"E. NOTA" denotes "None of the above answers is correct" For all problems, $i = \sqrt{-1}$

1. Find X if $7^{1/3}6^{2/3}5 = X^{1/3}$

A. 31.58 B. 210 C. 31500 D. 61740 E. NOTA

2. Find the infinite sum of the series 1 – 1/2 + 1/4 – 1/8 + 1/16 –...

A. 2/3 B. ln 2 C. 1 D. 2 E. NOTA

3. Solve for the positive real value of Y which satisfies the following equation for every real value of X:

$$Y^{X-3} = 4Y^{X+1}$$

A. 0 B. $\frac{\sqrt{2}}{2}$ C. $\frac{1}{4}$ D. Infinite solutions E. NOTA

4. The sequence log 2, log 4, log 8, log 16, log 32 ... is which of the following types?

A. Arithmetic B. Exponential C. Geometric D. Harmonic E. NOTA

5. Let $f(x) = \frac{13}{x+6}$, $x \neq -6$, and g(x) be the inverse of f(x). Find $f(g(f(8)))$.

A. 182/97 B. 13/14 C. 8 D. –35/8 E. NOTA

6. Which of the numbers in the set $\{\log 2^{2005}, \log 2005^{200}, \log 314^{271}, \log 271^{314}\}$ is the largest?

A. $\log 2^{2005}$ B. $\log 2005^{200}$ C. $\log 314^{271}$ D. $\log 271^{314}$ E. NOTA

7. Given that the area of a triangle with vertices (X_1, Y_1), (X_2, Y_2), and (X_3, Y_3) can be found using the formula

$$A = \pm 1/2 \begin{vmatrix} X_1 & Y_1 & 1 \\ X_2 & Y_2 & 1 \\ X_3 & Y_3 & 1 \end{vmatrix}$$

where the + or – sign is chosen to yield a positive A, find the area of a triangle with vertices (1, 5), (3, 8), and (9, 10).

A. 7 B. 14 C. 43 D. 50 E. NOTA

8. Find $f(1001) - f(1000)$ if $f(x) = 5x - 3 - 4|2 - 3x|$.

A. –7 B. –17 C. –12 D. 7 E. NOTA

9. Find the value of $(x^6 - y^2)^4$ if $x = i$ and $y = 3$

A. –6,560 B. 4096 C. 10,000 D. $6561 - 81i$ E. NOTA

10. If Larry and Moe together weigh 380 lbs, Moe and Curly together weigh 350 lbs, and Larry and Curly together weigh 400 lbs, how much do all three men weigh together?

A. 1130 lbs B. 530 lbs C. 565 lbs D. 575 lbs E. NOTA

11. What is the domain of $f(x) = \dfrac{1}{\ln(x^2 - 4)}$?

A. $[2, \infty)$
B. $(-\infty, -2) \cup (2, \infty)$
C. $(-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$
D. $(-\infty, -\sqrt{5}) \cup (-\sqrt{5}, -2) \cup (2, \sqrt{5}) \cup (\sqrt{5}, \infty)$
E. NOTA

12. Choose the only answer that is *BOTH* irrational *AND* complex.

A. πi B. $\sqrt{3136}$ C. $-e/4$ D. $\sqrt{3} - 2i$ E. NOTA

13. Box A has exactly 3 green balls and 3 red balls in it, as do also Box B and Box C. What is the probability that if we choose 1 ball randomly (all balls have equal chance of being chosen) from each box we will get 1 red ball and 2 green ones (regardless of which box each came from)?

