Authors

Raymond A. Serway, Ph.D.
Professor Emeritus
North Carolina State University

Jerry S. Faughn, Ph.D.
Professor Emeritus
Eastern Kentucky University

On the cover: The large blue image is an X ray of an energy-saving lightbulb. The leftmost, small image is a computer model of a torus-shaped magnet that is holding a hot plasma within its magnetic field, shown here as circular loops. The central small image is of a human eye overlying the visible light portion of the electromagnetic spectrum. The image on the right is of a worker inspecting the coating on a large turbine.
Acknowledgments

Contributing Writers

Robert W. Avakian
Instructor
Trinity School
Midland, Texas

David Bethel
Science Writer
San Lorenzo, New Mexico

David Bradford
Science Writer
Austin, Texas

Robert Davison
Science Writer
Delaware, Ohio

John Jewett Jr., Ph.D.
Professor of Physics
California State Polytechnic University
Pomona, California

Jim Metzner
Seth Madej
Pulse of the Planet radio series
Jim Metzner Productions, Inc.
Yorktown Heights, New York

John M. Stokes
Science Writer
Socorro, New Mexico

Salvatore Tocci
Science Writer
East Hampton, New York

Academic Reviewers

Mary L. Brake, Ph.D.
Physics Teacher
Mercy High School
Farmington Hills, Michigan

Mary L. Brake, Ph.D.
Assistant Professor
Department of Mechanical Engineering and Biomechanics
The University of Texas at San Antonio
San Antonio, Texas

Brad de Young
Professor
Department of Physics and Physical Oceanography
Memorial University
St. John's, Newfoundland, Canada

Bill Deutschmann, Ph.D.
President
Oregon Laser Consultants
Klamath Falls, Oregon

Arthur A. Few
Professor of Space Physics and Environmental Science
Rice University
Houston, Texas

Scott Fricke, Ph.D.
Schlumberger Oilfield Services
Sugarland, Texas

Simonetta Fritelli
Associate Professor of Physics
Duquesne University
Pittsburgh, Pennsylvania

David S. Hall, Ph.D.
Assistant Professor of Physics
Amherst College
Amherst, Massachusetts

Roy W. Hann, Jr., Ph.D.
Professor of Civil Engineering
Texas A & M University College Station, Texas

Sally Hicks, Ph.D.
Professor
Department of Physics
University of Dallas
Irving, Texas

Robert C. Hudson
Associate Professor Emeritus
Physics Department
Roanoke College
Salem, Virginia

William Ingham, Ph.D.
Professor of Physics
James Madison University
Harrisonburg, Virginia

Karen B. Kwitter, Ph.D.
Professor of Astronomy
Williams College
Williamstown, Massachusetts

Phillip LaRoe
Professor of Physics
Helena College of Technology
Helena, Montana

Joseph A. McClure, Ph.D.
Associate Professor Emeritus
Department of Physics
Georgetown University
Washington, DC

Ralph McGrew
Associate Professor
Engineering Science Department
Broome Community College
Binghamton, New York

Clement J. Moses, Ph.D.
Associate Professor of Physics
Utica College
Utica, New York
Acknowledgments, continued

Alvin M. Saperstein, Ph.D.
Professor of Physics; Fellow of Center for Peace and Conflict Studies
Department of Physics and Astronomy
Wayne State University
Detroit, Michigan

Donald E. Simanek, Ph.D.
Emeritus Professor of Physics
Lock Haven University
Lock Haven, Pennsylvania

H. Michael Sommermann, Ph.D.
Professor of Physics
Westmont College
Santa Barbara, California

Jack B. Swift, Ph.D.
Professor
Department of Physics
The University of Texas at Austin
Austin, Texas

Thomas H. Troland, Ph.D.
Physics Department
University of Kentucky
Lexington, Kentucky

Mary L. White
Coastal Ecology Institute
Louisiana State University
Baton Rouge, Louisiana

Jerome Williams M.S.
Professor Emeritus
Oceanography Department
US Naval Academy
Annapolis, MD

Carol J. Zimmerman, Ph.D.
Exxon Exploration Company
Houston, Texas

Teacher Reviewers

John Adamowski
Chairperson of Science Department
Fenton High School
Bensenville, Illinois

John Ahlquist, M.S.
Anoka High School
Anoka, Minnesota

Maurice Belanger
Science Department Head
Nashua High School
Nashua, New Hampshire

Larry G. Brown
Morgan Park Academy
Chicago, Illinois

William K. Conway, Ph.D.
Lake Forest High School
Lake Forest, Illinois

Jack Cooper
Ennis High School
Ennis, Texas

William D. Ellis
Chairman of Science Department
Butler Senior High School
Butler, Pennsylvania

Diego Enciso
Troy, Michigan

Ron Esman
Plano Senior High School
Plano, Texas

Bruce Esser
Marian High School
Omaha, Nebraska

Curtis Goehring
Palm Springs High School
Palm Springs, California

Herbert H. Gottlieb
Science Education Department
City College of New York
New York City, New York

David J. Hamilton, Ed.D.
Benjamin Franklin High School
Portland, Oregon

J. Philip Holden, Ph.D.
Physics Education Consultant
Michigan Dept. of Education
Lansing, Michigan

Joseph Hutchinson
Wichita High School East
Wichita, Kansas

Douglas C. Jenkins
Chairman, Science Department
Warren Central High School
Bowling Green, Kentucky

David S. Jones
Miami Sunset Senior High School
Miami, Florida

Roger Kassebaum
Millard North High School
Omaha, Nebraska

Mervin W. Koehlinger, M.S.
Concordia Lutheran High School
Fort Wayne, Indiana

Phillip LaRoe
Central Community College
Grand Island, Nebraska

William Lash
Westwood High School
Round Rock, Texas

Norman A. Mankins
Science Curriculum Specialist
Canton City Schools
Canton, Ohio

John McGehee
Palos Verdes Peninsula High School
Rolling Hills Estates, California

Debra Schell
Austintown Fitch High School
Austintown, Ohio

Edward Schweber
Solomon Schechter Day School
West Orange, New Jersey

Larry Stookey, P.E.
Science
Antigo High School
Antigo, Wisconsin

Mary R. Yeomans
Hopewell Valley Central High School
Pennington, New Jersey

G. Patrick Zober
Science Curriculum Coordinator
Yough Senior High School
Herminie, Pennsylvania

Patricia J. Zober
Ringgold High School
Monongahela, Pennsylvania

continued on page 973
# Contents

## CHAPTER

<table>
<thead>
<tr>
<th>1</th>
<th>The Science of Physics</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>What is Physics?</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>Measurements in Experiments</td>
<td>10</td>
</tr>
<tr>
<td>The Inside Story</td>
<td>The Mars Climate Orbiter Mission</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>The Language of Physics</td>
<td>21</td>
</tr>
<tr>
<td>Highlights and Review</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>Standardized Test Prep</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>Skills Practice Lab</td>
<td>Physics and Measurement</td>
<td>34</td>
</tr>
</tbody>
</table>

## CHAPTER

<table>
<thead>
<tr>
<th>2</th>
<th>Motion in One Dimension</th>
<th>38</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Displacement and Velocity</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>Acceleration</td>
<td>48</td>
</tr>
<tr>
<td>3</td>
<td>Falling Objects</td>
<td>60</td>
</tr>
<tr>
<td>The Inside Story</td>
<td>Sky Diving</td>
<td>64</td>
</tr>
<tr>
<td>Physics Careers</td>
<td>Science Writer</td>
<td>66</td>
</tr>
<tr>
<td>Highlights and Review</td>
<td>67</td>
<td></td>
</tr>
<tr>
<td>Standardized Test Prep</td>
<td>74</td>
<td></td>
</tr>
<tr>
<td>Skills Practice Lab</td>
<td>Free-Fall Acceleration</td>
<td>76</td>
</tr>
<tr>
<td>CBL™ Lab</td>
<td>Free-Fall Acceleration</td>
<td>932</td>
</tr>
<tr>
<td>Advanced Topics</td>
<td>Angular Kinematics</td>
<td>898</td>
</tr>
<tr>
<td></td>
<td>Relativity and Time Dilation</td>
<td>914</td>
</tr>
</tbody>
</table>

## CHAPTER

<table>
<thead>
<tr>
<th>3</th>
<th>Two-Dimensional Motion and Vectors</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Introduction to Vectors</td>
<td>82</td>
</tr>
<tr>
<td>2</td>
<td>Vector Operations</td>
<td>86</td>
</tr>
<tr>
<td>3</td>
<td>Projectile Motion</td>
<td>95</td>
</tr>
<tr>
<td>4</td>
<td>Relative Motion</td>
<td>102</td>
</tr>
<tr>
<td>Physics Careers</td>
<td>Kinesiology</td>
<td>106</td>
</tr>
<tr>
<td>Highlights and Review</td>
<td>107</td>
<td></td>
</tr>
<tr>
<td>Standardized Test Prep</td>
<td>114</td>
<td></td>
</tr>
<tr>
<td>Inquiry Lab</td>
<td>Velocity of a Projectile</td>
<td>116</td>
</tr>
<tr>
<td>Advanced Topics</td>
<td>Special Relativity and Velocities</td>
<td>916</td>
</tr>
</tbody>
</table>
### CHAPTER 10 Thermodynamics

<table>
<thead>
<tr>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Relationships Between Heat and Work</td>
</tr>
<tr>
<td>2 The First Law of Thermodynamics</td>
</tr>
<tr>
<td>\textbf{The Inside Story} Gasoline Engines</td>
</tr>
<tr>
<td>\textbf{The Inside Story} Refrigerators</td>
</tr>
<tr>
<td>3 The Second Law of Thermodynamics</td>
</tr>
<tr>
<td>\textbf{The Inside Story} Deep-Sea Air Conditioning</td>
</tr>
</tbody>
</table>

### Highlights and Review

- 359

### Standardized Test Prep

- 364

---

### CHAPTER 11 Vibrations and Waves

<table>
<thead>
<tr>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Simple Harmonic Motion</td>
</tr>
<tr>
<td>\textbf{The Inside Story} Shock Absorbers</td>
</tr>
<tr>
<td>2 Measuring Simple Harmonic Motion</td>
</tr>
<tr>
<td>3 Properties of Waves</td>
</tr>
<tr>
<td>4 Wave Interactions</td>
</tr>
</tbody>
</table>

### Highlights and Review

- 395

### Standardized Test Prep

- 400

### Inquiry Lab

- Simple Harmonic Motion of a Pendulum | 402

### Advanced Topics

- De Broglie Waves | 922

---

### Timeline—Physics and Its World: 1785–1830

<table>
<thead>
<tr>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>404</td>
</tr>
</tbody>
</table>

---

### CHAPTER 12 Sound

<table>
<thead>
<tr>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Sound Waves</td>
</tr>
<tr>
<td>\textbf{The Inside Story} Ultrasound Images</td>
</tr>
<tr>
<td>2 Sound Intensity and Resonance</td>
</tr>
<tr>
<td>\textbf{The Inside Story} Hearing Loss</td>
</tr>
<tr>
<td>3 Harmonics</td>
</tr>
<tr>
<td>\textbf{The Inside Story} Reverberation</td>
</tr>
<tr>
<td>\textbf{PHYSICS CAREERS} Piano Tuner</td>
</tr>
</tbody>
</table>

### Highlights and Review

- 433

### Standardized Test Prep

- 438

### Skills Practice Lab

- Speed of Sound | 440

### CBL™ Lab

- Speed of Sound | 938

### Advanced Topics

- The Doppler Effect and the Big Bang | 912

---

### Science, Technology and Society

<table>
<thead>
<tr>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise Pollution</td>
</tr>
</tbody>
</table>
CHAPTER 13 Light and Reflection 444

1 Characteristics of Light ........................................... 446
2 Flat Mirrors ........................................................... 451
3 Curved Mirrors ....................................................... 455
4 Color and Polarization ............................................. 469

Highlights and Review ............................................... 475
Standardized Test Prep .............................................. 482
Skills Practice Lab  Brightness of Light ....................... 484

CHAPTER 14 Refraction 486

1 Refraction .............................................................. 488
2 Thin Lenses ........................................................... 494
   The Inside Story  Cameras ....................................... 504
3 Optical Phenomena ................................................ 506
   The Inside Story  Fiber Optics ................................... 508
   Physics Careers  Optometrist .................................... 512

Highlights and Review ............................................... 513
Standardized Test Prep .............................................. 520
Skills Practice Lab  Converging Lenses ....................... 522

CHAPTER 15 Interference and Diffraction 524

1 Interference ........................................................... 526
2 Diffraction ............................................................. 532
3 Lasers ................................................................. 541
   The Inside Story  Compact Disc Players .................... 544
   Physics Careers  Laser Surgeon ............................... 546

Highlights and Review ............................................... 547
Standardized Test Prep .............................................. 552
Skills Practice Lab  Diffraction .................................... 554

CHAPTER 16 Electric Forces and Fields 556

1 Electric Charge ...................................................... 558
2 Electric Force ........................................................ 564
3 The Electric Field .................................................... 572
   The Inside Story  Microwave Ovens ......................... 579

Highlights and Review ............................................... 580
Standardized Test Prep .............................................. 586
Skills Practice Lab  Electrostatics ............................... 588
Contents

Appendix A Mathematical Review .................................................. 832
Appendix B Downloading Graphing Calculator Programs .................. 847
Appendix C Symbols ................................................................. 848
Appendix D Equations ............................................................... 854
Appendix E SI Units ................................................................. 866
Appendix F Useful Tables .......................................................... 868
Appendix G Periodic Table of the Elements .................................. 872
Appendix H Abbreviated Table of Isotopes and Atomic Masses ......... 874
Appendix I Additional Problems ................................................... 880
Appendix J Advanced Topics ....................................................... 897
   Angular Kinematics ............................................................. 898
   Tangential Speed and Acceleration ........................................ 902
   Rotation and Inertia ............................................................ 904
   Rotational Dynamics .......................................................... 906
   Properties of Gases ............................................................ 908
   Fluid Pressure ................................................................. 910
   The Doppler Effect and the Big Bang ..................................... 912
   Special Relativity and Time Dilation ...................................... 914
   Special Relativity and Velocities .......................................... 916
   The Equivalence of Mass and Energy ..................................... 918
   General Relativity ............................................................. 920
   De Broglie Waves .............................................................. 922
   Electron Tunneling ............................................................. 924
   Semiconductor Doping ......................................................... 926
   Superconductors and BCS Theory ......................................... 928
   Antimatter ........................................................................ 930
Appendix K Selected CBL™ Procedures ......................................... 932
   Free-Fall Acceleration (Chapter 2) ......................................... 932
   Force and Acceleration (Chapter 4) ........................................ 934
   Specific Heat Capacity (Chapter 9) ........................................ 936
   Speed of Sound (Chapter 12) ............................................... 938
   Magnetic Field of a Conducting Wire (Chapter 19) ..................... 940

Selected Answers ........................................................................ 942

Glossary ....................................................................................... 952

