ta07-01, ha07-01

One ticket to a show costs \$20 at full price. Susan buys 4 tickets using a coupon that gives her a 25% discount. Pam buys 5 tickets using a coupon that gives her a 30% discount. How many more dollars does Pam pay than Susan?

(A) 2 **(B)** 5 **(C)** 10 **(D)** 15 **(E)** 20

tb07-01, hb07-01

Isabella's house has 3 bedrooms. Each bedroom is 12 feet long, 10 feet wide, and 8 feet high. Isabella must paint the walls of all the bedrooms. Doorways and windows, which will not be painted, occupy 60 square feet in each bedroom. How many square feet of walls must be painted?

(A) 678 **(B)** 768 **(C)** 786 **(D)** 867 **(E)** 876

tb07-02

Define the operation $\star$ by $a \star b = (a+b)b$. What is $(3 \star 5) - (5 \star 3)$?

(A) -16 **(B)** -8 **(C)** 0 **(D)** 8 **(E)** 16

ta07-03, ha07-02

An aquarium has a rectangular base that measures 100 cm by 40 cm and has a height of 50 cm. It is filled with water to a height of 40 cm. A brick with a rectangular base that measures 40 cm by 20 cm and a height of 10 cm is placed in the aquarium. By how many centimeters does the water rise?

(A) 0.5 **(B)** 1 **(C)** 1.5 **(D)** 2 **(E)** 2.5

hb07-04

At Frank's Fruit Market, 3 bananas cost as much as 2 apples, and 6 apples cost as much as 4 oranges. How many oranges cost as much as 18 bananas?

(A) 6 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** 18

ta07-04, ha07-03

The larger of two consecutive odd integers is three times the smaller. What is their sum?

(A) 4 **(B)** 8 **(C)** 12 **(D)** 16 **(E)** 20

ta07-07, ha07-05

Last year Mr. John Q. Public received an inheritance. He paid 20% in federal taxes on the inheritance, and paid 10% of what he had left in state taxes. He paid a total of \$10,500 for both taxes. How many dollars was the inheritance?

(A) 30,000 **(B)** 32,500 **(C)** 35,000 **(D)** 37,500 **(E)** 40,000

tb07-07, hb07-07

All sides of the convex pentagon $ABCDE$ are of equal length, and $\angle A = \angle B = 90°$. What is the degree measure of $\angle E$?

(A) 90 **(B)** 108 **(C)** 120 **(D)** 144 **(E)** 150

ta07-08, ha07-06

Triangles ABC and ADC are isosceles with $AB = BC$ and $AD = DC$. Point D is inside $\triangle ABC$, $\angle ABC = 40°$, and $\angle ADC = 140°$. What is the degree measure of $\angle BAD$?

(A) 20 **(B)** 30 **(C)** 40 **(D)** 50 **(E)** 60

tb07-08

On the trip home from the meeting where this AMC10 was constructed, the Contest Chair noted that his airport parking receipt had digits of the form $bbcac$, where $0 \le a < b < c \le 9$, and b was the average of a and c. How many different five-digit numbers satisfy all these properties?

(A) 12 **(B)** 16 **(C)** 18 **(D)** 20 **(E)** 24

ta07-09

Real numbers a and b satisfy the equations $3^a = 81^{b+2}$ and $125^b = 5^{a-3}$. What is ab?

(A) -60 **(B)** -17 **(C)** 9 **(D)** 12 **(E)** 60

tb07-09

A cryptographic code is designed as follows. The first time a letter appears in a given message it is replaced by the letter that is 1 place to its right in the alphabet (assuming that the letter A is one place to the right of the letter Z). The second time this same letter appears in the given message, it is replaced by the letter that is $1+2$ places to the right, the third time it is replaced by the letter that is $1+2+3$ places to the right, and so on. For example, with this code the word "banana" becomes "cbodqg". What letter will replace the last letter s in the message

"Lee's sis is a Mississippi miss, Chriss!"?

(A) g **(B)** h **(C)** o **(D)** s **(E)** t

tb07-10

Two points B and C are in a plane. Let S be the set of all points A in the plane for which $\triangle ABC$ has area 1. Which of the following describes S?