A. 1/3 B. 1/4 C. 5/16 D. 3/8 E. NOTA

14. Let $\log_K (1/t) = A$ (for $t>0$, $K>0$, and $A\neq 0$). Express $\log_t K$ in terms of A.

A. –1/A B. 1/A C. A – 1 D. 1 E. NOTA

15. The roots of $x^3 - 8x^2 + 4x - 32 = 0$ are which of the following?

A. –2, 2 and 8 B. 2, 4 and 8 C. $-2i$, $2i$ and 8 D. $2 + i$, $2 - i$ and 8 E. NOTA

16. Find the value of the determinant as shown:

$$\begin{vmatrix} 2 & 9 & 1 & 4 \\ 0 & 3 & 2 & 1 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 6 \end{vmatrix}$$

A. 12 B. 36 C. 34 D. 0 E. NOTA

17. As x grows to infinity, the graph of $y = \frac{x^3+x^2+x+1}{5x^3+3x+4}$ approaches the line...

A. $y=5$ B. $y=1$ C. $y=4$ D. $y=\frac{1}{5}x$ E. NOTA

18. Find the length of the major axis of the ellipse with a minor axis endpoint at (8, 9) and foci located at (4, 6) and (12, 6).

A. 5 B. 6 C. $2\sqrt{7}$ D. 10 E. NOTA

19. Find the sum of the real solutions to the equation $2x-7\sqrt{x}+6=0$

A. $\frac{7}{2}$ B. 4 C. $\frac{11}{2}$ D. $\frac{9}{4}$ E. NOTA

20. An even function $f(x)$ is characterized by the property $f(x) = f(-x)$ for all x in its domain. An even function is then symmetric about which of the following?

A. The origin B. $x=0$ C. $y=0$ D. $y=x$ E. NOTA

21. The remainder when $(n^2+3n+1)^{2005}$ is divided by n, for all integers $n > 1$, is always...

A. $n-1$ B. 0 C. 1 D. $\frac{n^2+2}{3n}$ E. NOTA

22. How many complex roots does the equation $x^{2005} = 1$ have?

A. 1 B. 2 C. 2004 D. 2005 E. NOTA

23. Find the midpoint of the line segment joining the point (6, 3) and the point on the line $y=2x+1$ to which it is closest.

A. (2, 5) B. (4, 4) C. $(\sqrt{15}, 4)$ D. $(2\sqrt{5}, \sqrt{15})$ E. NOTA

24. Which of the following does not have the line $x = 2$ as an asymptote?

A. $y=\frac{3}{x-2}$ B. $y=\frac{2x^2+2}{x-2}$ C. $y=\frac{\log x}{x-2}$ D. $y=\frac{x^3-8}{x-2}$ E. NOTA

25. Find the following sum ("x" denotes multiplication):

1x1 + 1x2 + 1x3 + 1x4 + 1x5 + 1x6 + 1x7 + 1x8 + 1x9 +
2x1 + 2x2 + 2x3 + 2x4 + 2x5 + 2x6 + 2x7 + 2x8 + 2x9 +
3x1 + 3x2 + 3x3 + 3x4 + 3x5 + 3x6 + 3x7 + 3x8 + 3x9 +
...+
9x1 + 9x2 + 9x3 + 9x4 + 9x5 + 9x6 + 9x7 + 9x8 + 9x9 .

A. 1000 B. 2025 C. 4500 D. 6560 E. NOTA

26. The following three equations have exactly one real solution, n, common to ALL THREE (I wouldn't lie to you):

I. $2x^3 + 15x^2 + 16x - 12 = 0$
II. $6x^3 + 17x^2 + 6x - 8 = 0$
III. $6x^3 + 23x^2 - 73x + 30 = 0$

Find the digit in the hundredths place of e^n.

A. 0 B. 3 C. 4 D. 9 E. NOTA

27. The function $f(x) = (x-1)(x-2)(x-2)(x-3)$ is negative on the interval $x = \ldots$

A. (1,2) only
B. $(1,2) \cup (2,3)$ only
C. (2, 3) only
D. $(-\infty, 1) \cup (2, 3)$ only
E. NOTA

28. A certain function $f(x)$ has the following distribution of values:

x	2	5	6	9
$f(x)$	6	2	9	5
$g(x)$				

If $g(x)$ is the inverse of $f(x)$, which of the following completes the bottom row? (Assume both functions are defined for all real numbers)

A. 2, 5, 6, 9 B. 9, 2, 6, 5 C. 5, 9, 2, 6 D. 9, 6, 5, 2 E. NOTA

29. Combine the following into one fractional expression for $x \neq \pm 1$:

$$\frac{1}{x-1} - \frac{1}{x+1}$$

A. $\frac{2}{x^2-1}$ B. $\frac{2x}{x^2-1}$ C. $\frac{2x+2}{x^2-1}$ D. 0 E. NOTA

30. Find the sum of the first 50 terms of the recursive sequence whose n^{th} term is given by $a_n = a_{n-1} + n$ and whose first term is 1. The formula for the sum of the squares of the first n natural numbers, $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$, might be useful.