Index .......................................................................................... 958
<table>
<thead>
<tr>
<th>Feature Articles</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Mars Climate Orbiter Mission</td>
<td>13</td>
</tr>
<tr>
<td>Sky Diving</td>
<td>64</td>
</tr>
<tr>
<td>Seat Belts</td>
<td>128</td>
</tr>
<tr>
<td>Driving and Friction</td>
<td>142</td>
</tr>
<tr>
<td>The Energy in Food</td>
<td>168</td>
</tr>
<tr>
<td>Surviving a Collision</td>
<td>207</td>
</tr>
<tr>
<td>Black Holes</td>
<td>243</td>
</tr>
<tr>
<td>Climate and Clothing</td>
<td>312</td>
</tr>
<tr>
<td>Earth-Coupled Heat Pumps</td>
<td>316</td>
</tr>
<tr>
<td>Gasoline Engines</td>
<td>348</td>
</tr>
<tr>
<td>Refrigerators</td>
<td>350</td>
</tr>
<tr>
<td>Deep-Sea Air Conditioning</td>
<td>358</td>
</tr>
<tr>
<td>Shock Absorbers</td>
<td>372</td>
</tr>
<tr>
<td>Ultrasound Images</td>
<td>410</td>
</tr>
<tr>
<td>Hearing Loss</td>
<td>421</td>
</tr>
<tr>
<td>Reverberation</td>
<td>429</td>
</tr>
<tr>
<td>Cameras</td>
<td>504</td>
</tr>
<tr>
<td>Fiber Optics</td>
<td>508</td>
</tr>
<tr>
<td>Compact Disc Players</td>
<td>544</td>
</tr>
<tr>
<td>Microwave Ovens</td>
<td>579</td>
</tr>
<tr>
<td>Superconductors</td>
<td>617</td>
</tr>
<tr>
<td>Household Appliance Power Usage</td>
<td>622</td>
</tr>
<tr>
<td>Light Bulbs</td>
<td>643</td>
</tr>
<tr>
<td>Transistors and Integrated Circuits</td>
<td>646</td>
</tr>
<tr>
<td>Decorative Lights and Bulbs</td>
<td>662</td>
</tr>
<tr>
<td>Magnetic Resonance Imaging</td>
<td>683</td>
</tr>
<tr>
<td>Television Screens</td>
<td>688</td>
</tr>
<tr>
<td>Electric Guitar Pickups</td>
<td>715</td>
</tr>
<tr>
<td>Avoiding Electrocution</td>
<td>722</td>
</tr>
<tr>
<td>Radio and TV Broadcasts</td>
<td>734</td>
</tr>
<tr>
<td>Movie Theater Sound</td>
<td>761</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PHYSICS CAREERS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Science Writer</td>
<td>66</td>
</tr>
<tr>
<td>Kinesiologist</td>
<td>106</td>
</tr>
<tr>
<td>Roller Coaster Designer</td>
<td>182</td>
</tr>
<tr>
<td>High School Physics Teacher</td>
<td>221</td>
</tr>
<tr>
<td>HVAC Technician</td>
<td>320</td>
</tr>
<tr>
<td>Piano Tuner</td>
<td>432</td>
</tr>
<tr>
<td>Optometrist</td>
<td>512</td>
</tr>
<tr>
<td>Laser Surgeon</td>
<td>546</td>
</tr>
<tr>
<td>Electrician</td>
<td>624</td>
</tr>
<tr>
<td>Semiconductor Technician</td>
<td>664</td>
</tr>
<tr>
<td>Radiologist</td>
<td>818</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Science • Technology • Society</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Climatic Warming</td>
<td>332</td>
</tr>
<tr>
<td>Noise Pollution</td>
<td>442</td>
</tr>
<tr>
<td>Hybrid Electric Vehicles</td>
<td>636</td>
</tr>
<tr>
<td>Electromagnetic Fields:</td>
<td></td>
</tr>
<tr>
<td>Can They Affect Your Health?</td>
<td>704</td>
</tr>
<tr>
<td>What Can We Do With Nuclear Waste?</td>
<td>828</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Timelines</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Physics and Its World: 1540–1690</td>
<td>156</td>
</tr>
<tr>
<td>Physics and Its World: 1690–1785</td>
<td>294</td>
</tr>
<tr>
<td>Physics and Its World: 1785–1830</td>
<td>404</td>
</tr>
<tr>
<td>Physics and Its World: 1830–1890</td>
<td>748</td>
</tr>
<tr>
<td>Physics and Its World: 1890–1950</td>
<td>786</td>
</tr>
</tbody>
</table>
Safety Symbols

Remember that the safety symbols shown here apply to a specific activity, but the numbered rules on the following pages apply to all laboratory work.

Eye Protection

- Wear safety goggles when working around chemicals, acids, bases, flames or heating devices. Contents under pressure may become projectiles and cause serious injury.
- Never look directly at the sun through any optical device or use direct sunlight to illuminate a microscope.

Clothing Protection

- Secure loose clothing and remove dangling jewelry. Do not wear open-toed shoes or sandals in the lab.
- Wear an apron or lab coat to protect your clothing when you are working with chemicals.

Chemical Safety

- Always wear appropriate protective equipment. Always wear eye goggles, gloves, and a lab apron or lab coat when you are working with any chemical or chemical solution.
- Never taste, touch, or smell chemicals unless your instructor directs you to do so.
- Do not allow radioactive materials to come into contact with your skin, hair, clothing, or personal belongings. Although the materials used in this lab are not hazardous when used properly, radioactive materials can cause serious illness and may have permanent effects.

Electrical Safety

- Do not place electrical cords in walking areas or let cords hang over a table edge in a way that could cause equipment to fall if the cord is accidentally pulled.
- Do not use equipment that has frayed electrical cords or loose plugs.
- Be sure that equipment is in the “off” position before you plug it in.
- Never use an electrical appliance around water or with wet hands or clothing.
- Be sure to turn off and unplug electrical equipment when you are finished using it.
- Never close a circuit until it has been approved by your teacher. Never rewire or adjust any element of a closed circuit.
- If the pointer on any kind of meter moves off scale, open the circuit immediately by opening the switch.
- Do not work with any batteries, electrical devices, or magnets other than those provided by your teacher.

Heating Safety

- Avoid wearing hair spray or hair gel on lab days.
- Whenever possible, use an electric hot plate instead of an open flame as a heat source.
- When heating materials in a test tube, always angle the test tube away from yourself and others.
- Glass containers used for heating should be made of heat-resistant glass.

Sharp Object Safety

- Use knives and other sharp instruments with extreme care.

Hand Safety

- Perform this experiment in a clear area. Attach masses securely. Falling, dropped, or swinging objects can cause serious injury.
- Use a hot mitt to handle resistors, light sources, and other equipment that may be hot. Allow all equipment to cool before storing it.
- To avoid burns, wear heat-resistant gloves whenever instructed to do so.
- Always wear protective gloves when working with an open flame, chemicals, solutions, or wild or unknown plants.
- If you do not know whether an object is hot, do not touch it.
- Use tongs when heating test tubes. Never hold a test tube in your hand to heat the test tube.

Glassware Safety

- Check the condition of glassware before and after using it. Inform your teacher of any broken, chipped, or cracked glassware, because it should not be used.
- Do not pick up broken glass with your bare hands. Place broken glass in a specially designated disposal container.

Waste Disposal

- Clean and decontaminate all work surfaces and personal protective equipment as directed by your instructor.
- Dispose of all broken glass, contaminated sharp objects, and other contaminated materials (biological and chemical) in special containers as directed by your instructor.
Systematic, careful lab work is an essential part of any science program because lab work is the key to progress in science. In this class, you will practice some of the same fundamental laboratory procedures and techniques that experimental physicists use to pursue new knowledge.

The equipment and apparatus you will use involve various safety hazards, just as they do for working physicists. You must be aware of these hazards. Your teacher will guide you in properly using the equipment and carrying out the experiments, but you must also take responsibility for your part in this process. With the active involvement of you and your teacher, these risks can be minimized so that working in the physics laboratory can be a safe, enjoyable process of discovery.

These safety rules always apply in the lab:

1. **Always wear a lab apron and safety goggles.**

   Wear these safety devices whenever you are in the lab, not just when you are working on an experiment.

2. **No contact lenses in the lab.**

   Contact lenses should not be worn during any investigations using chemicals (even if you are wearing goggles). In the event of an accident, chemicals can get behind contact lenses and cause serious damage before the lenses can be removed. If your doctor requires that you wear contact lenses instead of glasses, you should wear eye-cup safety goggles in the lab. Ask your doctor or your teacher how to use this very important and special eye protection.

3. **Personal apparel should be appropriate for laboratory work.**

   On lab days avoid wearing long necklaces, dangling bracelets, bulky jewelry, and bulky or loose-fitting clothing. Loose, flopping, or dangling items may get caught in moving parts, accidentally contact electrical connections, or interfere with the investigation in some potentially hazardous manner. In addition, chemical fumes may react with some jewelry, such as pearl jewelry, and ruin them. Cotton clothing is preferable to clothes made of wool, nylon, or polyester. Tie back long hair. Wear shoes that will protect your feet from chemical spills and falling objects. Do not wear open-toed shoes or sandals or shoes with woven leather straps.

4. **NEVER work alone in the laboratory.**

   Work in the lab only while under the supervision of your teacher. Do not leave equipment unattended while it is in operation.

5. **Only books and notebooks needed for the experiment should be in the lab.**

   Only the lab notebook and perhaps the textbook should be in the lab. Keep other books, backpacks, purses, and similar items in your desk, locker, or designated storage area.

6. **Read the entire experiment before entering the lab.**

   Your teacher will review any applicable safety precautions before the lab. If you are not sure of something, ask your teacher.
7. Heed all safety symbols and cautions written in the experimental investigations and handouts, posted in the room, and given verbally by your teacher.
They are provided for a reason: YOUR SAFETY.

8. Know the proper fire-drill procedures and the locations of fire exits and emergency equipment.
Make sure you know the procedures to follow in case of a fire or emergency.

9. If your clothing catches on fire, do not run; WALK to the safety shower, stand under it, and turn it on.
Call to your teacher while you do this.

10. Report all accidents to the teacher immediately, no matter how minor.
In addition, if you get a headache, feel sick to your stomach, or feel dizzy, tell your teacher immediately.

11. Report all spills to your teacher immediately.
Call your teacher rather than trying to clean a spill yourself. Your teacher will tell you if it is safe for you to clean up the spill; if not, your teacher will know how the spill should be cleaned up safely.

12. Student-designed inquiry investigations, such as the Invention Labs in the Laboratory Experiments manual, must be approved by the teacher before being attempted by the student.

13. DO NOT perform unauthorized experiments or use equipment and apparatus in a manner for which they are not intended.
Use only materials and equipment listed in the activity equipment list or authorized by your teacher. Steps in a procedure should only be performed as described in the book or lab manual or as approved by your teacher.

14. Stay alert in the lab, and proceed with caution.
Be aware of others near you or your equipment when you are about to do something in the lab. If you are not sure of how to proceed, ask your teacher.

15. Horseplay and fooling around in the lab are very dangerous.
Laboratory equipment and apparatus are not toys; never play in the lab or use lab time or equipment for anything other than their intended purpose.

16. Food, beverages, chewing gum, and tobacco products are NEVER permitted in the laboratory.

17. NEVER taste chemicals. Do not touch chemicals or allow them to contact areas of bare skin.

18. Use extreme CAUTION when working with hot plates or other heating devices.
Keep your head, hands, hair, and clothing away from the flame or heating area, and turn the devices off when they are not in use. Remember that metal surfaces connected to the heated area will become hot by conduction. Gas burners should only be lit with a spark lighter. Make sure all heating devices and gas valves are turned off before leaving the laboratory. Never leave a hot plate or other heating device unattended when it is in use. Remember that many metal, ceramic, and glass items do not always look hot when they are hot. Allow all items to cool before storing.

19. Exercise caution when working with electrical equipment.
Do not use electrical equipment with frayed or twisted wires. Be sure your hands are dry before using electrical equipment. Do not let electrical cords dangle from work stations; dangling cords can cause tripping or electrical shocks.

20. Keep work areas and apparatus clean and neat.
Always clean up any clutter made during the course of lab work, rearrange apparatus in an orderly manner, and report any damaged or missing items.

21. Always thoroughly wash your hands with soap and water at the conclusion of each investigation.
How to Use this Textbook

Your Roadmap for Success with *Holt Physics*

**Get Organized**
Read *What to Expect* and *Why It Matters* at the beginning of each chapter to understand what you will learn in the chapter and how it applies to real situations and systems.

**STUDY TIP** Use the *Chapter Preview* outline at the beginning of the chapter to organize your notes on the chapter content in a way that you understand.

**Read for Meaning**
Read the *Section Objectives* at the beginning of each section because they will tell you what you’ll need to learn. Each *Key Term* is highlighted in the text and defined in the margin. After reading each chapter, turn to the *Chapter Highlights* page and review the Key Terms and the *Key Ideas*, which are brief summaries of the chapter’s main concepts. You may want to do this even before you read the chapter.

Use the charts at the bottom of the Chapter Highlights page to review important variable symbols and diagram symbols introduced in the chapter.

**STUDY TIP** If you don’t understand a definition, reread the page on which the term is introduced. The surrounding text should help make the definition easier to understand.

---

**Internet Connect**
boxes in your textbook take you to resources that you can use for science projects, reports, and research papers. Go to *scilinks.org*, and type in the SciLinks code to get information on a topic.

**Visit go.hrw.com**
Find resources and reference materials that go with your textbook at *go.hrw.com*. Enter the keyword *HF6 HOME* to access the home page for your textbook.
Work the Problems

Sample Problems, followed by associated Practice problems, build your reasoning and problem-solving skills by guiding you through explicit example problems.

Prepare for Tests

Section Reviews and Chapter Reviews test your knowledge of the main points of the chapter. Critical Thinking items challenge you to think about the material in different ways and in greater depth. The Standardized Test Prep that is located after each Chapter Review helps you sharpen your test-taking abilities.

STUDY TIP Reread the Section Objectives and Chapter Highlights when studying for a test to be sure you know the material.

Use the Appendix

Your Appendix contains a variety of resources designed to enhance your learning experience. A Mathematical Review sharpens your math skills. The appendices Symbols, Equations, SI Units, and Useful Tables summarize essential problem-solving information. Additional Problems provides more practice in math and problem-solving skills. Advanced Topics allows you to delve deeper into areas of physics that lie beyond material presented in the chapters.

Visit Holt Online Learning

If your teacher gives you a special password to log onto the Holt Online Learning site, you'll find your complete textbook on the Web. In addition, you'll find some great learning tools and practice quizzes. You'll be able to see how well you know the material from your textbook.
The runner in this photograph is participating in sports science research at the National Institute of Sport and Physical Education in France. The athlete is being filmed by a video camera. The white reflective patches enable researchers to generate a computer model from the video, similar to the diagram. Researchers use the model to analyze his technique and to help him improve his performance.

**WHAT TO EXPECT**

In this chapter, you will learn about the branches of physics, the scientific method, and the use of models in physics. You will also learn some useful tools for working with measurements and data.

**WHY IT MATTERS**

Physics develops powerful models that can be used to describe many things in the physical world, including the movements of an athlete in training.

**CHAPTER PREVIEW**

1. **What Is Physics?**
   - The Topics of Physics
   - The Scientific Method

2. **Measurements in Experiments**
   - Numbers as Measurements
   - Accuracy and Precision

3. **The Language of Physics**
   - Mathematics and Physics
   - Evaluating Physics Equations

For advanced project ideas from *Scientific American*, visit [go.hrw.com](http://go.hrw.com) and type in the keyword **HF6SAA**.
What Is Physics?

SECTION 1

THE TOPICS OF PHYSICS

Many people consider physics to be a difficult science that is far removed from their lives. This may be because many of the world’s most famous physicists study topics such as the structure of the universe or the incredibly small particles within an atom, often using complicated tools to observe and measure what they are studying.

But everything around you can be described by using the tools of physics. The goal of physics is to use a small number of basic concepts, equations, and assumptions to describe the physical world. These physics principles can then be used to make predictions about a broad range of phenomena. For example, the same physics principles that are used to describe the interaction between two planets can be used to describe the motion of a satellite orbiting Earth.

Many physicists study the laws of nature simply to satisfy their curiosity about the world we live in. Learning the laws of physics can be rewarding just for its own sake. Also, many of the inventions, appliances, tools, and buildings we live with today are made possible by the application of physics principles. Physics discoveries often turn out to have unexpected practical applications, and advances in technology can in turn lead to new physics discoveries. Figure 1 indicates how the areas of physics apply to building and operating a car.

**SECTION OBJECTIVES**

- Identify activities and fields that involve the major areas within physics.
- Describe the processes of the scientific method.
- Describe the role of models and diagrams in physics.

**Figure 1**

Without knowledge of many of the areas of physics, making cars would be impossible.

- **Thermodynamics** Efficient engines, use of coolants
- **Electromagnetism** Battery, starter, headlights
- **Optics** Headlights, rearview mirrors
- **Mechanics** Spinning motion of the wheels, tires that provide enough friction for traction
- **Vibrations and mechanical waves** Shock absorbers, radio speakers
Physics is everywhere

We are surrounded by principles of physics in our everyday lives. In fact, most people know much more about physics than they realize. For example, when you buy a carton of ice cream at the store and put it in the freezer at home, you do so because from past experience you know enough about the laws of physics to know that the ice cream will melt if you leave it on the counter.

Any problem that deals with temperature, size, motion, position, shape, or color involves physics. Physicists categorize the topics they study in a number of different ways. Table 1 shows some of the major areas of physics that will be described in this book.

People who design, build, and operate sailboats, such as the ones shown in Figure 2, need a working knowledge of the principles of physics. Designers figure out the best shape for the boat’s hull so that it remains stable and floating yet quick-moving and maneuverable. This design requires knowledge of the physics of fluids. Determining the most efficient shapes for the sails and how to arrange them requires an understanding of the science of motion and its causes. Balancing loads in the construction of a sailboat requires knowledge of mechanics. Some of the same physics principles can also explain how the keel keeps the boat moving in one direction even when the wind is from a slightly different direction.

