Apollonius and Conic Sections

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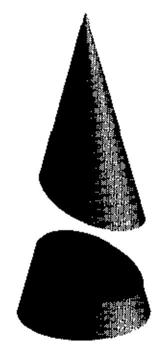
The comet of 1066, as pictured in the Bayeaux Tapestry (ca A.D. 1080) by Matilda of Flanders.

Greek astronomer **Apollonius** (ca 262 B.C.) is best known for his work *Conic Sections,* in which he describes the series of graceful curves formed when a plane surface intersects a cone. Different figures are formed by this intersection, depending on *where* the plane intersects the cone: parallel to the base (a circle), oblique to the base (an ellipse), such that it intersects the base (a parabola), and parallel to the altitude of the cone (a hyperbola)—in this last case the plane also intersects a mirror image of the cone atop the given cone. The Greeks studied the conics out of interest and curiosity, and used them in problems involving geometric constructions.

Today, we know that conic sections are part of the reality that represents and describes our modern world. They are everywhere! We can see this, for example, when we observe the paths of our galaxy's planets. With their finite knowledge of the universe, early scholars probably didn't envision that seventeenth-century mathematicians and scientists would use the conics to represent the paths that projectiles, satellites, planets, and stars follow under the influence of gravity. **Nicholas Copernicus** (1473–1543) thought that our solar system's planets traveled in circular paths, then **Johannes Kepler** (1571–1630) discovered that the ellipse better represents their journey around the sun. **Galileo Galilei** (1564–1642) found that the parabola describes the motions of trajectiles on the earth. (For more on Nicholas Copernicus, Johannes Kepler, and Galileo Galilei, see vignettes 41 and 42.) In 1704, **Edmond Halley** (1656–1743) used data from comets observed in 1456, 1531, 1607, and 1682 to conclude that they represented a single comet orbiting the sun every 76 years in an elliptical path. (Ancient documents suggest the Chinese observed this comet in 240 B.C.) Halley then correctly predicted that this comet named after him would return in 1758.

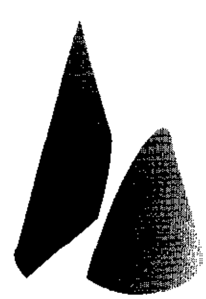
There are many other examples of the presence of conic sections in our world. A sonic boom wave has the shape of a cone, which, as it expands, intersects the ground in a hyperbolic curve. At any given instant, people along the curve are hearing the boom at the same time. Satellite dishes and reflecting telescopes are examples of the parabola. On a "no lose" elliptical pool table, a ball shot through one focus always makes it into the pocket at the other focus. ★

Circle



Ellipse

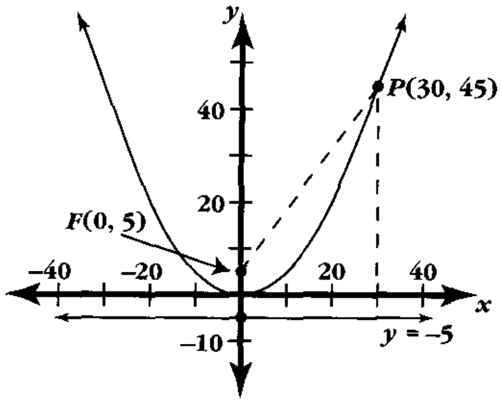
Hyperbola



Parabola

Activities

1. In a plane, a parabola is the set of points equidistant from a line, called the directrix, and a point not on the line, called the focus. The parabola $y = \frac{1}{20}x^2$ with focus (0, 5) and directrix $y = -5$ is shown below. Point $P = (30, 45)$.

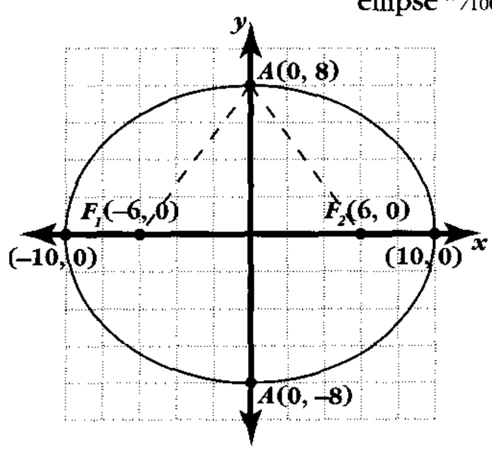


a. What is the distance from point P to the directrix? What is the distance PF?

b. Based on what you discovered in 1a, is point P on the parabola?

c. Do the coordinates of point P satisfy the equation of the parabola? Find two other points that satisfy the equation. In each case, check to see if the points are equidistant from the directrix and the focus.

2. In a plane, an ellipse is the set of points such that the sum of the distances from each point to two fixed points (the foci) is a constant. The ellipse $x^2/100 + y^2/64 = 1$ with foci at (-6, 0) and (6, 0) is shown at left.



a. Consider the point $A(0, 8)$. What is the total distance $AF_1 + AF_2$?

b. Show that the point $P(6, 6.4)$ satisfies the equation of the ellipse.

c. Calculate $PF_1 + PF_2$ and compare the total to that in 2a.

d. Find two other points that satisfy the equation of the ellipse. For each point, calculate the sum of its distances from the foci. Compare your results to those in 2a.

3. Research how parabolic surfaces are used in automobile headlights and in French solar furnaces.

Related Reading

Boyer, Carl. *A History of Mathematics*, 2nd ed rev. Uta C. Merzbach. New York: John Wiley, 1991.

Eves, Howard. *An Introduction to the History of Mathematics*. New York: Holt, Rinehart and Winston, 1990.

Jacobs, Harold. *Mathematics: A Human Endeavor*. San Francisco: W.H. Freeman, 1987.

Pappas, Theoni. *The Joy of Mathematics*. San Carlos, CA: Wide World/Tetra, 1989.