A. 22,051 B. 22,100 C. 25,000 D. 27,250 E. NOTA

March Algebra II Regional

1. Let $h(x) = f(g(x))$. Assume f, g, and h exist for all real values of x and each possesses an inverse function defined for all x. Using the values given below, find the value of $h^{-1}(5)$.

$f(5) = 11$	$g(2) = 3$
$f(4) = 2$	$g(7) = 2$
$f(3) = 7$	$g(11) = 4$
$f(2) = 5$	$g(5) = 1$
$f(1) = 8$	$g(8) = 5$

2. A quadratic function $f(x)$ has the following properties:
   a) When $f(x)$ is divided by $x - 4$, the remainder is 10
   b) $f(1) = f(9)$
   c) $f(0) = 16$

Find $f(12)$.

3. Find the value of xyz (with x, y, and z each greater than zero) if

$$16^{\frac{3}{4}} = x^{\frac{3}{7}}$$

$$y \log_3 9 = 3^{\log_9 y}$$

$$z = 4 + 2 + 1 + \frac{1}{2} + \ldots$$

4. Let all of the following be true:

$$A = 2B + 3C - 5$$
$$B = 3A - 3C - 3$$
$$C = -2B - 3A - 7$$

Find the value of A+B+C.

5. Let $f(3x-2) = 1/x$.

$A = f(5)$
B = the value of k where $f(k) = 5$
C = 5 if $f(x)$ is an odd function, 4 if even, 3 if neither

Find the value of ABC.

6. Let A = the coefficient on the x^2 term in the expansion of $(x-4)^4$

Let B = the number of terms in the expansion of $(x^2 + x + 1)^{45}$

Let C = the sum of the coefficients in the expansion of $(3x - y)^8$ (Hint: try substituting some particular values for x and y)

Find A+B+C.

7. Let A = the maximum value of $y = 9 - 4|x^2 - 5|$

Let B = the maximum value of $y = -x^2 + 4x - 7$

Let C = the minimum value of $y = x^6 - 2x^3 + 5$

Find A + B + C.

8. A = the value of $x^2 + y^2$ if $xy = 5$ and $x + y = 16$
B = the distance between the points (6, 3) and (3, 6)
C = the radius of a circle whose area is half that of the circle given by the equation $x^2 + y^2 + 2x - 8y + 16 = 0$
Find ABC.

9. Let $A = \begin{bmatrix} 5 & 3 & 1 \\ 9 & 2 & 2 \\ 0 & x & y \end{bmatrix}$. Let also $A \times \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 20 \\ 27 \\ 33 \end{bmatrix}$ and $|A| = -71$. Find the product xy.

10. Let A = the number of committees of three people able to be formed by choosing from a group of eleven people.
Let B = the number of distinct ways the letters in the word "AARDVARK" can be arranged.
Let C = the number of 4-digit codes able to be formed using the digits 0-9 if each digit can be used only once and the first digit cannot be 0.
Find A+B+C.

11. An ellipse centered at the origin (whose vertices lie on the coordinate axes) contains the points $(5\sqrt{3}, 4)$ and (10, 0).
Let A = the eccentricity of this ellipse.
Let B = the perimeter of any triangle with both foci of the ellipse for two of its vertices and a third vertex anywhere on the ellipse (except the x-axis).
Find AB.

12. Let A = the maximum possible number of intersections of any two distinct quartic polynomials of the form $f(x) = x^4 + ax^3 + bx^2 + cx + d$.
Let B = the maximum number of times any sixth degree polynomial can cross the line $y = 15$.
Let C = the maximum number of non-real roots a polynomial of degree 9 with real coefficients can have.
Let D = the maximum number of times a cubic polynomial can intersect the parabola $y = x^2 + 3x + 1$.
Find ABCD.