<table>
<thead>
<tr>
<th>Name</th>
<th>Subjects</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanics</td>
<td>motion and its causes, interactions between objects</td>
<td>falling objects, friction, weight, spinning objects</td>
</tr>
<tr>
<td>Thermodynamics</td>
<td>heat and temperature</td>
<td>melting and freezing processes, engines, refrigerators</td>
</tr>
<tr>
<td>Vibrations and wave phenomena</td>
<td>specific types of repetitive motions</td>
<td>springs, pendulums, sound</td>
</tr>
<tr>
<td>Optics</td>
<td>light</td>
<td>mirrors, lenses, color, astronomy</td>
</tr>
<tr>
<td>Electromagnetism</td>
<td>electricity, magnetism, and light</td>
<td>electrical charge, circuitry, permanent magnets, electromagnets</td>
</tr>
<tr>
<td>Relativity</td>
<td>particles moving at any speed, including very high speeds</td>
<td>particle collisions, particle accelerators, nuclear energy</td>
</tr>
<tr>
<td>Quantum mechanics</td>
<td>behavior of submicroscopic particles</td>
<td>the atom and its parts</td>
</tr>
</tbody>
</table>
THE SCIENTIFIC METHOD

When scientists look at the world, they see a network of rules and relationships that determine what will happen in a given situation. Everything you will study in this course was learned because someone looked out at the world and asked questions about how things work.

There is no single procedure that scientists follow in their work. However, there are certain steps common to all good scientific investigations. These steps, called the scientific method, are summarized in Figure 3. This simple chart is easy to understand; but, in reality, most scientific work is not so easily separated. Sometimes, exploratory experiments are performed as a part of the first step in order to generate observations that can lead to a focused question. A revised hypothesis may require more experiments.

Physics uses models that describe phenomena

Although the physical world is very complex, physicists often use models to explain the most fundamental features of various phenomena. Physics has developed powerful models that have been very successful in describing nature. Many of the models currently used in physics are mathematical models. Simple models are usually developed first. It is often easier to study and model parts of a system or phenomenon one at a time. These simple models can then be synthesized into more-comprehensive models.

When developing a model, physicists must decide which parts of the phenomenon are relevant and which parts can be disregarded. For example, let’s say you wish to study the motion of the ball shown in Figure 4. Many observations
can be made about the situation, including the ball’s surroundings, size, spin, weight, color, time in the air, speed, and sound when hitting the ground. The first step toward simplifying this complicated situation is to decide what to study, that is, to define the system. Typically, a single object and the items that immediately affect it are the focus of attention. For instance, suppose you decide to study the ball’s motion in the air (before it potentially reaches any of the other players), as shown in Figure 5(a). To study this situation, you can eliminate everything except information that affects the ball’s motion.

![Figure 5(a)](image)

**Figure 5**
To analyze the basketball’s motion, (a) isolate the objects that will affect its motion. Then, (b) draw a diagram that includes only the motion of the object of interest.

You can disregard characteristics of the ball that have little or no effect on its motion, such as the ball’s color. In some studies of motion, even the ball’s spin and size are disregarded, and the change in the ball’s position will be the only quantity investigated, as shown in Figure 5(b).

In effect, the physicist studies the motion of a ball by first creating a simple model of the ball and its motion. Unlike the real ball, the model object is isolated; it has no color, spin, or size, and it makes no noise on impact. Frequently, a model can be summarized with a diagram, like the one in Figure 5(b). Another way to summarize these models is to build a computer simulation or small-scale replica of the situation.

Without models to simplify matters, situations such as building a car or sailing a boat would be too complex to study. For instance, analyzing the motion of a sailboat is made easier by imagining that the push on the boat from the wind is steady and consistent. The boat is also treated as an object with a certain mass being pushed through the water. In other words, the color of the boat, the model of the boat, and the details of its shape are left out of the analysis. Furthermore, the water the boat moves through is treated as if it were a perfectly smooth-flowing liquid with no internal friction. In spite of these simplifications, the analysis can still make useful predictions of how the sailboat will move.
Models can help build hypotheses

A scientific hypothesis is a reasonable explanation for observations—one that can be tested with additional experiments. The process of simplifying and modeling a situation can help you determine the relevant variables and identify a hypothesis for testing.

Consider the example of Galileo’s “thought experiment,” in which he modeled the behavior of falling objects in order to develop a hypothesis about how objects fell. At the time Galileo published his work on falling objects, in 1638, scientists believed that a heavy object would fall faster than a lighter object.

Galileo imagined two objects of different masses tied together and released at the same time from the same height, such as the two bricks of different masses shown in Figure 6. Suppose that the heavier brick falls faster than the lighter brick when they are separate, as in (a). When tied together, the heavier brick will speed up the fall of the lighter brick somewhat, and the lighter brick will slow the fall of the heavier brick somewhat. Thus, the tied bricks should fall at a rate in between that of either brick alone, as in (b).

However, the two bricks together have a greater mass than the heavier brick alone. For this reason, the tied bricks should fall faster than the heavier brick, as in (c). Galileo used this logical contradiction to refute the idea that different masses fall at different rates. He hypothesized instead that all objects fall at the same rate in the absence of air resistance, as in (d).

Models help guide experimental design

Galileo performed many experiments to test his hypothesis. To be certain he was observing differences due to weight, he kept all other variables the same: the objects he tested had the same size (but different weights) and were measured falling from the same point.

The measuring devices at that time were not precise enough to measure the motion of objects falling in air. So, Galileo used the motion of a ball rolling down a ramp as a model of the motion of a falling ball. The steeper the ramp, the closer the model came to representing a falling object. These ramp experiments provided data that matched the predictions Galileo made in his hypothesis.
Like Galileo’s hypothesis, any hypothesis must be tested in a **controlled experiment**. In an experiment to test a hypothesis, you must change one variable at a time to determine what influences the phenomenon you are observing. Galileo performed a series of experiments using balls of different weights on one ramp before determining the time they took to roll down a steeper ramp.

**The best physics models can make predictions in new situations**

Until the invention of the air pump, it was not possible to perform direct tests of Galileo’s model by observing objects falling in the absence of air resistance. But even though it was not completely testable, Galileo’s model was used to make reasonably accurate predictions about the motion of many objects, from raindrops to boulders (even though they all experience air resistance).

Even if some experiments produce results that support a certain model, at any time another experiment may produce results that do not support the model. When this occurs, scientists repeat the experiment until they are sure that the results are not in error. If the unexpected results are confirmed, the model must be abandoned or revised. That is why the last step of the scientific method is so important. A conclusion is valid only if it can be verified by other people.

---

**SECTION REVIEW**

1. Name the major areas of physics.

2. Identify the area of physics that is most relevant to each of the following situations. Explain your reasoning.
   a. a high school football game
   b. food preparation for the prom
   c. playing in the school band
   d. lightning in a thunderstorm
   e. wearing a pair of sunglasses outside in the sun

3. What are the activities involved in the scientific method?

4. Give two examples of ways that physicists model the physical world.

5. **Critical Thinking** Identify the area of physics involved in each of the following tests of a lightweight metal alloy proposed for use in sailboat hulls:
   a. testing the effects of a collision on the alloy
   b. testing the effects of extreme heat and cold on the alloy
   c. testing whether the alloy can affect a magnetic compass needle

---

**Did you know?**

In addition to conducting experiments to test their hypotheses, scientists also research the work of other scientists. The steps of this type of research include

- identifying reliable sources
- searching the sources to find references
- checking for opposing views
- documenting sources
- presenting findings to other scientists for review and discussion
Measurements in Experiments

SECTION OBJECTIVES

- List basic SI units and the quantities they describe.
- Convert measurements into scientific notation.
- Distinguish between accuracy and precision.
- Use significant figures in measurements and calculations.

NUMBERS AS MEASUREMENTS

Physicists perform experiments to test hypotheses about how changing one variable in a situation affects another variable. An accurate analysis of such experiments requires numerical measurements.

Numerical measurements are different from the numbers used in a mathematics class. In mathematics, a number like 7 can stand alone and be used in equations. In science, measurements are more than just a number. For example, a measurement reported as 7 leads to several questions. What physical quantity is being measured—length, mass, time, or something else? If it is length that is being measured, what units were used for the measurement—meters, feet, inches, miles, or light-years?

The description of what kind of physical quantity is represented by a certain measurement is called dimension. In the next several chapters, you will encounter three basic dimensions: length, mass, and time. Many other measurements can be expressed in terms of these three dimensions. For example, physical quantities such as force, velocity, energy, volume, and acceleration can all be described as combinations of length, mass, and time. In later chapters, we will need to add two other dimensions to our list, for temperature and for electric current.

The description of how much of a physical quantity is represented by a certain numerical measurement depends on the units with which the quantity is measured. For example, small distances are more easily measured in millimeters than in kilometers or light-years.

SI is the standard measurement system for science

When scientists do research, they must communicate the results of their experiments with each other and agree on a system of units for their measurements. In 1960, an international committee agreed on a system of standards, such as the standard shown in Figure 7. They also agreed on designations for the fundamental quantities needed for measurements. This system of units is called the Système International d’Unités (SI). In SI, there are only seven base units. Each base unit describes a single dimension, such as length, mass, or time.

Figure 7
The official standard kilogram mass is a platinum-iridium cylinder kept in a sealed container at the International Bureau of Weights and Measures at Sèvres, France.
The units of length, mass, and time are the meter, kilogram, and second, respectively. In most measurements, these units will be abbreviated as m, kg, and s, respectively.

These units are defined by the standards described in Table 2 and are reproduced so that every meterstick, kilogram mass, and clock in the world is calibrated to give consistent results. We will use SI units throughout this book because they are almost universally accepted in science and industry.

Not every observation can be described using one of these units, but the units can be combined to form derived units. Derived units are formed by combining the seven base units with multiplication or division. For example, speeds are typically expressed in units of meters per second (m/s).

In other cases, it may appear that a new unit that is not one of the base units is being introduced, but often these new units merely serve as shorthand ways to refer to combinations of units. For example, forces and weights are typically measured in units of newtons (N), but a newton is defined as being exactly equivalent to one kilogram multiplied by meters per second squared (1 kg•m/s²). Derived units, such as newtons, will be explained throughout this book as they are introduced.

### SI uses prefixes to accommodate extremes

Physics is a science that describes a broad range of topics and requires a wide range of measurements, from very large to very small. For example, distance measurements can range from the distances between stars (about 100 000 000 000 000 000 m) to the distances between atoms in a solid (0.000 000 001 m). Because these numbers can be extremely difficult to read and write, they are often expressed in powers of 10, such as $1 \times 10^{17}$ m or $1 \times 10^{-9}$ m.

Another approach commonly used in SI is to combine the units with prefixes that symbolize certain powers of 10, as illustrated in Figure 8.

### Table 2: SI Standards

<table>
<thead>
<tr>
<th>Unit</th>
<th>Original standard</th>
<th>Current standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>meter (length)</td>
<td>$\frac{1}{10000000}$ distance from equator to North Pole</td>
<td>the distance traveled by light in a vacuum in $3.33564095 \times 10^{-9}$ s</td>
</tr>
<tr>
<td>kilogram (mass)</td>
<td>mass of 0.001 cubic meters of water</td>
<td>the mass of a specific platinum-iridium alloy cylinder</td>
</tr>
<tr>
<td>second (time)</td>
<td>$(\frac{1}{60})(\frac{1}{60})(\frac{1}{24}) = 0.000\ 011\ 574$ average solar days</td>
<td>9 192 631 770 times the period of a radio wave emitted from a cesium-133 atom</td>
</tr>
</tbody>
</table>

### Did you know?

NIST-FI, an atomic clock at the National Institute of Standards and Technology in Colorado, is one of the most accurate timing devices in the world. NIST-FI is so accurate that it will not gain or lose a second in nearly 20 million years. As a public service, the Institute broadcasts the time given by NIST-FI through the Internet, radio stations WWV and WWVB, and satellite signals.

**Figure 8**

The mass of this mosquito can be expressed several different ways: $1 \times 10^{-5}$ kg, 0.01 g, or 10 mg.
The most common prefixes and their symbols are shown in Table 3. For example, the length of a housefly, $5 \times 10^{-3}$ m, is equivalent to 5 millimeters (mm), and the distance of a satellite $8.25 \times 10^5$ m from Earth’s surface can be expressed as 825 kilometers (km). A year, which is about $3.2 \times 10^7$ s, can also be expressed as 32 megaseconds (Ms).

Converting a measurement from its prefix form is easy to do. You can build conversion factors from any equivalent relationship, including those in Table 3. Just put the quantity on one side of the equation in the numerator and the quantity on the other side in the denominator, as shown below for the case of the conversion $1 \text{ mm} = 1 \times 10^{-3} \text{ m}$. Because these two quantities are equal, the following equations are also true:

\[
\frac{1 \text{ mm}}{10^{-3} \text{ m}} = 1 \quad \text{and} \quad \frac{10^{-3} \text{ m}}{1 \text{ mm}} = 1
\]

Thus, any measurement multiplied by either one of these fractions will be multiplied by 1. The number and the unit will change, but the quantity described by the measurement will stay the same.

To convert measurements, use the conversion factor that will cancel with the units you are given to provide the units you need, as shown in Table 3. Just put the quantity on one side of the equation in the numerator and the quantity on the other side in the denominator, as shown below for the case of the conversion $1 \text{ mm} = 1 \times 10^{-3} \text{ m}$. Because these two quantities are equal, the following equations are also true:

**Units don't cancel:** $37.2 \text{ mm} \times \frac{1 \text{ mm}}{10^{-3} \text{ m}} = 3.72 \times 10^4 \text{ mm}^2 \text{ m}^{-1}$

**Units do cancel:** $37.2 \text{ mm} \times \frac{10^{-3} \text{ m}}{1 \text{ mm}} = 3.72 \times 10^{-2} \text{ m}$

### Table 3

<table>
<thead>
<tr>
<th>Power</th>
<th>Prefix</th>
<th>Abbreviation</th>
<th>Power</th>
<th>Prefix</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-18}$</td>
<td>atto-</td>
<td>a</td>
<td>$10^{-1}$</td>
<td>deci-</td>
<td>d</td>
</tr>
<tr>
<td>$10^{-15}$</td>
<td>femto-</td>
<td>f</td>
<td>$10^{1}$</td>
<td>deka-</td>
<td>da</td>
</tr>
<tr>
<td>$10^{-12}$</td>
<td>pico-</td>
<td>p</td>
<td>$10^{3}$</td>
<td>kilo-</td>
<td>k</td>
</tr>
<tr>
<td>$10^{-9}$</td>
<td>nano-</td>
<td>n</td>
<td>$10^{6}$</td>
<td>mega-</td>
<td>M</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>micro-</td>
<td>µ (Greek letter mu)</td>
<td>$10^{9}$</td>
<td>giga-</td>
<td>G</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>milli-</td>
<td>m</td>
<td>$10^{12}$</td>
<td>tera-</td>
<td>T</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>centi-</td>
<td>c</td>
<td>$10^{15}$</td>
<td>peta-</td>
<td>P</td>
</tr>
<tr>
<td>$10^{0}$</td>
<td></td>
<td></td>
<td>$10^{18}$</td>
<td>exa-</td>
<td>E</td>
</tr>
</tbody>
</table>

The most common prefixes and their symbols are shown in Table 3. For example, the length of a housefly, $5 \times 10^{-3}$ m, is equivalent to 5 millimeters (mm), and the distance of a satellite $8.25 \times 10^5$ m from Earth’s surface can be expressed as 825 kilometers (km). A year, which is about $3.2 \times 10^7$ s, can also be expressed as 32 megaseconds (Ms).

> Converting a measurement from its prefix form is easy to do. You can build conversion factors from any equivalent relationship, including those in Table 3. Just put the quantity on one side of the equation in the numerator and the quantity on the other side in the denominator, as shown below for the case of the conversion $1 \text{ mm} = 1 \times 10^{-3} \text{ m}$. Because these two quantities are equal, the following equations are also true:

\[
\frac{1 \text{ mm}}{10^{-3} \text{ m}} = 1 \quad \text{and} \quad \frac{10^{-3} \text{ m}}{1 \text{ mm}} = 1
\]

Thus, any measurement multiplied by either one of these fractions will be multiplied by 1. The number and the unit will change, but the quantity described by the measurement will stay the same.