(A) two parallel lines **(B)** a parabola **(C)** a circle **(D)** a line segment **(E)** two points

ta07-11

The numbers from 1 to 8 are placed at the vertices of a cube in such a manner that the sum of the four numbers on each face is the same. What is this common sum?

(A) 14 **(B)** 16 **(C)** 18 **(D)** 20 **(E)** 24

tb07-12, hb07-08

Tom's age is T years, which is also the sum of the ages of his three children. His age N years ago was twice the sum of their ages then. What is T/N?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

ta07-12

Two tour guides are leading six tourists. The guides decide to split up. Each tourist must choose one of the guides, but with the stipulation that each guide must take at least one tourist. How many different groupings of guides and tourists are possible?

(A) 56 **(B)** 58 **(C)** 60 **(D)** 62 **(E)** 64

ta07-13, ha07-09

Yan is somewhere between his home and the stadium. To get to the stadium he can walk directly to the stadium, or else he can walk home and then ride his bicycle to the stadium. He rides 7 times as fast as he walks, and both choices require the same amount of time. What is the ratio of Yan's distance from his home to his distance from the stadium?

(A) $\frac{2}{3}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{4}{5}$ **(D)** $\frac{5}{6}$ **(E)** $\frac{6}{7}$

tb07-13

Two circles of radius 2 are centered at $(2,0)$ and at $(0,2)$. What is the area of the intersection of the interiors of the two circles?

(A) $\pi - 2$ **(B)** $\frac{\pi}{2}$ **(C)** $\frac{\pi\sqrt{3}}{3}$ **(D)** $2(\pi - 2)$ **(E)** π

ta07-14, ha07-10

A triangle with side lengths in the ratio 3:4:5 is inscribed in a circle of radius 3. What is the area of the triangle?

(A) 8.64 **(B)** 12 **(C)** 5π **(D)** 17.28 **(E)** 18

tb07-14, hb07-10

Some boys and girls are having a car wash to raise money for a class trip to China. Initially 40% of the group are girls. Shortly thereafter two girls leave and two boys arrive, and then 30% of the group are girls. How many girls were initially in the group?

(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

hb07-15

The geometric series $a + ar + ar^2 + \cdots$ has a sum of 7, and the terms involving odd powers of r have a sum of 3. What is $a + r$?

(A) $\frac{4}{3}$ **(B)** $\frac{12}{7}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{7}{3}$ **(E)** $\frac{5}{2}$

tb07-15, hb07-11

The angles of quadrilateral $ABCD$ satisfy $\angle A = 2\angle B = 3\angle C = 4\angle D$. What is the degree measure of $\angle A$, rounded to the nearest whole number?

(A) 125 **(B)** 144 **(C)** 153 **(D)** 173 **(E)** 180

ha07-16

How many three-digit numbers are composed of three distinct digits such that one digit is the average of the other two?

(A) 96 **(B)** 104 **(C)** 112 **(D)** 120 **(E)** 256

hb07-16

Each face of a regular tetrahedron is painted either red, white, or blue. Two colorings are considered indistinguishable if two congruent tetrahedra with those colorings can be rotated so that their appearances are identical. How many distinguishable colorings are possible?

(A) 15 **(B)** 18 **(C)** 27 **(D)** 54 **(E)** 81

tb07-16, hb07-12

A teacher gave a test to a class in which 10% of the students are juniors and 90% are seniors. The average score on the test was 84. The juniors all received the same score, and the average score of the seniors was 83. What score did each of the juniors receive on the test?

(A) 85 **(B)** 88 **(C)** 93 **(D)** 94 **(E)** 98

ha07-17

Suppose that $\sin a + \sin b = \sqrt{5/3}$ and $\cos a + \cos b = 1$. What is $\cos(a - b)$?

(A) $\sqrt{\frac{5}{3}} - 1$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** 1

ta07-17

Suppose that m and n are positive integers such that $75m = n^3$. What is the minimum possible value of $m + n$?

(A) 15 **(B)** 30 **(C)** 50 **(D)** 60 **(E)** 5700

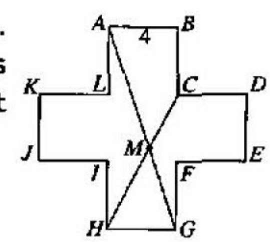
ha07-18

The polynomial $f(x) = x^4 + ax^3 + bx^2 + cx + d$ has real coefficients, and $f(2i) = f(2+i) = 0$. What is $a+b+c+d$?