13. Let $f(x) = x^3 - 2$ and let $g(x)$ be the inverse of $f(x)$. The graphs of the two functions $f(x)$ and $g(x)$ meet at the point (a, b). Find $\frac{a}{b}$.

14. Let A = log 1 + log 2 + log 3 + log 4 +...+ log 2005
Let B = ln 1 + ln 2 + ln 3 + ln 4 +...+ ln 2005
Let $C = 1 + \frac{e-1}{e} + \frac{(e-1)^2}{e^2} + \frac{(e-1)^3}{e^3} + \ldots$
Find $C^{B/A}$.

15. Find the largest possible value of $Z(X, Y) = 50X + 30Y$ if $(X+Y)$ must be less than or equal to 15, $(X - Y)$ must be no greater than 6, and X can be no greater than 10. ($X \geq 0$, $Y \geq 0$).

Algebra II March Regional

ALGEBRA II INDIVIDUAL KEY	ALGEBRA II TEAM ROUND KEY
1. C	1. 7
2. A	2. 22
3. B	3. 256
4. A	4. -15
5. B	5. -9/5 (or -1.8)
6. D	6. 443
7. A	7. 10
8. A	8. 738
9. C	9. 12
10.C	10. 8061
11.D	11. 96/5 (or 19.2)
12.C	12. 432
13.D	13. 1
14.A	14. 10
15.C	15. 650
16.B	
17.E	
18.D	
19.E	
20.B	
21.C	
22.D	
23.B	
24.D	
25.B	
26.C	
27.B	
28.C	
29.A	
30.B	