To convert measurements, use the conversion factor that will cancel with the units you are given to provide the units you need, as shown in the example below. Typically, the units to which you are converting should be placed in the numerator. It is useful to cross out units that cancel to help keep track of them.

**Units don't cancel:** $37.2 \text{ mm} \times \frac{1 \text{ mm}}{10^{-3} \text{ m}} = 3.72 \times 10^4 \text{ mm}^2 \text{ m}^{-1}$

**Units do cancel:** $37.2 \text{ mm} \times \frac{10^{-3} \text{ m}}{1 \text{ mm}} = 3.72 \times 10^{-2} \text{ m}$
The Mars Climate Orbiter was a NASA spacecraft designed to take pictures of the Martian surface, generate daily weather maps, and analyze the Martian atmosphere from an orbit about 80 km (50 mi) above Mars. It was also supposed to relay signals from its companion, the Mars Polar Lander, which was scheduled to land near the edge of the southern polar cap of Mars shortly after the orbiter arrived.

The orbiter was launched from Cape Canaveral, Florida, on December 11, 1998. Its thrusters were fired several times along the way to direct it along its path. The orbiter reached Mars nine and a half months later, on September 23, 1999. A signal was sent to the orbiter to fire the thrusters a final time in order to push the spacecraft into orbit around the planet. However, the orbiter did not respond to this final signal. NASA soon determined that the orbiter had passed closer to the planet than intended, as close as 60 km (36 mi). The orbiter most likely overheated because of friction in the Martian atmosphere and then passed beyond the planet into space, fatally damaged.

The Mars Climate Orbiter was built by Lockheed Martin in Denver, Colorado, while the mission was run by a NASA flight control team at Jet Propulsion Laboratory in Pasadena, California. Review of the failed mission revealed that engineers at Lockheed Martin sent thrust specifications to the flight control team in English units of pounds of force, while the flight control team assumed that the thrust specifications were in newtons, the SI unit for force. Such a problem normally would be caught by others checking and double-checking specifications, but somehow the error escaped notice until it was too late.

Unfortunately, communication with the Mars Polar Lander was also lost as the lander entered the Martian atmosphere on December 3, 1999. The failure of these and other space exploration missions reveals the inherent difficulty in sending complex technology into the distant, harsh, and often unknown conditions in space and on other planets. However, NASA has had many more successes than failures. A later Mars mission, the Exploration Rover mission, successfully placed two rovers named Spirit and Opportunity on the surface of Mars, where they collected a wide range of data. Among other things, the rovers found convincing evidence that liquid water once flowed on the surface of Mars. Thus, it is possible that Mars supported life sometime in the past.
Both dimension and units must agree

Measurements of physical quantities must be expressed in units that match the dimensions of that quantity. For example, measurements of length cannot be expressed in units of kilograms because units of kilograms describe the dimension of mass. It is very important to be certain that a measurement is expressed in units that refer to the correct dimension. One good technique for avoiding errors in physics is to check the units in an answer to be certain they are appropriate for the dimension of the physical quantity that is being sought in a problem or calculation.

In addition to having the correct dimension, measurements used in calculations should also have the same units. As an example, consider Figure 9(a), which shows two people measuring a room to determine the room’s area. Suppose one person measures the length in meters and the other person measures the width in centimeters. When the numbers are multiplied to find the area, they will give a difficult-to-interpret answer in units of cm•m, as shown in Figure 9(b). On the other hand, if both measurements are made using the same units, the calculated area is much easier to interpret because it is expressed in units of m², as shown in Figure 9(c). Even if the measurements were made in different units, as in the example above, one unit can be easily converted to the other because centimeters and meters are both units of length. It is also necessary to convert one unit to another when working with units from two different systems, such as meters and feet. In order to avoid confusion, it is better to make the conversion to the same units before doing any more arithmetic.
**SAMPLE PROBLEM A**

**Metric Prefixes**

**PROBLEM**

A typical bacterium has a mass of about 2.0 fg. Express this measurement in terms of grams and kilograms.

**SOLUTION**

**Given:** mass = 2.0 fg  
**Unknown:** mass = ? g  
mass = ? kg

Build conversion factors from the relationships given in Table 3. Two possibilities are shown below.

\[
\frac{1 \times 10^{-15} \text{ g}}{1 \text{ fg}} \quad \text{and} \quad \frac{1 \text{ fg}}{1 \times 10^{-15} \text{ g}}
\]

Only the first one will cancel the units of femtograms to give units of grams.

\[
(2.0 \text{ fg}) \left( \frac{1 \times 10^{-15} \text{ g}}{1 \text{ fg}} \right) = 2.0 \times 10^{-15} \text{ g}
\]

Then, take this answer and use a similar process to cancel the units of grams to give units of kilograms.

\[
(2.0 \times 10^{-15} \text{ g}) \left( \frac{1 \text{ kg}}{1 \times 10^3 \text{ g}} \right) = 2.0 \times 10^{-18} \text{ kg}
\]

**PRACTICE A**

**Metric Prefixes**

1. A human hair is approximately 50 µm in diameter. Express this diameter in meters.

2. If a radio wave has a period of 1 µs, what is the wave’s period in seconds?

3. A hydrogen atom has a diameter of about 10 nm.
   a. Express this diameter in meters.
   b. Express this diameter in millimeters.
   c. Express this diameter in micrometers.

4. The distance between the sun and Earth is about \(1.5 \times 10^{11}\) m. Express this distance with an SI prefix and in kilometers.

5. The average mass of an automobile in the United States is about \(1.440 \times 10^6\) g. Express this mass in kilograms.
ACCURACY AND PRECISION

Because theories are based on observation and experiment, careful measurements are very important in physics. But no measurement is perfect. In describing the imperfection, one must consider both a measurement’s **accuracy** and a measurement’s **precision**. Although these terms are often used interchangeably in everyday speech, they have specific meanings in a scientific discussion. A numeric measure of confidence in a measurement or result is known as **uncertainty**. A lower uncertainty indicates greater confidence. Uncertainties are usually expressed by using statistical methods.

**Error in experiments must be minimized**

Experimental work is never free of error, but it is important to minimize error in order to obtain accurate results. An error can occur, for example, if a mistake is made in reading an instrument or recording the results. One way to minimize error from human oversight or carelessness is to take repeated measurements to be certain they are consistent.

If some measurements are taken using one method and some are taken using a different method, a type of error called **method error** will result. Method error can be greatly reduced by standardizing the method of taking measurements. For example, when measuring a length with a meterstick, choose a line of sight directly over what is being measured, as shown in **Figure 10(a)**. If you are too far to one side, you are likely to overestimate or underestimate the measurement, as shown in **Figure 10(b)** and (c).

Another type of error is **instrument error**. If a meterstick or balance is not in good working order, this will introduce error into any measurements made with the device. For this reason, it is important to be careful with lab equipment. Rough handling can damage balances. If a wooden meterstick gets wet, it can warp, making accurate measurements difficult.

---

**Figure 10**

If you measure this window by keeping your line of sight directly over the measurement (a), you will find that it is 165.2 cm long. If you do not keep your eye directly above the mark, as in (b) and (c), you may report a measurement with significant error.

---
Because the ends of a meterstick can be easily damaged or worn, it is best to minimize instrument error by making measurements with a portion of the scale that is in the middle of the meterstick. Instead of measuring from the end (0 cm), try measuring from the 10 cm line.

**Precision describes the limitations of the measuring instrument**

Poor accuracy involves errors that can often be corrected. On the other hand, precision describes how exact a measurement can possibly be. For example, a measurement of 1.325 m is more precise than a measurement of 1.3 m. A lack of precision is typically due to limitations of the measuring instrument and is not the result of human error or lack of calibration. For example, if a meterstick is divided only into centimeters, it will be difficult to measure something only a few millimeters thick with it.

In many situations, you can improve the precision of a measurement. This can be done by making a reasonable estimation of where the mark on the instrument would have been. Suppose that in a laboratory experiment you are asked to measure the length of a pencil with a meterstick marked in centimeters, as shown in Figure 11. The end of the pencil lies somewhere between 18 cm and 18.5 cm. The length you have actually measured is slightly more than 18 cm. You can make a reasonable estimation of how far between the two marks the end of the pencil is and add a digit to the end of the actual measurement. In this case, the end of the pencil seems to be less than halfway between the two marks, so you would report the measurement as 18.2 cm.

**Significant figures help keep track of imprecision**

It is important to record the precision of your measurements so that other people can understand and interpret your results. A common convention used in science to indicate precision is known as **significant figures**.

In the case of the measurement of the pencil as about 18.2 cm, the measurement has three significant figures. The significant figures of a measurement include all the digits that are actually measured (18 cm), plus one estimated digit. Note that the number of significant figures is determined by the precision of the markings on the measuring scale.

The last digit is reported as a 0.2 (for the estimated 0.2 cm past the 18 cm mark). Because this digit is an estimate, the true value for the measurement is actually somewhere between 18.15 cm and 18.25 cm.

When the last digit in a recorded measurement is a zero, it is difficult to tell whether the zero is there as a place holder or as a significant digit. For example, if a length is recorded as 230 mm, it is impossible to tell whether this number has two or three significant digits. In other words, it can be difficult to know whether the measurement of 230 mm means the measurement is known to be between 225 mm and 235 mm or is known more precisely to be between 229.5 mm and 230.5 mm.
One way to solve such problems is to report all values using scientific notation. In scientific notation, the measurement is recorded to a power of 10, and all of the figures given are significant. For example, if the length of 230 cm has two significant figures, it would be recorded in scientific notation as $2.3 \times 10^2$ cm. If it has three significant figures, it would be recorded as $2.30 \times 10^2$ cm.

Scientific notation is also helpful when the zero in a recorded measurement appears in front of the measured digits. For example, a measurement such as 0.000 15 cm should be expressed in scientific notation as $1.5 \times 10^{-4}$ cm if it has two significant figures. The three zeros between the decimal point and the digit 1 are not counted as significant figures because they are present only to locate the decimal point and to indicate the order of magnitude. The rules for determining how many significant figures are in a measurement that includes zeros are shown in Table 4.

**Significant figures in calculations require special rules**

In calculations, the number of significant figures in your result depends on the number of significant figures in each measurement. For example, if someone reports that the height of a mountaintop, like the one shown in Figure 12, is 1710 m, that implies that its actual height is between 1705 and 1715 m. If another person builds a pile of rocks 0.20 m high on top of the mountain, that would not suddenly make the mountain’s new height known accurately enough to be measured as 1710.20 m. The final reported height cannot be more precise than the least precise measurement used to find the answer. Therefore, the reported height should be rounded off to 1710 m even if the pile of rocks is included.

---

**Table 4  Rules for Determining Whether Zeros Are Significant Figures**

<table>
<thead>
<tr>
<th>Rule</th>
<th>Examples</th>
</tr>
</thead>
</table>
| 1. Zeros between other nonzero digits are significant. | a. 50.3 m has three significant figures.  
  b. 3.0025 s has five significant figures. |
| 2. Zeros in front of nonzero digits are not significant. | a. 0.892 kg has three significant figures.  
  b. 0.0008 ms has one significant figure. |
| 3. Zeros that are at the end of a number and also to the right of the decimal are significant. | a. 57.00 g has four significant figures.  
  b. 2.000 000 kg has seven significant figures. |
| 4. Zeros at the end of a number but to the left of a decimal are significant if they have been measured or are the first estimated digit; otherwise, they are not significant. In this book, they will be treated as not significant. (Some books place a bar over a zero at the end of a number to indicate that it is significant. This textbook will use scientific notation for these cases instead.) | a. 1000 m may contain from one to four significant figures, depending on the precision of the measurement, but in this book it will be assumed that measurements like this have one significant figure.  
  b. 20 m may contain one or two significant figures, but in this book it will be assumed to have one significant figure. |
Similar rules apply to multiplication. Suppose that you calculate the area of a room by multiplying the width and length. If the room's dimensions are 4.6 m by 6.7 m, the product of these values would be 30.82 m$^2$. However, this answer contains four significant figures, which implies that it is more precise than the measurements of the length and width. Because the room could be as small as 4.55 m by 6.65 m or as large as 4.65 m by 6.75 m, the area of the room is known only to be between 30.26 m$^2$ and 31.39 m$^2$. The area of the room can have only two significant figures because each measurement has only two. So, the area must be rounded off to 31 m$^2$. Table 5 summarizes the two basic rules for determining significant figures when you are performing calculations.

### Table 5  **Rules for Calculating with Significant Figures**

<table>
<thead>
<tr>
<th>Type of calculation</th>
<th>Rule</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>addition or subtraction</td>
<td>Given that addition and subtraction take place in columns, round the final answer to the first column from the left containing an estimated digit.</td>
<td>$97.3 + 5.85 \rightarrow 103.15 \text{ round off } 103.2$</td>
</tr>
<tr>
<td>multiplication or division</td>
<td>The final answer has the same number of significant figures as the measurement having the smallest number of significant figures.</td>
<td>$123 \times 5.35 \rightarrow 658.05 \text{ round off } 658$</td>
</tr>
</tbody>
</table>

**Calculators do not pay attention to significant figures**

When you use a calculator to analyze problems or measurements, you may be able to save time because the calculator can compute faster than you can. However, the calculator does not keep track of significant figures.

Calculators often exaggerate the precision of your final results by returning answers with as many digits as the display can show. To reinforce the correct approach, the answers to the sample problems in this book will always show only the number of significant figures that the measurements justify.

Providing answers with the correct number of significant figures often requires rounding the results of a calculation. The rules listed in Table 6 on the next page will be used in this book for rounding, and the results of a calculation will be rounded after each type of mathematical operation. For example, the result of a series of multiplications should be rounded using the multiplication/division rule before it is added to another number. Similarly, the sum of several numbers should be rounded according to the addition/subtraction rule before the sum is multiplied by another number. Multiple roundings can increase the error in a calculation, but with this method there is no ambiguity about which rule to apply. You should consult your teacher to find out whether to round this way or to delay rounding until the end of all calculations.
Table 6  Rules for Rounding in Calculations

<table>
<thead>
<tr>
<th>What to do</th>
<th>When to do it</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>round down</td>
<td>• whenever the digit following the last significant figure is a 0, 1, 2, 3, or 4</td>
<td>30.24 becomes 30.2</td>
</tr>
<tr>
<td></td>
<td>• if the last significant figure is an even number and the next digit is a 5, with no other nonzero digits</td>
<td>32.25 becomes 32.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>32.65000 becomes 32.6</td>
</tr>
<tr>
<td>round up</td>
<td>• whenever the digit following the last significant figure is a 6, 7, 8, or 9</td>
<td>22.49 becomes 22.5</td>
</tr>
<tr>
<td></td>
<td>• if the digit following the last significant figure is a 5 followed by a nonzero digit</td>
<td>54.75II becomes 54.8</td>
</tr>
<tr>
<td></td>
<td>• if the last significant figure is an odd number and the next digit is a 5, with no other nonzero digits</td>
<td>54.75 becomes 54.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>79.3500 becomes 79.4</td>
</tr>
</tbody>
</table>

SECTION REVIEW

1. Which SI units would you use for the following measurements?
   a. the length of a swimming pool
   b. the mass of the water in the pool
   c. the time it takes a swimmer to swim a lap

2. Express the following measurements as indicated.
   a. 6.20 mg in kilograms
   b. $3 \times 10^{-9}$ s in milliseconds
   c. 88.0 km in meters

3. Perform these calculations, following the rules for significant figures.
   a. $26 \times 0.02584 = ?$
   b. $15.3 + 1.1 = ?$
   c. $782.45 - 3.5328 = ?$
   d. $63.258 + 734.2 = ?$

4. Critical Thinking  The following students measure the density of a piece of lead three times. The density of lead is actually 11.34 g/cm$^3$. Considering all of the results, which person’s results were accurate? Which were precise? Were any both accurate and precise?
   a. Rachel: 11.32 g/cm$^3$, 11.35 g/cm$^3$, 11.33 g/cm$^3$
   b. Daniel: 11.43 g/cm$^3$, 11.44 g/cm$^3$, 11.42 g/cm$^3$
   c. Leah: 11.55 g/cm$^3$, 11.34 g/cm$^3$, 11.04 g/cm$^3$
The Language of Physics

MATHEMATICS AND PHYSICS

Just as physicists create simplified models to better understand the real world, they use the tools of mathematics to analyze and summarize their observations. Then they can use the mathematical relationships among physical quantities to help predict what will happen in new situations.