(A) 0 **(B)** 1 **(C)** 4 **(D)** 9 **(E)** 16

ta07-18

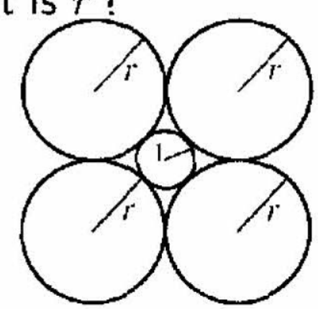
Consider the 12-sided polygon $ABCDEFGHIJKL$, as shown. Each of its sides has length 4, and each two consecutive sides form a right angle. Suppose that $\overline{AG}$ and $\overline{CH}$ meet at M. What is the area of quadrilateral $ABCM$?



(A) $\frac{44}{3}$ **(B)** 16 **(C)** $\frac{88}{5}$ **(D)** 20 **(E)** $\frac{62}{3}$

tb07-18

A circle of radius 1 is surrounded by 4 circles of radius r as shown. What is r?



(A) $\sqrt{2}$ **(B)** $1+\sqrt{2}$ **(C)** $\sqrt{6}$ **(D)** 3 **(E)** $2+\sqrt{2}$

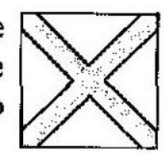
hb07-19

Rhombus $ABCD$, with side length 6, is rolled to form a cylinder of volume 6 by taping $\overline{AB}$ to $\overline{DC}$. What is $\sin(\angle ABC)$?

(A) $\frac{\pi}{9}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{\pi}{6}$ **(D)** $\frac{\pi}{4}$ **(E)** $\frac{\sqrt{3}}{2}$

ta07-19

A paint brush is swept along both diagonals of a square to produce the symmetric painted area, as shown. Half the area of the square is painted. What is the ratio of the side length of the square to the brush width?



(A) $2\sqrt{2}+1$ **(B)** $3\sqrt{2}$ **(C)** $2\sqrt{2}+2$ **(D)** $3\sqrt{2}+1$ **(E)** $3\sqrt{2}+2$

ha07-20

Corners are sliced off a unit cube so that the six faces each become regular octagons. What is the total volume of the removed tetrahedra?

(A) $\frac{5\sqrt{2}-7}{3}$ **(B)** $\frac{10-7\sqrt{2}}{3}$ **(C)** $\frac{3-2\sqrt{2}}{3}$ **(D)** $\frac{8\sqrt{2}-11}{3}$
(E) $\frac{6-4\sqrt{2}}{3}$

hb07-20

The parallelogram bounded by the lines $y = ax + c$, $y = ax + d$, $y = bx + c$, and $y = bx + d$ has area 18. The parallelogram bounded by the lines $y = ax + c$, $y = ax - d$, $y = bx + c$, and $y = bx - d$ has area 72. Given that a, b, c, and d are positive integers, what is the smallest possible value of $a + b + c + d$?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

tb07-20

A set of 25 square blocks is arranged into a 5×5 square. How many different combinations of 3 blocks can be selected from that set so that no two are in the same row or column?

(A) 100 **(B)** 125 **(C)** 600 **(D)** 2300 **(E)** 3600

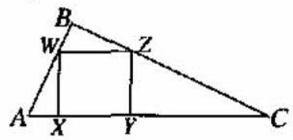
ha07-21

The sum of the zeros, the product of the zeros, and the sum of the coefficients of the function $f(x) = ax^2 + bx + c$ are equal. Their common value must also be which of the following?

(A) the coefficient of x^2 **(B)** the coefficient of x
(C) the y-intercept of the graph of $y = f(x)$
(D) one of the x-intercepts of the graph of $y = f(x)$
(E) the mean of the x-intercepts of the graph of $y = f(x)$

tb07-21

Right $\triangle ABC$ has $AB = 3$, $BC = 4$, and $AC = 5$. Square $XYZW$ is inscribed in $\triangle ABC$ with X and Y on $\overline{AC}$, W on $\overline{AB}$, and Z on $\overline{BC}$. What is the side length of the square?



