28
\end{aligned}
$$

$$
\begin{aligned}
& 6 A+D+E \\
& =13+7 \\
& =20
\end{aligned}
$$

$$
\text { 12. } \begin{aligned}
\mathbf{A} & =-1 \\
\mathbf{B} & =50 \\
\mathbf{C} & =3 \\
\mathbf{D} & =2 \mathrm{x}^{2}-\mathrm{y}^{2}=12, \mathrm{xy}=7, \mathrm{y}=\frac{7}{\mathrm{x}} . \\
& 2 x^{4}-7 x^{2}-12=0 . \text { Let } \mathrm{p}=\mathrm{x}^{2} .
\end{aligned}
$$

can determine that the roots of this equation are opposites, which will yield a sum of zero.
13. True $=2$. False $=-3$.

0 ! Is equal to one. [2]
The quotient of the eccentricity of a hyperbola and the eccentricity of a circle is negative. [-3]
$\pi$ is a real number. [2]
$\sqrt{\frac{4}{9}}$ is an irrational number. [-3]
For any real number $\mathrm{p}, p \bullet \frac{1}{p}=1$ by the identity property of equality. [-3]
14. $\log 2+\frac{\log x}{2 \log 2}=1$
denominator gives $\frac{2 \log x}{2 \log 2}+\frac{\log x}{2 \log 2}=1$
and so $\log \left(x^{3}\right)=\log 4$ and $x^{3}=4, x^{6}=16$. So
$\mathrm{A}=16$.
For part II, multiply by the conjugate to get
$\frac{\sqrt{6}+\sqrt{3}+\sqrt{2}+\sqrt{1}}{2}$.
15.Get a common denominator to get $4 x+1=A(x+3)+B(x-3)$ and set the coefficient of the x's equal to get $\mathrm{A}+\mathrm{B}=4$, and the constants are equal to get $3 \mathrm{~A}-3 \mathrm{~B}=1$. We get $6 \mathrm{~A}=13$.
On part II, square both sides to get $\mathrm{D}+\mathrm{E}=7$ and $\mathrm{DE}=10$ so $D$ and $E$ are 5 and 2 in any order.