Tables, graphs, and equations can make data easier to understand

There are many ways to organize data. Consider the experiment shown in Figure 13, which tests Galileo’s hypothesis that all objects fall at the same rate in the absence of air resistance (see Section 2). In this experiment, a table-tennis ball and a golf ball are dropped in a vacuum. The results are recorded as a set of numbers corresponding to the times of the fall and the distance each ball falls. A convenient way to organize the data is to form a table like Table 7. A clear trend can be seen in the data. The more time that passes after each ball is dropped, the farther the ball falls.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Distance golf ball falls (cm)</th>
<th>Distance table-tennis ball falls (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.067</td>
<td>2.20</td>
<td>2.20</td>
</tr>
<tr>
<td>0.133</td>
<td>8.67</td>
<td>8.67</td>
</tr>
<tr>
<td>0.200</td>
<td>19.60</td>
<td>19.59</td>
</tr>
<tr>
<td>0.267</td>
<td>34.93</td>
<td>34.92</td>
</tr>
<tr>
<td>0.333</td>
<td>54.34</td>
<td>54.33</td>
</tr>
<tr>
<td>0.400</td>
<td>78.40</td>
<td>78.39</td>
</tr>
</tbody>
</table>

One method for analyzing the data in Table 7 is to construct a graph of the distance the balls have fallen versus the elapsed time since they were released. This graph is shown in Figure 14 on the next page. Because the graph shows an obvious pattern, we can draw a smooth curve through the data points to make estimations for times when we have no data. The shape of the graph also provides information about the relationship between time and distance.

Figure 13
This experiment tests Galileo’s hypothesis by having two balls with different masses dropped simultaneously in a vacuum.
We can also use the following equation to describe the relationship between the variables in the experiment:

\[(\text{change in position in meters}) = 4.9 \times (\text{time of fall in seconds})^2\]

This equation allows you to reproduce the graph and make predictions about the change in position for any arbitrary time during the fall.

**Physics equations describe relationships**

While mathematicians use equations to describe relationships between variables, physicists use the tools of mathematics to describe measured or predicted relationships between physical quantities in a situation. For example, one or more variables may affect the outcome of an experiment. In the case of a prediction, the physical equation is a compact statement based on a model of the situation. It shows how two or more variables are thought to be related. Many of the equations in physics represent a simple description of the relationship between physical quantities.

To make expressions as simple as possible, physicists often use letters to describe specific quantities in an equation. For example, the letter \(v\) is used to denote speed. Sometimes, Greek letters are used to describe mathematical operations. For example, the Greek letter \(\Delta\) (delta) is often used to mean “difference or change in,” and the Greek letter \(\Sigma\) (sigma) is used to mean “sum” or “total.”

With these conventions, the word equation above can be written as follows:

\[\Delta y = 4.9(\Delta t)^2\]

The abbreviation \(\Delta y\) indicates the vertical change in a ball’s position from its starting point, and \(\Delta t\) indicates the time elapsed.

As you saw in Section 2, the units in which these quantities are measured are also often abbreviated with symbols consisting of a letter or two. Most physics books provide some clues to help you keep track of which letters refer to quantities and variables and which letters are used to indicate units. Typically, variables and other specific quantities are abbreviated with letters that are
boldfaced or italicized. (You will learn the difference between the two in the chapter “Two-Dimensional Motion and Vectors.”) Units are abbreviated with regular letters (sometimes called roman letters). Some examples of variable symbols and the abbreviations for the units that measure them are shown in Table 8.

As you continue to study physics, carefully note the introduction of new variable quantities, and recognize which units go with them. The tables provided in Appendices C–E can help you keep track of these abbreviations.

<table>
<thead>
<tr>
<th>Table 8 Abbreviations for Variables and Units</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quantity</strong></td>
</tr>
<tr>
<td>change in vertical position</td>
</tr>
<tr>
<td>time interval</td>
</tr>
<tr>
<td>mass</td>
</tr>
</tbody>
</table>

EVALUATING PHYSICS EQUATIONS

Like most models physicists build to describe the world around them, physics equations are valid only if they can be used to make correct predictions about situations. Although an experiment is the ultimate way to check the validity of a physics equation, several techniques can be used to evaluate whether an equation or result can possibly be valid.

Dimensional analysis can weed out invalid equations

Suppose a car, such as the one in Figure 15, is moving at a speed of 88 km/h and you want to know how much time it will take it to travel 725 km. How can you decide a good way to solve the problem?

You can use a powerful procedure called *dimensional analysis*. Dimensional analysis makes use of the fact that *dimensions can be treated as algebraic quantities*. For example, quantities can be added or subtracted only if they have the same dimensions, and the two sides of any given equation must have the same dimensions.

Let us apply this technique to the problem of the car moving at a speed of 88 km/h. This measurement is given in dimensions of length over time. The total distance traveled has the dimension of length. Multiplying these numbers together gives the dimensions indicated below. Clearly, the result of this calculation does not have the dimensions of time, which is what you are trying to calculate. That is,

$$\frac{\text{length}}{\text{time}} \times \text{length} = \frac{\text{length}^2}{\text{time}} \quad \text{or} \quad \frac{88 \text{ km}}{1.0 \text{ h}} \times 725 \text{ km} = \frac{6.4 \times 10^4 \text{ km}^2}{1.0 \text{ h}}$$

Figure 15

Dimensional analysis can be a useful check for many types of problems, including those involving how much time it would take for this car to travel 725 km if it moves with a speed of 88 km/h.
To calculate an answer that will have the dimension of time, you should take the distance and divide it by the speed of the car, as follows:

\[
\frac{\text{length}}{\text{length/time}} = \frac{\text{length} \times \text{time}}{\text{length}} = \text{time} \quad \frac{725 \text{ km} \times 1.0 \text{ h}}{88 \text{ km}} = 8.2 \text{ h}
\]

In a simple example like this one, you might be able to identify the invalid equation without dimensional analysis. But with more-complicated problems, it is a good idea to check your final equation with dimensional analysis. This step will prevent you from wasting time computing an invalid equation.

**Order-of-magnitude estimations check answers**

Because the scope of physics is so wide and the numbers may be astronomically large or subatomically small, it is often useful to estimate an answer to a problem before trying to solve the problem exactly. This kind of estimate is called an order-of-magnitude calculation, which means determining the power of 10 that is closest to the actual numerical value of the quantity. Once you have done this, you will be in a position to judge whether the answer you get from a more exacting procedure is correct.

For example, consider the car trip described in the discussion of dimensional analysis. We must divide the distance by the speed to find the time. The distance, 725 km, is closer to \(10^3 \text{ km}\) (or 1000 km) than to \(10^2 \text{ km}\) (or 100 km), so we use \(10^3 \text{ km}\). The speed, 88 km/h, is about \(10^2 \text{ km/h}\) (or 100 km/h).

\[
\frac{10^3 \text{ km}}{10^2 \text{ km/h}} = 10 \text{ h}
\]

This estimate indicates that the answer should be closer to 10 than to 1 or to 100 (or \(10^2\)). The correct answer (8.2 h) certainly fits this range.

Order-of-magnitude estimates can also be used to estimate numbers in situations in which little information is given. For example, how could you estimate how many gallons of gasoline are used annually by all of the cars in the United States?

First, consider that the United States has almost 300 million people. Assuming that each family of about five people has two cars, an estimate of the number of cars in the country is 120 million.

Next, decide the order of magnitude of the average distance each car travels every year. Some cars travel as few as 1000 mi per year, while others travel more than 100 000 mi per year. The appropriate order of magnitude to include in the estimate is \(10^4 \text{ mi}\), or \(10^4 \text{ mi}\), per year.

If we assume that cars average 20 mi for every gallon of gas, each car needs about 500 gal per year.

\[
\left( \frac{10 000 \text{ mi}}{1 \text{ year}} \right) \left( \frac{1 \text{ gal}}{20 \text{ mi}} \right) = 500 \text{ gal/year for each car}
\]
Multiplying this by the estimate of the total number of cars in the United States gives an annual consumption of 6 × 10^{10} gal.

\[(12 \times 10^7 \text{ cars}) \left(\frac{500 \text{ gal}}{1 \text{ car}}\right) = 6 \times 10^{10} \text{ gal}\]

Note that this estimate depends on the assumptions made about the average household size, the number of cars per household, the distance traveled, and the average gas mileage.

### SECTION REVIEW

1. Indicate which of the following physics symbols denote units and which denote variables or quantities.
   - C  
   - c  
   - C  
   - t  
   - T  
   - T

2. Determine the units of the quantity described by each of the following combinations of units:
   - kg (m/s) (1/s)
   - (kg/s) (m/s^2)
   - (kg/s) (m/s)^2
   - (kg/s) (m/s)

3. Which of the following is the best order-of-magnitude estimate in meters of the height of a mountain?
   - 1 m  
   - 10 m  
   - 100 m  
   - 1000 m

4. **Interpreting Graphics** Which graph best matches the data?

<table>
<thead>
<tr>
<th>Volume of air (m^3)</th>
<th>Mass of air (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>0.644</td>
</tr>
<tr>
<td>1.50</td>
<td>1.936</td>
</tr>
<tr>
<td>2.25</td>
<td>2.899</td>
</tr>
<tr>
<td>4.00</td>
<td>5.159</td>
</tr>
<tr>
<td>5.50</td>
<td>7.096</td>
</tr>
</tbody>
</table>

5. **Critical Thinking** Which of the following equations best matches the data from item 4?
   - \((\text{mass})^2 = 1.29 \times (\text{volume})\)
   - \((\text{mass})(\text{volume}) = 1.29\)
   - \(\text{mass} = 1.29 \times (\text{volume})\)
   - \(\text{mass} = 1.29 \times (\text{volume})^2\)
KEY IDEAS

Section 1 What Is Physics?
• Physics is the study of the physical world, from motion and energy to light and electricity.
• Physics uses the scientific method to discover general laws that can be used to make predictions about a variety of situations.
• A common technique in physics for analyzing a complex situation is to disregard irrelevant factors and create a model that describes the essence of a system or situation.

Section 2 Measurements in Experiments
• Physics measurements are typically made and expressed in SI, a system that uses a set of base units and prefixes to describe measurements of physical quantities.
• **Accuracy** describes how close a measurement is to reality. **Precision** results from the limitations of the measuring device used.
• The significant figures of a measurement include all of the digits that are actually measured plus one estimated digit.
• Significant-figure rules provide a means to ensure that calculations do not report results that are more precise than the data used to make them.

Section 3 The Language of Physics
• Physicists make their work easier by summarizing data in tables and graphs and by abbreviating quantities in equations.
• Dimensional analysis can help identify whether a physics equation is invalid.
• Order-of-magnitude calculations provide a quick way to evaluate the appropriateness of an answer.

### Variable Symbols

<table>
<thead>
<tr>
<th>Quantities</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta y$</td>
<td>m (meters)</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>s (seconds)</td>
</tr>
<tr>
<td>$m$</td>
<td>kg (kilograms)</td>
</tr>
</tbody>
</table>

If you need more problem-solving practice, see Appendix I: Additional Problems.
Review Questions

1. Refer to Table 1 of this chapter to identify at least two areas of physics involved in the following:
   a. building a louder stereo system in your car
   b. bungee jumping
   c. judging how hot an electric stove burner is by looking at it
   d. cooling off on a hot day by diving into a swimming pool

2. Which of the following scenarios fit the approach of the scientific method?
   a. An auto mechanic listens to how a car runs and comes up with an idea of what might be wrong. The mechanic tests the idea by adjusting the idle speed. Then the mechanic decides his idea was wrong based on this evidence. Finally, the mechanic decides the only other problem could be the fuel pump, and he consults with the shop’s other mechanics about his conclusion.
   b. Because of a difference of opinions about where to take the class trip, the class president holds an election. The majority of the students decide to go to the amusement park instead of to the shore.
   c. Your school’s basketball team has advanced to the regional playoffs. A friend from another school says their team will win because their players want to win more than your school’s team does.
   d. A water fountain does not squirt high enough. The handle on the fountain seems loose, so you try to push the handle in as you turn it. When you do this, the water squirts high enough that you can get a drink. You make sure to tell all your friends how you did it.

3. You have decided to select a new car by using the scientific method. How might you proceed?

4. Consider the phrase, “The quick brown fox jumped over the lazy dog.” Which details of this situation would a physicist who is modeling the path of a fox ignore?

SI Units

Review Questions

5. List an appropriate SI base unit (with a prefix as needed) for measuring the following:
   a. the time it takes to play a CD in your stereo
   b. the mass of a sports car
   c. the length of a soccer field
   d. the diameter of a large pizza
   e. the mass of a single slice of pepperoni
   f. a semester at your school
   g. the distance from your home to your school
   h. your mass
   i. the length of your physics lab room
   j. your height

6. If you square the speed expressed in meters per second, in what units will the answer be expressed?

7. If you divide a force measured in newtons (1 newton = 1 kg • m/s²) by a speed expressed in meters per second, in what units will the answer be expressed?

Conceptual Questions

8. The height of a horse is sometimes given in units of “hands.” Why was this a poor standard of length before it was redefined to refer to exactly 4 in.?

9. Explain the advantages in having the meter officially defined in terms of the distance light travels in a given time rather than as the length of a specific metal bar.

10. Einstein’s famous equation indicates that \( E = mc^2 \), where \( c \) is the speed of light and \( m \) is the object’s mass. Given this, what is the SI unit for \( E \)?
Practice Problems

For problems 11–14, see Sample Problem A.

11. Express each of the following as indicated:
   a. 2 dm expressed in millimeters
   b. 2 h 10 min expressed in seconds
   c. 16 g expressed in micrograms
   d. 0.75 km expressed in centimeters
   e. 0.675 mg expressed in grams
   f. 462 μm expressed in centimeters
   g. 35 km/h expressed in meters per second

12. Use the SI prefixes in Table 3 of this chapter to convert these hypothetical units of measure into appropriate quantities:
   a. 10 rations
   b. 2000 mockingbirds
   c. $10^{-6}$ phones
   d. $10^{-9}$ goats
   e. $10^{18}$ miners

13. Use the fact that the speed of light in a vacuum is about $3.00 \times 10^8$ m/s to determine how many kilometers a pulse from a laser beam travels in exactly one hour.

14. If a metric ton is $1.000 \times 10^3$ kg, how many 85 kg people can safely occupy an elevator that can hold a maximum mass of exactly 1 metric ton?

ACCURACY, PRECISION, AND SIGNIFICANT FIGURES

Review Questions

15. Can a set of measurements be precise but not accurate? Explain.

16. How many significant figures are in the following measurements?
   a. 300 000 000 m/s
   b. $3.00 \times 10^8$ m/s
   c. 25.030°C
   d. 0.006 070°C
   e. 1.004 J
   f. 1.305 20 MHz

17. The photographs below show unit conversions on the labels of some grocery-store items. Check the accuracy of these conversions. Are the manufacturers using significant figures correctly?

18. The value of the speed of light is now known to be $2.997 924 58 \times 10^8$ m/s. Express the speed of light in the following ways:
   a. with three significant figures
   b. with five significant figures
   c. with seven significant figures

19. How many significant figures are there in the following measurements?
   a. $78.9 \pm 0.2$ m
   b. $3.788 \times 10^9$ s
   c. $2.46 \times 10^6$ kg
   d. 0.0032 mm

20. Carry out the following arithmetic operations:
   a. find the sum of the measurements 756 g, 37.2 g, 0.83 g, and 2.5 g
   b. find the quotient of $3.2$ m/3.563 s
   c. find the product of $5.67$ mm $\times \pi$
   d. find the difference of $27.54$ s and $3.8$ s

21. A fisherman catches two sturgeons. The smaller of the two has a measured length of 93.46 cm (two decimal places and four significant figures), and the larger fish has a measured length of 135.3 cm (one decimal place and four significant figures). What is the total length of the two fish?

22. A farmer measures the distance around a rectangular field. The length of each long side of the rectangle is found to be 38.44 m, and the length of each short side is found to be 19.5 m. What is the total distance around the field?
Note: In developing answers to order-of-magnitude calculations, you should state your important assumptions, including the numerical values assigned to parameters used in the solution. Since only order-of-magnitude results are expected, do not be surprised if your results differ from those of other students.