(A) $\frac{3}{2}$ **(B)** $\frac{60}{37}$ **(C)** $\frac{12}{7}$ **(D)** $\frac{23}{13}$ **(E)** 2

hb07-22

Two particles move along the edges of equilateral $\triangle ABC$ in the direction

$$A \to B \to C \to A,$$

starting simultaneously and moving at the same speed. One starts at A, and the other starts at the midpoint of $\overline{BC}$. The midpoint of the line segment joining the two particles traces out a path that encloses a region R. What is the ratio of the area of R to the area of $\triangle ABC$?

(A) $\frac{1}{16}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{9}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{4}$

tb07-22

A player chooses one of the numbers 1 through 4. After the choice has been made, two regular four-sided (tetrahedral) dice are rolled, with the sides of the dice numbered 1 through 4. If the number chosen appears on the bottom of exactly one die after it is rolled, then the player wins \$1. If the number chosen appears on the bottom of both of the dice, then the player wins \$2. If the number chosen does not appear on the bottom of either of the dice, the player loses \$1. What is the expected return to the player, in dollars, for one roll of the dice?

(A) $-\frac{1}{8}$ **(B)** $-\frac{1}{16}$ **(C)** 0 **(D)** $\frac{1}{16}$ **(E)** $\frac{1}{8}$

ha07-23

Square $ABCD$ has area 36, and $\overline{AB}$ is parallel to the x-axis. Vertices A, B, and C are on the graphs of $y = \log_a x$, $y = 2\log_a x$, and $y = 3\log_a x$, respectively. What is a?

(A) $\sqrt[6]{3}$ **(B)** $\sqrt{3}$ **(C)** $\sqrt[3]{6}$ **(D)** $\sqrt{6}$ **(E)** 6

hb07-23

How many non-congruent right triangles with positive integer leg lengths have areas that are numerically equal to 3 times their perimeters?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 10 **(E)** 12

tb07-23

A pyramid with a square base is cut by a plane that is parallel to its base and is 2 units from the base. The surface area of the smaller pyramid that is cut from the top is half the surface area of the original pyramid. What is the altitude of the original pyramid?

(A) 2 **(B)** $2+\sqrt{2}$ **(C)** $1+2\sqrt{2}$ **(D)** 4 **(E)** $4+2\sqrt{2}$

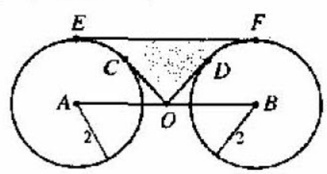
ha07-24

For each integer $n > 1$, let $F(n)$ be the number of solutions of the equation $\sin x = \sin nx$ on the interval $[0, \pi]$. What is $\sum_{n=2}^{2007} F(n)$?

(A) 2,014,524 **(B)** 2,015,028 **(C)** 2,015,033 **(D)** 2,016,532
(E) 2,017,033

ta07-24

Circles centered at A and B each have radius 2, as shown. Point O is the midpoint of $\overline{AB}$, and $OA = 2\sqrt{2}$. Segments OC and OD are tangent to the circles centered at A and B, respectively, and $\overline{EF}$ is a common tangent. What is the area of the shaded region $ECODF$?



(A) $\frac{8\sqrt{2}}{3}$ **(B)** $8\sqrt{2}-4-\pi$ **(C)** $4\sqrt{2}$ **(D)** $4\sqrt{2}+\frac{\pi}{8}$ **(E)** $8\sqrt{2}-2-\frac{\pi}{2}$

tb07-24

Let n denote the smallest positive integer that is divisible by both 4 and 9, and whose base-10 representation consists of only 4's and 9's, with at least one of each. What are the last four digits of n?

(A) 4444 **(B)** 4494 **(C)** 4944 **(D)** 9444 **(E)** 9944

hb07-25

Points A,B,C,D, and E are located in 3-dimensional space with $AB = BC = CD = DE = EA = 2$ and $\angle ABC = \angle CDE = \angle DEA = 90^\circ$. The plane of $\triangle ABC$ is parallel to $\overline{DE}$. What is the area of $\triangle BDE$?

(A) $\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** 2 **(D)** $\sqrt{5}$ **(E)** $\sqrt{6}$