Solutions to Algebra II Individual Test March Regional

1. 7^1/3 * 6^2/3 * 5 = 7^1/3 * 36^1/3 * 125^1/3 = 31500^1/3 C
2. a/(1-r) → 1/(1-(-1/2)) = 2/3 A
3. Dividing both sides by Y^{X-3}, we arrive at $1 = 4Y^4$. Hence, $Y = \frac{\sqrt{2}}{2}$. B
4. The sequence is equivalent to log 2, 2log 2, 3log 2, 4log 2... which is arithmetic. A
5. Since *f* and *g* are inverses, *g(f(8))* = 8, and so we only need to find *f*(8) = 13/14. B
6. 2005log2 ~ 604 200log2005 ~ 660 271log314 ~ 677 314log271 ~ 764 Choice D
7. Plug values into formula to get 7 Choice A
8. *f*(1001) – *f*(1000) will be the slope of the function. In the range we're dealing with, *f(x)* can be rewritten as 5*x* – 3 – 4(2-3*x*)*(-1) = 5*x* – 3 + 8 – 12*x* = -7*x* + 5. Slope is –7. A
9. $(i^6 - 3^2)^4$ = (-1-9)^4 = 10,000. C
10. Adding 380+350+400 = 1130, we have counted each man twice → 1130/2 = 565. C
11. First, x^2 – 4 must be positive, so x<-2 or x>2. Also, ln(x^2 – 4) cannot equal 0, which yields $x \neq \pm\sqrt{5}$. Choice D gives the correct domain.
12. All numbers are complex; irrational numbers are a subset of the real numbers. C
13. There are 3 ways for this to happen, and 2*2*2 total outcomes. Probability is 3/8. D
14. $\log_k(1/t) = \log_k t$^-1 = $-\log_k t$, so $\log_k t$ = -A. $1/\log_k t = \log_t k$, so answer is –1/A. A
15. Factor by grouping: x^2(x-8) +4(x-8) = (x-8)(x^2+4)—roots are 8, ±2*i* C
16. Using the expansion algorithm along the first column, we have 2x(determinant of minor) - 0x(det. of minor) + 0x(det. of minor) - 0x(det. of minor). The determinant of the minor works out similarly, so the determinant is the product of the diagonal = 2*3*1*6 = 36. B (There is a theorem that states if a square matrix has all entries 0 to the left of the main diagonal, then the determinant is the product of the entries on the diagonal).
17. y = 1/5 → E
18. The sum of the distances from both foci to a point on the ellipse is always 2a, which is the length of the major axis. The distance from a minor axis endpoint to either focus is the same, so we only need to double the distance between (8,9) and (4,6) → 2*5 = 10. D
19. Substitute $U = \sqrt{x}$. Then equation becomes 2U^2 – 7U + 6 = 0 → (2U-3)(U-2)=0 and hence U = 3/2 or 2. Solving for x in each case, we get 9/4 and 4, sum 25/4. E
20. An even function is symmetric over x=0. B
21. Use remainder theorem→(0^2 + 3*0 + 1)^2005 = 1^2005 = 1. C
22, Every polynomial of degree *n* has exactly *n* complex zeros→2005. D
23. The closest point lies on the line perpendicular to Y = 2X+1 that passes through (6,3). The equation of this line is then y=-(1/2)x+b, and plugging in (6,3), b=6. The intersection of y=2x+1 and y=-(1/2)x+6 is (2,5). Midpoint of (2,5) and (6,3) is (4,4). B
24. D is not an asymptote because (x-2) is a factor of the top, and x=2 is only a hole. D
25. Work backwards from distributive property → $(1+2+3+...+9)^2 = 2025$. B
26. We may treat these equations as a system in 3 variables. Let A = x^3, B = x^2, and C = x, just solve the system to find C = *x* = 1/2, and e^(1/2) ~ 1.6487... Choice C
27. Between 1 and 2, 3 factors are negative, so function is. Between 2 and 3, 1 factor is, so function is. All other points either 0 or an even # of factors negative, so positive B
28. g(f(2)) = 2, so g(6) = 2. Similarly, g(2) = 5, g(5) = 9, g(9) = 6. C
29. (x+1)/(x^2-1) – (x-1)/(x^2-1) = (x+1-x +1)/(x^2-1) = 2/(x^2-1) A
30. The sequence is 1, 3, 6, 10, 15... The second differences are equal, so it can be modeled by a quadratic function ax^2 + bx + c. To make it easier, we will make the sequence begin with the 0th term and find the sum of the 0th through 49th terms. a(0)^2 + b(0) + c = 1, so c = 1. Also, a(1)^2 + b(1) + 1 = 3, so a+b = 2. a(2)^2 + b(2) + 1 = 6, so 4a + 2b = 5. Solving, a=1/2 and b=3/2. After finding the function 1/2x^2 + 3/2x + 1, we need to find the sum $\sum_{x=0}^{49} \frac{1}{2}x^2 + \frac{3}{2}x + 1 = \frac{1}{2}\frac{49(50)(99)}{6} + \frac{3}{2}\frac{49(50)}{2} + 50 = 22100$ B

1. If h(x) = f(g(x)), then go backwards to find $h^{-1}(x) = g^{-1}(f^{-1}(x))$. Then $h^{-1}(5) = g^{-1}(f^{-1}(5)) = g^{-1}(2)$ [since f(2) = 5], and $g^{-1}(2) = \boxed{7}$ [since g(7) = 2]

2. Write $f(x) = a(x-h)^2 + k$. Since b) is true, the parabola is symmetric about x=5, so h = 5. a) tells us *f(4)* = 10 and c) tells us *f(0)* = 16. Plugging in these values, we wind up with a system of equations in two variables (*a* and *k*). Solving, *a* = 1/4 and *k* = 39/4. Then $f(12) = 1/4(12\text{-}5)^2 + 39/4 = \boxed{22}$

3. x = 2^7 = 128 2y = 9^(1/2log₉y) = y^1/2→y = 1/4 z = 4/(1-1/2) = 8 128*1/4*8 = $\boxed{256}$