Review Questions

23. Suppose that two quantities, $A$ and $B$, have different dimensions. Which of the following arithmetic operations could be physically meaningful?

a. $A + B$

b. $A/B$

c. $A \times B$

d. $A - B$

24. Estimate the order of magnitude of the length in meters of each of the following:

a. a ladybug

b. your leg

c. your school building

d. a giraffe

e. a city block

25. If an equation is dimensionally correct, does this mean that the equation is true?

26. The radius of a circle inscribed in any triangle whose sides are $a$, $b$, and $c$ is given by the following equation, in which $s$ is an abbreviation for $(a + b + c)/2$. Check this formula for dimensional consistency.

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

27. The period of a simple pendulum, defined as the time necessary for one complete oscillation, is measured in time units and is given by the equation

$$T = 2\pi \sqrt{\frac{L}{a_g}}$$

where $L$ is the length of the pendulum and $a_g$ is the acceleration due to gravity, which has units of length divided by time squared. Check this equation for dimensional consistency.

Conceptual Questions

28. In a desperate attempt to come up with an equation to solve a problem during an examination, a student tries the following: $(\text{velocity in m/s})^2 = (\text{acceleration in m/s}^2) \times (\text{time in s})$. Use dimensional analysis to determine whether this equation might be valid.

29. Estimate the number of breaths taken by a person during 70 years.

30. Estimate the number of times your heart beats in an average day.

31. Estimate the magnitude of your age, as measured in units of seconds.

32. An automobile tire is rated to last for 50,000 mi. Estimate the number of revolutions the tire will make in its lifetime.

33. Imagine that you are the equipment manager of a professional baseball team. One of your jobs is to keep a supply of baseballs for games in your home ballpark. Balls are sometimes lost when players hit them into the stands as either home runs or foul balls. Estimate how many baseballs you have to buy per season in order to make up for such losses. Assume your team plays an 81-game home schedule in a season.

34. A chain of hamburger restaurants advertises that it has sold more than 50 billion hamburgers over the years. Estimate how many pounds of hamburger meat must have been used by the restaurant chain to make 50 billion hamburgers and how many head of cattle were required to furnish the meat for these hamburgers.

35. Estimate the number of piano tuners living in New York City. (The population of New York City is approximately 8 million.) This problem was first proposed by the physicist Enrico Fermi, who was well known for his ability to quickly make order-of-magnitude calculations.

36. Estimate the number of table-tennis balls that would fit (without being crushed) into a room that is 4 m long, 4 m wide, and 3 m high. Assume that the diameter of a ball is 3.8 cm.
MIXED REVIEW

37. Calculate the circumference and area for the following circles. (Use the following formulas: circumference = \(2\pi r\) and area = \(\pi r^2\).)
   a. a circle of radius 3.5 cm
   b. a circle of radius 4.65 cm

38. A billionaire offers to give you (1) $5 billion if you will count out the amount in $1 bills or (2) a lump sum of $5000. Which offer should you accept? Explain your answer. (Assume that you can count at an average rate of one bill per second, and be sure to allow for the fact that you need about 10 hours a day for sleeping and eating. Your answer does not need to be limited to one significant figure.)

39. Exactly 1 quart of ice cream is to be made in the form of a cube. What should be the length of one side in meters for the container to have the appropriate volume? (Use the following conversion: 4 qt = 3.786 \times 10^{-3} \text{ m}^3.)

40. You can obtain a rough estimate of the size of a molecule with the following simple experiment: Let a droplet of oil spread out on a fairly large but smooth water surface. The resulting “oil slick” that forms on the surface of the water will be approximately one molecule thick. Given an oil droplet with a mass of 9.00 \times 10^{-7} \text{ kg} and a density of 918 kg/m^3 that spreads out to form a circle with a radius of 41.8 cm on the water surface, what is the approximate diameter of an oil molecule?

Graphing Calculator Practice

Mass Versus Length
What is the relationship between the mass and length of three wires, each of which is made of a different substance? All three wires have the same diameter. Because the wires have the same diameter, their cross-sectional areas are the same. The cross-sectional area of any circle is equal to \(\pi r^2\). Consider a wire with a diameter of 0.50 cm and a density of 8.96 g/cm^3. The following equation describes the mass of the wire as a function of the length:

\[ Y_1 = 8.96X \times \pi(0.25)^2 \]

In this equation, \(Y_1\) represents the mass of the wire in grams, and \(X\) represents the length of the wire in centimeters. Each of the three wires is made of a different substance, so each wire has a different density and a different relationship between its mass and length.

In this graphing calculator activity, you will
- use dimensional analysis
- observe the relationship between a mathematical function and a graph
- determine values from a graph
- gain a better conceptual understanding of density

Visit go.hrw.com and type in the keyword HF6SOPX to find this graphing calculator activity. Refer to Appendix B for instructions on downloading the program for this activity.
41. An ancient unit of length called the cubit was equal to approximately 50 centimeters, which is, of course, approximately 0.50 meters. It has been said that Noah’s ark was 300 cubits long, 50 cubits wide, and 30 cubits high. Estimate the volume of the ark in cubic meters. Also estimate the volume of a typical home, and compare it with the ark’s volume.

42. If one micrometeorite (a sphere with a diameter of $1.0 \times 10^{-6}$ m) struck each square meter of the moon each second, it would take many years to cover the moon with micrometeorites to a depth of 1.0 m. Consider a cubic box, 1.0 m on a side, on the moon. Estimate how long it would take to completely fill the box with micrometeorites.

43. One cubic centimeter (1.0 cm$^3$) of water has a mass of $1.0 \times 10^{-3}$ kg at 25°C. Determine the mass of 1.0 m$^3$ of water at 25°C.

44. Assuming biological substances are 90 percent water and the density of water is $1.0 \times 10^3$ kg/m$^3$, estimate the masses (density multiplied by volume) of the following:
   a. a spherical cell with a diameter of 1.0 µm (volume = $\frac{4}{3}\pi r^3$)
   b. a fly, which can be approximated by a cylinder 4.0 mm long and 2.0 mm in diameter (volume = $\ell \pi r^2$)

45. The radius of the planet Saturn is $6.03 \times 10^7$ m, and its mass is $5.68 \times 10^{26}$ kg.
   a. Find the density of Saturn (its mass divided by its volume) in grams per cubic centimeter. (The volume of a sphere is given by $\frac{4}{3}\pi r^3$.)
   b. Find the surface area of Saturn in square meters. (The surface area of a sphere is given by $4\pi r^2$.)

---

**Alternative Assessment**

1. Imagine that you are a member of your state’s highway board. In order to comply with a bill passed in the state legislature, all of your state’s highway signs must show distances in miles and kilometers. Two plans are before you. One plan suggests adding metric equivalents to all highway signs as follows: Dallas 300 mi (483 km). Proponents of the other plan say that the first plan makes the metric system seem more cumbersome, so they propose replacing the old signs with new signs every 50 km as follows: Dallas 300 km (186 mi). Participate in a class debate about which plan should be followed.

2. Can you measure the mass of a five-cent coin with a bathroom scale? Record the mass in grams displayed by your scale as you place coins on the scale, one at a time. Then, divide each measurement by the number of coins to determine the approximate mass of a single five-cent coin, but remember to follow the rules for significant figures in calculations. Which estimate do you think is the most accurate? Which is the most precise?

3. Find out who were the Nobel laureates for physics last year, and research their work. Alternatively, explore the history of the Nobel Prizes. Who founded the awards? Why? Who delivers the award? Where? Document your sources and present your findings in a brochure, poster, or presentation.

4. You have a clock with a second hand, a ruler marked in millimeters, a graduated cylinder marked in milliliters, and scales sensitive to 1 mg. How would you measure the mass of a drop of water? How would you measure the period of a swing? How would you measure the volume of a paper clip? How can you improve the accuracy of your measurements? Write the procedures clearly so that a partner can follow them and obtain reasonable results.

5. Create a poster or other presentation depicting the possible ranges of measurement for a dimension, such as distance, time, temperature, speed, or mass. Depict examples ranging from the very large to the very small. Include several examples that are typical of your own experiences.
MULTIPLE CHOICE

1. What area of physics deals with the subjects of heat and temperature?
   A. mechanics
   B. thermodynamics
   C. electrodynamics
   D. quantum mechanics

2. What area of physics deals with the behavior of subatomic particles?
   F. mechanics
   G. thermodynamics
   H. electrodynamics
   J. quantum mechanics

3. What term describes a set of particles or interacting components considered to be a distinct physical entity for the purpose of study?
   A. system
   B. model
   C. hypothesis
   D. controlled experiment

4. What is the SI base unit for length?
   F. inch
   G. foot
   H. meter
   J. kilometer

5. A light-year (ly) is a unit of distance defined as the distance light travels in one year. Numerically, 1 ly = 9 500 000 000 000 km. How many meters are in a light-year?
   A. $9.5 \times 10^{10}$ m
   B. $9.5 \times 10^{12}$ m
   C. $9.5 \times 10^{15}$ m
   D. $9.5 \times 10^{18}$ m

6. If you do not keep your line of sight directly over a length measurement, how will your measurement most likely be affected?
   F. Your measurement will be less precise.
   G. Your measurement will be less accurate.
   H. Your measurement will have fewer significant figures.
   J. Your measurement will suffer from instrument error.

7. If you measured the length of a pencil by using the meterstick shown in the figure below and you report your measurement in centimeters, how many significant figures should your reported measurement have?
   A. one
   B. two
   C. three
   D. four

8. A room is measured to be 3.6 m by 5.8 m. What is the area of the room? (Keep significant figures in mind.)
   F. 20.88 m$^2$
   G. $2 \times 10^1$ m$^2$
   H. $2.0 \times 10^1$ m$^2$
   J. 21 m$^2$

9. What technique can help you determine the power of 10 closest to the actual numerical value of a quantity?
   A. rounding
   B. order-of-magnitude estimation
   C. dimensional analysis
   D. graphical analysis
10. Which of the following statements is true of any valid physical equation?
   F. Both sides have the same dimensions.
   G. Both sides have the same variables.
   H. There are variables but no numbers.
   J. There are numbers but no variables.

The graph below shows the relationship between time and distance for a ball dropped vertically from rest. Use the graph to answer questions 11–12.

11. About how far has the ball fallen after 0.200 s?
   A. 5.00 cm
   B. 10.00 cm
   C. 20.00 cm
   D. 30.00 cm

12. Which of the following statements best describes the relationship between the variables?
   F. For equal time intervals, the change in position is increasing.
   G. For equal time intervals, the change in position is decreasing.
   H. For equal time intervals, the change in position is constant.
   J. There is no clear relationship between time and change in position.

13. Determine the number of significant figures in each of the following measurements.
   A. 0.0057 kg
   B. 5.70 g
   C. 6070 m
   D. $6.070 \times 10^3$ m

14. Calculate the following sum, and express the answer in meters. Follow the rules for significant figures.
   $(25.873 \text{ km}) + (1024 \text{ m}) + (3.0 \text{ cm})$

15. Demonstrate how dimensional analysis can be used to find the dimensions that result from dividing distance by speed.

16. You have decided to test the effects of four different garden fertilizers by applying them to four separate rows of vegetables. What factors should you control? How could you measure the results?

17. In a paragraph, describe how you could estimate the number of blades of grass on a football field.
In this laboratory exercise, you will gain experience making measurements as a physicist does. All measurements will be made using units to the precision allowed by your instruments.

**SAFETY**
- Perform this lab in a clear area. Falling or dropped masses can cause serious injury.

**PROCEDURE**

**Preparation**

1. Read the entire lab procedure, and plan the steps you will take.

**Measuring Length, Width, Thickness, and Mass**

2. If you are not using a datasheet provided by your teacher, prepare a data table in your lab notebook with seven columns and five rows, as shown below. In the first row, label the second through seventh columns \textit{Trial 1}, \textit{Trial 2}, \textit{Trial 3}, \textit{Trial 4}, \textit{Trial 5}, and \textit{Trial 6}. In the first column, label the second through fifth rows \textit{Length (cm)}, \textit{Width (cm)}, \textit{Thickness (cm)}, and \textit{Mass (kg)}.

<table>
<thead>
<tr>
<th></th>
<th>Trial 1</th>
<th>Trial 2</th>
<th>Trial 3</th>
<th>Trial 4</th>
<th>Trial 5</th>
<th>Trial 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (cm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Width (cm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thickness (cm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass (kg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**MATERIALS LIST**
- 2 rectangular wooden blocks
- 15 cm metric ruler
- balance
- meterstick
- rectangular wooden block
- stopwatch
3. Use a meterstick to measure the length of the wooden block. Record all measured digits plus one estimated digit.

4. Follow the same procedure to measure the width and thickness of the block. Repeat all measurements two more times. Record your data.

5. Carefully adjust the balance to obtain an average zero reading when there is no mass on it. Your teacher will show you how to adjust the balances in your classroom to obtain an average zero reading. Use the balance to find the mass of the block, as shown in Figure 1. Record the measurement in your data table.

6. Repeat the mass measurement two more times, and record the values in your data table. Each time, move the block so that it rests on a different side.

7. For trials 4–6, repeat steps 3 through 6 with the second wooden block.

Measuring Time and Distance

8. If you are not using a datasheet provided by your teacher, prepare a second data table in your lab notebook with three columns and seven rows, as shown below. In the first row, label the columns Trial, Distance (m), and Time (s). Label the second through seventh rows 1, 2, 3, 4, 5, and 6.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Distance (m)</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
9. Perform this exercise with a partner. One partner will drop the wooden block from a measured height, and the other partner will measure the time it takes the block to fall to the floor. Perform this in a clear area away from other groups.

10. One student should hold the wooden block straight out in front of him or her at shoulder height. Hold the block between your hands, as shown in Figure 2. Use the meterstick to measure the height of the wooden block above the floor. Record this distance in your data table.

11. Use the stopwatch to time the fall of the block. Make sure the area is clear, and inform nearby groups that you are about to begin. The student holding the block should release it by pulling both hands straight out to the sides. The student with the stopwatch should begin timing the instant the block is released and stop timing as soon as the block hits the floor. In your data table, record the time required for the block to fall.

12. Repeat for two more trials, recording all data in your data table. Try to drop the block from exactly the same height each time.

ANALYSIS

1. Organizing Data Using your data from the first data table, calculate the volume of the wooden block for each trial. The equation for the volume of a rectangular block is \( \text{volume} = \text{length} \times \text{width} \times \text{thickness} \).

2. Analyzing Data Use your data from the first table and your results from item 1 above to answer the following questions.
   
   a. For each block, what is the difference between the smallest length measurement and the largest length measurement?
   
   b. For each block, what is the difference between the smallest calculated volume and the largest calculated volume?
   
   c. Based on your answers to (a) and (b), how does multiplying several length measurements together to find the volume affect the precision of the result?

3. Analyzing Data Did the block always fall from the same height in the same amount of time? Explain how you found the answer to this question.

4. Constructing Graphs Using the data from all trials, make a scatter plot of the distance versus the time of the block’s fall. Use a graphing calculator, computer, or graph paper.

CONCLUSIONS

5. Drawing Conclusions For each trial in the first data table, find the ratio between the mass and the volume. Based on your data, what is the relationship between the mass and volume?

6. Evaluating Methods For each type of measurement you made, explain how error could have affected your results. Consider method error and instrument error. How could you find out whether error had a significant effect on your results for each part of the lab? Explain the role of human reaction time in your measurements.

EXTENSION

7. Evaluating Data If there is time and your teacher approves, conduct the following experiment. Have one student drop the wooden block from shoulder height while all other class members time the fall. Perform three trials. Compare results each time. What does this exercise suggest about accuracy and precision in the laboratory?
High-speed passenger trains such as the one shown here are used in many countries, including Japan, France, England, Germany, and South Korea. These trains have operational speeds from 200 to 300 km/h. A train moving along a straight track is an example of one-dimensional motion. The train in the diagram is covering greater distances in equal time intervals—in other words, it is accelerating.

WHAT TO EXPECT
In this chapter, you will learn how to analyze one-dimensional motion in terms of displacement, time, speed, and velocity. You will also learn how to distinguish between accelerated and nonaccelerated motion.

WHY IT MATTERS
Velocity and acceleration are involved in many aspects of everyday life, from riding a bicycle to driving a car to traveling on a high-speed train. The definitions and equations you will study in this chapter allow you to make predictions about these aspects of motion, given certain initial conditions.

CHAPTER PREVIEW
1 Displacement and Velocity
   Motion
   Displacement
   Velocity
2 Acceleration
   Changes in Velocity
   Motion with Constant Acceleration
3 Falling Objects
   Free Fall
Motion happens all around us. Every day, we see objects such as cars, people, and soccer balls move in different directions with different speeds. We are so familiar with the idea of motion that it requires a special effort to analyze motion as a physicist does.

One-dimensional motion is the simplest form of motion

One way to simplify the concept of motion is to consider only the kinds of motion that take place in one direction. An example of this one-dimensional motion is the motion of a commuter train on a straight track, as in Figure 1.

In this one-dimensional motion, the train can move either forward or backward along the tracks. It cannot move left and right or up and down. This chapter deals only with one-dimensional motion. In later chapters, you will learn how to describe more complicated motions such as the motion of thrown baseballs and other projectiles.

Motion takes place over time and depends upon the frame of reference

It seems simple to describe the motion of the train. As the train in Figure 1 begins its route, it is at the first station. Later, it will be at another station farther down the tracks. But Earth is spinning on its axis, so the train, stations, and the tracks are also moving around the axis. At the same time, Earth is moving around the sun. The sun and the rest of the solar system are moving through our galaxy. This galaxy is traveling through space as well.

When faced with a complex situation like this, physicists break it down into simpler parts. One key approach is to choose a frame of reference against which you can measure changes in position. In the case of the train, any of the stations along its route could serve as a convenient frame of reference. When you select a reference frame, note that it remains fixed for the problem in question and has an origin, or starting point, from which the motion is measured.
If an object is at rest (not moving), its position does not change with respect to a fixed frame of reference. For example, the benches on the platform of one subway station never move down the tracks to another station.

In physics, any frame of reference can be chosen as long as it is used consistently. If you are consistent, you will get the same results, no matter which frame of reference you choose. But some frames of reference can make explaining things easier than other frames of reference.

For example, when considering the motion of the gecko in Figure 2, it is useful to imagine a stick marked in centimeters placed under the gecko’s feet to define the frame of reference. The measuring stick serves as an x-axis. You can use it to identify the gecko’s initial position and its final position.

**DISPLACEMENT**

As any object moves from one position to another, the length of the straight line drawn from its initial position to the object’s final position is called the displacement of the object.

**Displacement is a change in position**

The gecko in Figure 2 moves from left to right along the x-axis from an initial position, $x_i$, to a final position, $x_f$. The gecko’s displacement is the difference between its final and initial coordinates, or $x_f - x_i$. In this case, the displacement is about 61 cm ($85 \text{ cm} - 24 \text{ cm}$). The Greek letter delta ($\Delta$) before the $x$ denotes a change in the position of an object.

$$\Delta x = x_f - x_i$$

**DISPLACEMENT**

| Displacement = change in position = final position – initial position |

A change in any quantity, indicated by the Greek symbol delta ($\Delta$), is equal to the final value minus the initial value. When calculating displacement, always be sure to subtract the initial position from the final position so that your answer has the correct sign.

**1. Space Shuttle**

A space shuttle takes off from Florida and circles Earth several times, finally landing in California. While the shuttle is in flight, a photographer flies from Florida to California to take pictures of the astronauts when they step off the shuttle. Who undergoes the greater displacement, the photographer or the astronauts?

**2. Roundtrip**

What is the difference between the displacement of the photographer flying from Florida to California and the displacement of the astronauts flying from California back to Florida?
Now suppose the gecko runs up a tree, as shown in Figure 3. In this case, we place the measuring stick parallel to the tree. The measuring stick can serve as the \( y \)-axis of our coordinate system. The gecko’s initial and final positions are indicated by \( y_i \) and \( y_f \), respectively, and the gecko’s displacement is denoted as \( \Delta y \).

**Displacement is not always equal to the distance traveled**

Displacement does not always tell you the distance an object has moved. For example, what if the gecko in Figure 3 runs up the tree from the 20 cm marker (its initial position) to the 80 cm marker. After that, it retreats down the tree to the 50 cm marker (its final position). It has traveled a total distance of 90 cm. However, its displacement is only 30 cm (\( y_f - y_i = 50 \text{ cm} - 20 \text{ cm} = 30 \text{ cm} \)). If the gecko were to return to its starting point, its displacement would be zero because its initial position and final position would be the same.

**Displacement can be positive or negative**

Displacement also includes a description of the direction of motion. In one-dimensional motion, there are only two directions in which an object can move, and these directions can be described as positive or negative.

In this book, unless otherwise stated, the right (or east) will be considered the positive direction and the left (or west) will be considered the negative direction. Similarly, upward (or north) will be considered positive and downward (or south) will be considered negative. Table 1 gives examples of determining displacements for a variety of situations.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Positive and Negative Displacements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td><img src="image1" alt="Positive Example" /></td>
</tr>
<tr>
<td>( \Delta x = x_f - x_i = 80 \text{ cm} - 10 \text{ cm} = +70 \text{ cm} )</td>
<td></td>
</tr>
<tr>
<td><img src="image2" alt="Positive Example" /></td>
<td></td>
</tr>
<tr>
<td>( \Delta x = x_f - x_i = 12 \text{ cm} - 3 \text{ cm} = +9 \text{ cm} )</td>
<td></td>
</tr>
<tr>
<td><img src="image3" alt="Positive Example" /></td>
<td></td>
</tr>
<tr>
<td>( \Delta x = x_f - x_i = 6 \text{ cm} - (-10 \text{ cm}) = +16 \text{ cm} )</td>
<td></td>
</tr>
<tr>
<td>Negative</td>
<td><img src="image4" alt="Negative Example" /></td>
</tr>
<tr>
<td>( \Delta x = x_f - x_i = 20 \text{ cm} - 80 \text{ cm} = -60 \text{ cm} )</td>
<td></td>
</tr>
<tr>
<td><img src="image5" alt="Negative Example" /></td>
<td></td>
</tr>
<tr>
<td>( \Delta x = x_f - x_i = 0 \text{ cm} - 15 \text{ cm} = -15 \text{ cm} )</td>
<td></td>
</tr>
<tr>
<td><img src="image6" alt="Negative Example" /></td>
<td></td>
</tr>
<tr>
<td>( \Delta x = x_f - x_i = -20 \text{ cm} - (-10 \text{ cm}) = -10 \text{ cm} )</td>
<td></td>
</tr>
</tbody>
</table>
VELOCITY

Where an object started and where it stopped does not completely describe the motion of the object. For example, the ground that you’re standing on may move 8.0 cm to the left. This motion could take several years and be a sign of the normal slow movement of Earth’s tectonic plates. If this motion takes place in just a second, however, you may be experiencing an earthquake or a landslide. Knowing the speed is important when evaluating motion.

Average velocity is displacement divided by the time interval

Consider the car in Figure 4. The car is moving along a highway in a straight line (the x-axis). Suppose that the positions of the car are $x_i$ at time $t_i$ and $x_f$ at time $t_f$. In the time interval $\Delta t = t_f - t_i$, the displacement of the car is $\Delta x = x_f - x_i$. The average velocity, $v_{avg}$, is defined as the displacement divided by the time interval during which the displacement occurred. In SI, the unit of velocity is meters per second, abbreviated as m/s.

**AVERAGE VELOCITY**

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

average velocity $= \frac{\text{change in position}}{\text{change in time}} = \frac{\text{displacement}}{\text{time interval}}$

The average velocity of an object can be positive or negative, depending on the sign of the displacement. (The time interval is always positive.) As an example, consider a car trip to a friend’s house 370 km to the west (the negative direction) along a straight highway. If you left your house at 10 A.M. and arrived at your friend’s house at 3 P.M., your average velocity would be as follows:

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{370 \text{ km}}{5.0 \text{ h}} = -74 \text{ km/h} = 74 \text{ km/h west}$$

This value is an average. You probably did not travel exactly 74 km/h at every moment. You may have stopped to buy gas or have lunch. At other times, you may have traveled more slowly as a result of heavy traffic. To make up for such delays, when you were traveling slower than 74 km/h, there must also have been other times when you traveled faster than 74 km/h.

The average velocity is equal to the constant velocity needed to cover the given displacement in a given time interval. In the example above, if you left your house and maintained a velocity of 74 km/h to the west at every moment, it would take you 5.0 h to travel 370 km.

AVERAGE VELOCITY $= \frac{\text{total displacement divided by the time interval during which the displacement occurred}}{\text{the time interval during which the displacement occurred}}$

**Did you know?**

The branch of physics concerned with motion and forces is called mechanics. The subset of mechanics that describes motion without regard to its causes is called kinematics.
SAMPLE PROBLEM A

Average Velocity and Displacement

**PROBLEM**  
During a race on level ground, Andra runs with an average velocity of 6.02 m/s to the east. What is Andra’s displacement after 137 s?

**SOLUTION**  
Given:  
\[ v_{avg} = 6.02 \text{ m/s} \]  
\[ \Delta t = 137 \text{ s} \]  
Unknown:  
\[ \Delta x = ? \]  
Rearrange the average velocity equation to solve for displacement.  
\[ v_{avg} = \frac{\Delta x}{\Delta t} \]  
\[ \Delta x = v_{avg} \Delta t \]  
\[ \Delta x = (6.02 \text{ m/s})(137 \text{ s}) = 825 \text{ m to the east} \]

**CALCULATOR SOLUTION**  
The calculator answer is 824.74 m, but both the values for velocity and time have three significant figures, so the displacement must be reported as 825 m.

**PRACTICE A**

Average Velocity and Displacement

1. Heather and Matthew walk with an average velocity of 0.98 m/s eastward. If it takes them 34 min to walk to the store, what is their displacement?

2. If Joe rides his bicycle in a straight line for 15 min with an average velocity of 12.5 km/h south, how far has he ridden?

3. It takes you 9.5 min to walk with an average velocity of 1.2 m/s to the north from the bus stop to the museum entrance. What is your displacement?

4. Simpson drives his car with an average velocity of 48.0 km/h to the east. How long will it take him to drive 144 km on a straight highway?

5. Look back at item 4. How much time would Simpson save by increasing his average velocity to 56.0 km/h to the east?

6. A bus travels 280 km south along a straight path with an average velocity of 88 km/h to the south. The bus stops for 24 min. Then, it travels 210 km south with an average velocity of 75 km/h to the south.  
   **a.** How long does the total trip last?  
   **b.** What is the average velocity for the total trip?
**Velocity is not the same as speed**

In everyday language, the terms speed and velocity are used interchangeably. In physics, however, there is an important distinction between these two terms. As we have seen, velocity describes motion with both a direction and a numerical value (a magnitude) indicating how fast something moves. However, speed has no direction, only magnitude. An object’s average speed is equal to the distance traveled divided by the time interval for the motion.

\[
\text{average speed} = \frac{\text{distance traveled}}{\text{time of travel}}
\]

**Velocity can be interpreted graphically**

The velocity of an object can be determined if the object’s position is known at specific times along its path. One way to determine this is to make a graph of the motion. Figure 5 represents such a graph. Notice that time is plotted on the horizontal axis and position is plotted on the vertical axis.

The object moves 4.0 m in the time interval between \( t = 0 \) s and \( t = 4.0 \) s. Likewise, the object moves an additional 4.0 m in the time interval between \( t = 4.0 \) s and \( t = 8.0 \) s. From these data, we see that the average velocity for each of these time intervals is +1.0 m/s (because \( v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{4.0 \text{ m}}{4.0 \text{ s}} \)). Because the average velocity does not change, the object is moving with a constant velocity of +1.0 m/s, and its motion is represented by a straight line on the position-time graph.

For any position-time graph, we can also determine the average velocity by drawing a straight line between any two points on the graph. The slope of this line indicates the average velocity between the positions and times represented by these points. To better understand this concept, compare the equation for the slope of the line with the equation for the average velocity.

<table>
<thead>
<tr>
<th>Slope of a Line</th>
<th>Average Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in vertical coordinates}}{\text{change in horizontal coordinates}} )</td>
<td>( v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} )</td>
</tr>
</tbody>
</table>

---

**Conceptual Challenge**

1. **Book on a Table** A book is moved once around the edge of a tabletop with dimensions 1.75 m × 2.25 m. If the book ends up at its initial position, what is its displacement? If it completes its motion in 23 s, what is its average velocity? What is its average speed?

2. **Travel** Car A travels from New York to Miami at a speed of 25 m/s. Car B travels from New York to Chicago, also at a speed of 25 m/s. Are the velocities of the cars equal? Explain.
Figure 6 represents straight-line graphs of position-versus-time for three different objects. Object 1 has a constant positive velocity because its position increases uniformly with time. Thus, the slope of this line is positive. Object 2 has zero velocity because its position does not change (the object is at rest). Hence, the slope of this line is zero. Object 3 has a constant negative velocity because its position decreases with time. As a result, the slope of this line is negative.

**Instantaneous velocity may not be the same as average velocity**

Now consider an object whose position versus time graph is not a straight line, but a curve, as in Figure 7. The object moves through larger and larger displacements as each second passes. Thus, its velocity increases with time.

For example, between $t = 0$ s and $t = 2.0$ s, the object moves 8.0 m, and its average velocity in this time interval is 4.0 m/s (because $v_{avg} = 8.0$ m/2.0 s). However, between $t = 0$ s and $t = 4.0$ s, it moves 32 m, so its average velocity in this time interval is 8.0 m/s (because $v_{avg} = 32$ m/4.0 s). We obtain different average velocities, depending on the time interval we choose. But how can we find the velocity at an instant of time?

To determine the velocity at some instant, such as $t = 3.0$ s, we study a small time interval near that instant. As the intervals become smaller and smaller, the average velocity over that interval approaches the exact velocity at $t = 3.0$ s. This is called the **instantaneous velocity**.

One way to determine the instantaneous velocity is to construct a straight line that is tangent to the position-versus-time graph at that instant. The slope of this tangent line is equal to the value of the instantaneous velocity at that point. For example, the instantaneous velocity of the object in Figure 7 at $t = 3.0$ s is 12 m/s. Table 2 lists the instantaneous velocities of the object described by the graph in Figure 7. You can verify some of these values by carefully measuring the slope of the curve.

<table>
<thead>
<tr>
<th>$t$ (s)</th>
<th>$v$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1.0</td>
<td>4.0</td>
</tr>
<tr>
<td>2.0</td>
<td>8.0</td>
</tr>
<tr>
<td>3.0</td>
<td>12.0</td>
</tr>
<tr>
<td>4.0</td>
<td>16.0</td>
</tr>
</tbody>
</table>

**Figure 7**
The instantaneous velocity at a given time can be determined by measuring the slope of the line that is tangent to that point on the position-versus-time graph.
1. What is the shortest possible time in which a bacterium could travel a distance of 8.4 cm across a Petri dish at a constant speed of 3.5 mm/s?

2. A child is pushing a shopping cart at a speed of 1.5 m/s. How long will it take this child to push the cart down an aisle with a length of 9.3 m?

3. An athlete swims from the north end to the south end of a 50.0 m pool in 20.0 s and makes the return trip to the starting position in 22.0 s.
   a. What is the average velocity for the first half of the swim?
   b. What is the average velocity for the second half of the swim?
   c. What is the average velocity for the roundtrip?

4. Two students walk in the same direction along a straight path, at a constant speed—one at 0.90 m/s and the other at 1.90 m/s.
   a. Assuming that they start at the same point and the same time, how much sooner does the faster student arrive at a destination 780 m away?
   b. How far would the students have to walk so that the faster student arrives 5.50 min before the slower student?

5. Critical Thinking Does knowing the distance between two objects give you enough information to locate the objects? Explain.

6. Interpreting Graphics Figure 8 shows position-time graphs of the straight-line movement of two brown bears in a wildlife preserve. Which bear has the greater average velocity over the entire period? Which bear has the greater velocity at \( t = 8.0 \) min? Is the velocity of bear A always positive? Is the velocity of bear B ever negative?
Acceleration

SECTION 2

CHANGES IN VELOCITY

Many bullet trains have a top speed of about 300 km/h. Because a train stops to load and unload passengers, it does not always travel at that top speed. For some of the time the train is in motion, its velocity is either increasing or decreasing. It loses speed as it slows down to stop and gains speed as it pulls away and heads for the next station.

Acceleration is the rate of change of velocity with respect to time

Similarly, when a shuttle bus approaches a stop, the driver begins to apply the brakes to slow down 5.0 s before actually reaching the stop. The speed changes from 9.0 m/s to 0 m/s over a time interval of 5.0 s. Sometimes, however, the shuttle stops much more quickly. For example, if the driver slams on the brakes to avoid hitting a dog, the bus slows from 9.0 m/s to 0 m/s in just 1.5 s.