4. Summing all three, we have A+B+C = $\boxed{-15}$

5. *f*(5) = *f*(3(7/3)-2) = 1/(7/3) = 3/7 A = 3/7
5 = 1/(1/5) = *f*(3(1/5)-2) = *f*(-7/5) = *f*(*k*), so B = -7/5
f(x) = *f*(3((x+2)/3) -2) = 1/((x+2)/3) = 3/(x+2) which is neither odd nor even
Thus, ABC = 3/7*-7/5*3 = $\boxed{-9/5}$

6. A) 4C2*(-4)^2 = 96
B) x^90 will be the term with the largest exponent, and it's easy to see there will be an x^89, an x^88, etc., through to the constant at the end, giving 91 terms
C) Letting x=y=1 will leave the sum of the coefficients in the expansion, so (3*1-1)^8 = 256
Thus, A+B+C = 96+91+256 = $\boxed{443}$

7. A) Minimum is where x^2 - 5 = 0, which does happen, so 9 - 4(0) = 9
B) Find the vertex, (2, -3). The max is then -3.
C) Let x^3 = u, then Y = u^2 - 2u + 5, and it follows the minimum of this is the minimum of the original. The vertex is at (1, 4), so minimum is 4. 9 - 3 + 4 = $\boxed{10}$

8. A) x^2+y^2 = (x + y)^2 – 2xy = 16^2 – 2*5 = 246
B) $3\sqrt{2}$
C) The circle given has radius 1, and since the ratio of the areas is equal to the ratio of the squares of the radii, it follows the ratio of the radii is the square root of the ratio of the areas, or $\frac{\sqrt{2}}{2}$ (since the ratio of the areas is ½). ABC = $246*3\sqrt{2}*\frac{\sqrt{2}}{2} = \boxed{738}$

9. We have 0 + 3*x* + 6*y* = 33 from the multiplication and 10*y*+9*x*–10*x*–27*y* = -17*y*-*x* = -71 from the determinant. Solving, *x=3* and *y=4*, so *xy* = $\boxed{12}$.

10. A = 11C3 = 165 B = 8!/3!/2! = 3360 C = 9*9*8*7 = 4536, and A+B+C = 165+3360+4536 = $\boxed{8061}$

11. Because (10, 0) is on the ellipse, we know a = 10. Substituting the other point in our partial equation *x*^2/100 + *y*^2/b^2 = 1 yields b = 8. Then *e* = c/a = sqrt(a^2 - b^2)/a = 6/10 = 3/5. For part B, recall the definition of an ellipse..."the set of points the sum of

whose distances from two fixed points is a constant". This constant is 2a, and so the 2 sides of the triangle connecting the point on the ellipse to the foci will sum to 2a = 20. The third side will be the distance between the 2 foci, or 2c = 12. So perimeter = 32, and 3/5*32 = **96/5**

12. A) The intersections will occur where $f(x) - g(x) = 0$, and the x^4 term on each will be eliminated, leaving at most a cubic equation with at most 3 real roots, and so the maximum # of intersections is 3. (Students cannot argue ∞ because the functions are said to be distinct)
B) 6
C) Since the non-real roots occur in pairs, there are at most 8
D) Same reasoning as in part A). A cubic minus a quadratic leaves a cubic with at most 3 real roots, so maximum # is 3.
3*6*8*3 = **432**

13. We need only realize inverses are symmetric over y=x, so if the functions intersect, the intersection must be on that line, and it follows a=b, so a/b = **1**. (A quick check of the graphs on a TI-83 shows they do intersect at one place in the first quadrant.)

14. B/A = ln 2005! / log 2005! = ln 2005! / (ln2005!/ln 10) = ln 10
C = 1/(1 - (e - 1)/e) = 1/(1/e) = e C^(B/A) = e^ln 10 = **10**.

15. This is a standard problem where the region of possible x,y values is sketched and the vertices are tested to find the max. Our x and y values must satisfy 1) $x \le 10$, 2) $y \le -x + 15$, 3) $y \ge x - 6$, and x,y positive. This gives us a polygonal region with vertices at (0,0), (0, 15), (6,0), (10, 4), and (10, 5). Plugging these values into Z, we see the max occurs at Z(10,5) = 50*10 + 30*5 = **650**.