Clearly, these two stops are very different, even though the shuttle’s velocity changes by the same amount in both cases. What is different in these two examples is the time interval during which the change in velocity occurs. As you can imagine, this difference has a great effect on the motion of the bus, as well as on the comfort and safety of the passengers. A sudden change in velocity feels very different from a slow, gradual change.

The quantity that describes the rate of change of velocity in a given time interval is called acceleration. The magnitude of the average acceleration is calculated by dividing the total change in an object’s velocity by the time interval in which the change occurs.

Average acceleration

\[ a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \]

The units of acceleration in SI are meters per second per second, as shown below. When measured in these units, acceleration describes how much the velocity changes in each second.

\[ \frac{m}{s^2} = \frac{m}{s \times s} = \frac{m}{s^2} \]
SAMPLE PROBLEM B

**AVERAGE ACCELERATION**

**Problem**
A shuttle bus slows down with an average acceleration of $-1.8 \text{ m/s}^2$. How long does it take the bus to slow from 9.0 m/s to a complete stop?

**Solution**
Given: \[ v_i = 9.0 \text{ m/s} \]
\[ v_f = 0 \text{ m/s} \]
\[ a_{\text{avg}} = -1.8 \text{ m/s}^2 \]

Unknown: \[ \Delta t = ? \]

Rearrange the average acceleration equation to solve for the time interval.

\[
\Delta t = \frac{\Delta v}{a_{\text{avg}}} = \frac{v_f - v_i}{a_{\text{avg}}} = \frac{0 \text{ m/s} - 9.0 \text{ m/s}}{-1.8 \text{ m/s}^2}
\]

\[ \Delta t = 5.0 \text{ s} \]

**Practice B**

Average Acceleration

1. As the shuttle bus comes to a sudden stop to avoid hitting a dog, it accelerates uniformly at $-4.1 \text{ m/s}^2$ as it slows from 9.0 m/s to 0.0 m/s. Find the time interval of acceleration for the bus.

2. A car traveling at 7.0 m/s accelerates uniformly at 2.5 m/s$^2$ to reach a speed of 12.0 m/s. How long does it take for this acceleration to occur?

3. With an average acceleration of $-1.2 \text{ m/s}^2$, how long will it take a cyclist to bring a bicycle with an initial speed of 6.5 m/s to a complete stop?

4. Turner’s treadmill runs with a velocity of $-1.2 \text{ m/s}$ and speeds up at regular intervals during a half-hour workout. After 25 min, the treadmill has a velocity of $-6.5 \text{ m/s}$. What is the average acceleration of the treadmill during this period?

5. Suppose a treadmill has an average acceleration of $4.7 \times 10^{-3} \text{ m/s}^2$.
   a. How much does its speed change after 5.0 min?
   b. If the treadmill’s initial speed is 1.7 m/s, what will its final speed be?
Acceleration has direction and magnitude

Figure 9 shows a high-speed train leaving a station. Imagine that the train is moving to the right so that the displacement and the velocity are positive. The velocity increases in magnitude as the train picks up speed. Therefore, the final velocity will be greater than the initial velocity, and $\Delta v$ will be positive. When $\Delta v$ is positive, the acceleration is positive.

On long trips with no stops, the train may travel for a while at a constant velocity. In this situation, because the velocity is not changing, $\Delta v = 0 \text{ m/s}$. When the velocity is constant, the acceleration is equal to zero.

Imagine that the train, still traveling in the positive direction, slows down as it approaches the next station. In this case, the velocity is still positive, but the initial velocity is larger than the final velocity, so $\Delta v$ will be negative. When $\Delta v$ is negative, the acceleration is negative.

The slope and shape of the graph describe the object’s motion

As with all motion graphs, the slope and shape of the velocity-time graph in Figure 10 allow a detailed analysis of the train’s motion over time. When the train leaves the station, its speed is increasing over time. The line on the graph plotting this motion slopes up and to the right, as at point A on the graph.

When the train moves with a constant velocity, the line on the graph continues to the right, but it is horizontal, with a slope equal to zero. This indicates that the train’s velocity is constant, as at point B on the graph.

Finally, as the train approaches the station, its velocity decreases over time. The graph segment representing this motion slopes down to the right, as at point C on the graph. This downward slope indicates that the velocity is decreasing over time.

A negative value for the acceleration does not always indicate a decrease in speed. For example, if the train were moving in the negative direction, the acceleration would be negative when the train gained speed to leave a station and positive when the train lost speed to enter a station.

Conceptual Challenge

1. **Fly Ball**  If a baseball has zero velocity at some instant, is the acceleration of the baseball necessarily zero at that instant? Explain, and give examples.

2. **Runaway Train**  If a passenger train is traveling on a straight track with a negative velocity and a positive acceleration, is it speeding up or slowing down?

3. **Hike-and-Bike Trail**  When Jennifer is out for a ride, she slows down on her bike as she approaches a group of hikers on a trail. Explain how her acceleration can be positive even though her speed is decreasing.
Table 3 shows how the signs of the velocity and acceleration can be combined to give a description of an object’s motion. From this table, you can see that a negative acceleration can describe an object that is speeding up (when the velocity is negative) or an object that is slowing down (when the velocity is positive). Use this table to check your answers to problems involving acceleration.

For example, in Figure 10 the initial velocity $v_i$ of the train is positive. At point A on the graph, the train’s velocity is still increasing, so its acceleration is positive as well. The first entry in Table 3 shows that in this situation, the train is speeding up. At point C, the velocity is still positive, but it is decreasing, so the train’s acceleration is negative. Table 3 tells you that in this case, the train is slowing down.

<table>
<thead>
<tr>
<th>$v_i$</th>
<th>$a$</th>
<th>Motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>speeding up</td>
</tr>
<tr>
<td>−</td>
<td>−</td>
<td>speeding up</td>
</tr>
<tr>
<td>+</td>
<td>−</td>
<td>slowing down</td>
</tr>
<tr>
<td>−</td>
<td>+</td>
<td>slowing down</td>
</tr>
<tr>
<td>− or +</td>
<td>0</td>
<td>constant velocity</td>
</tr>
<tr>
<td>0</td>
<td>− or +</td>
<td>speeding up from rest</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>remaining at rest</td>
</tr>
</tbody>
</table>

**MOTION WITH CONSTANT ACCELERATION**

Figure 11 is a strobe photograph of a ball moving in a straight line with constant acceleration. While the ball was moving, its image was captured ten times in one second, so the time interval between successive images is 0.10 s. As the ball’s velocity increases, the ball travels a greater distance during each time interval. In this example, the velocity increases by exactly the same amount during each time interval. Thus, the acceleration is constant. Because the velocity increases for each time interval, the successive change in displacement for each time interval increases. You can see this in the photograph by noting that the distance between images increases while the time interval between images remains constant. The relationships between displacement, velocity, and constant acceleration are expressed by equations that apply to any object moving with constant acceleration.
Displacement depends on acceleration, initial velocity, and time

Figure 12 is a graph of the ball’s velocity plotted against time. The initial, final, and average velocities are marked on the graph. We know that the average velocity is equal to displacement divided by the time interval.

\[ \bar{v} = \frac{\Delta x}{\Delta t} \]

For an object moving with constant acceleration, the average velocity is equal to the average of the initial velocity and the final velocity.

\[ \bar{v} = \frac{v_i + v_f}{2} \]

To find an expression for the displacement in terms of the initial and final velocity, we can set the expressions for average velocity equal to each other.

\[ \frac{\Delta x}{\Delta t} = \bar{v} = \frac{v_i + v_f}{2} \]

Multiplying both sides of the equation by \(\Delta t\) gives us an expression for the displacement as a function of time. This equation can be used to find the displacement of any object moving with constant acceleration.

\[ \Delta x = \frac{1}{2}(v_i + v_f)\Delta t \]

**Did you know?**

Decreases in speed are sometimes called **decelerations**. Despite the sound of the name, decelerations are really a special case of acceleration in which the magnitude of the velocity—and thus the speed—decreases with time.

**ADVANCED TOPICS**

See “Angular Kinematics” in Appendix J: Advanced Topics to learn how displacement, velocity, and acceleration can be used to describe circular motion.
SAMPLE PROBLEM C

Displacement with Constant Acceleration

**PROBLEM**

A racing car reaches a speed of 42 m/s. It then begins a uniform negative acceleration, using its parachute and braking system, and comes to rest 5.5 s later. Find the distance that the car travels during braking.

**SOLUTION**

Given: 
\[ v_i = 42 \text{ m/s} \quad v_f = 0 \text{ m/s} \]
\[ \Delta t = 5.5 \text{ s} \]

Unknown: \[ \Delta x = ? \]

Use the equation that relates displacement, initial and final velocities, and the time interval.

\[ \Delta x = \frac{1}{2} (v_i + v_f) \Delta t \]

\[ \Delta x = \frac{1}{2} (42 \text{ m/s} + 0 \text{ m/s}) (5.5 \text{ s}) \]

\[ \Delta x = 120 \text{ m} \]

**TIP**

Remember that this equation applies only when acceleration is constant. In this problem, you know that acceleration is constant by the phrase “uniform negative acceleration.” All of the kinematic equations introduced in this chapter are valid only for constant acceleration.

**CALCULATOR SOLUTION**

The calculator answer is 115.5. However, the velocity and time values have only two significant figures each, so the answer must be reported as 120 m.

PRACTICE C

Displacement with Constant Acceleration

1. A car accelerates uniformly from rest to a speed of 6.6 m/s in 6.5 s. Find the distance the car travels during this time.

2. When Maggie applies the brakes of her car, the car slows uniformly from 15.0 m/s to 0.0 m/s in 2.50 s. How many meters before a stop sign must she apply her brakes in order to stop at the sign?

3. A driver in a car traveling at a speed of 21.8 m/s sees a cat 101 m away on the road. How long will it take for the car to accelerate uniformly to a stop in exactly 99 m?

4. A car enters the freeway with a speed of 6.4 m/s and accelerates uniformly for 3.2 km in 3.5 min. How fast (in m/s) is the car moving after this time?
**Final velocity depends on initial velocity, acceleration, and time**

What if the final velocity of the ball is not known but we still want to calculate the displacement? If we know the initial velocity, the acceleration, and the elapsed time, we can find the final velocity. We can then use this value for the final velocity to find the total displacement of the ball.

By rearranging the equation for acceleration, we can find a value for the final velocity.

\[
a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}
\]

\[
a\Delta t = v_f - v_i
\]

By adding the initial velocity to both sides of the equation, we get an equation for the final velocity of the ball.

\[
a\Delta t + v_i = v_f
\]

---

**VELOCITY WITH CONSTANT ACCELERATION**

\[
v_f = v_i + a\Delta t
\]

final velocity = initial velocity + (acceleration $\times$ time interval)

You can use this equation to find the final velocity of an object after it has accelerated at a constant rate for any time interval.

If you want to know the displacement of an object moving with constant acceleration over some certain time interval, you can obtain another useful expression for displacement by substituting the expression for $v_f$ into the expression for $\Delta x$.

\[
\Delta x = \frac{1}{2}(v_i + v_f)\Delta t
\]

\[
\Delta x = \frac{1}{2}(v_i + v_i + a\Delta t)\Delta t
\]

\[
\Delta x = \frac{1}{2}[2v_i\Delta t + a(\Delta t)^2]
\]

---

**DISPLACEMENT WITH CONSTANT ACCELERATION**

\[
\Delta x = v_i\Delta t + \frac{1}{2}a(\Delta t)^2
\]

displacement = (initial velocity $\times$ time interval) +

$\frac{1}{2}$ acceleration $\times$ (time interval)$^2$

This equation is useful not only for finding the displacement of an object moving with constant acceleration but also for finding the displacement required for an object to reach a certain speed or to come to a stop. For the latter situation, you need to use both this equation and the equation given above.
SAMPLE PROBLEM D

Velocity and Displacement with Constant Acceleration

PROBLEM
A plane starting at rest at one end of a runway undergoes a uniform acceleration of 4.8 m/s² for 15 s before takeoff. What is its speed at takeoff? How long must the runway be for the plane to be able to take off?

SOLUTION
Given: \(v_i = 0\) m/s \(a = 4.8\) m/s² \(\Delta t = 15\) s
Unknowns: \(v_f = ?\) \(\Delta x = ?\)
First, use the equation for the velocity of a uniformly accelerated object.

\[
v_f = v_i + a\Delta t
\]
\[
= 0\ m/s + (4.8\ m/s^2)(15\ s)
\]
\[v_f = 72\ m/s\]

Then, use the displacement equation that contains the given variables.

\[
\Delta x = v_i\Delta t + \frac{1}{2}a(\Delta t)^2
\]
\[
= (0\ m/s)(15\ s) + \frac{1}{2}(4.8\ m/s^2)(15\ s)^2
\]
\[\Delta x = 540\ m\]

Because you now know \(v_f\), you could also use the equation

\[
\Delta x = \frac{1}{2}(v_i + v_f)(\Delta t),\ or
\]
\[
\Delta x = \frac{1}{2}(72\ m/s)(15\ s) = 540\ m.
\]

PRACTICE D

Velocity and Displacement with Constant Acceleration

1. A car with an initial speed of 6.5 m/s accelerates at a uniform rate of 0.92 m/s² for 3.6 s. Find the final speed and the displacement of the car during this time.

2. An automobile with an initial speed of 4.30 m/s accelerates uniformly at the rate of 3.00 m/s². Find the final speed and the displacement after 5.00 s.

3. A car starts from rest and travels for 5.0 s with a constant acceleration of \(-1.5\) m/s². What is the final velocity of the car? How far does the car travel in this time interval?

4. A driver of a car traveling at 15.0 m/s applies the brakes, causing a uniform acceleration of \(-2.0\) m/s². How long does it take the car to accelerate to a final speed of 10.0 m/s? How far has the car moved during the braking period?
Final velocity depends on initial velocity, acceleration, and displacement

So far, all of the equations for motion under uniform acceleration have required knowing the time interval. We can also obtain an expression that relates displacement, velocity, and acceleration without using the time interval. This method involves rearranging one equation to solve for Δt and substituting that expression in another equation, making it possible to find the final velocity of a uniformly accelerated object without knowing how long it has been accelerating. Start with the following equation for displacement:

\[ \Delta x = \frac{1}{2}(v_i + v_f)\Delta t \]

Now, multiply both sides by 2.

\[ 2\Delta x = (v_i + v_f)\Delta t \]

Next, divide both sides by \((v_i + v_f)\)
to solve for \(\Delta t\).

\[ \left( \frac{2\Delta x}{v_i + v_f} \right) = \Delta t \]

Now that we have an expression for \(\Delta t\), we can substitute this expression into the equation for the final velocity.

\[ v_f = v_i + a(\Delta t) \]

\[ v_f = v_i + a \left( \frac{2\Delta x}{v_i + v_f} \right) \]

In its present form, this equation is not very helpful because \(v_f\) appears on both sides. To solve for \(v_f\), first subtract \(v_i\) from both sides of the equation.

\[ v_f - v_i = a \left( \frac{2\Delta x}{v_i + v_f} \right) \]

Next, multiply both sides by \((v_i + v_f)\) to get all the velocities on the same side of the equation.

\[ (v_f - v_i) (v_f + v_i) = 2a\Delta x = v_f^2 - v_i^2 \]

Add \(v_i^2\) to both sides to solve for \(v_f^2\).

**FINAL VELOCITY AFTER ANY DISPLACEMENT**

\[ v_f^2 = v_i^2 + 2a\Delta x \]

\[ (\text{final velocity})^2 = (\text{initial velocity})^2 + 2(\text{acceleration})(\text{displacement}) \]

When using this equation, you must take the square root of the right side of the equation to find the final velocity. Remember that the square root may be either positive or negative. If you have been consistent in your use of the sign convention, you will be able to determine which value is the right answer by reasoning based on the direction of the motion.
SAMPLE PROBLEM E

Final Velocity After Any Displacement

PROBLEM
A person pushing a stroller starts from rest, uniformly accelerating at a rate of 0.500 m/s². What is the velocity of the stroller after it has traveled 4.75 m?

SOLUTION

1. DEFINE

Given:

\[ v_i = 0 \text{ m/s} \quad a = 0.500 \text{ m/s}^2 \]

\[ \Delta x = 4.75 \text{ m} \]

Unknown:

\[ v_f = ? \]

Diagram:

Choose a coordinate system. The most convenient one has an origin at the initial location of the stroller. The positive direction is to the right.

2. PLAN

Choose an equation or situation:

Because the initial velocity, acceleration, and displacement are known, the final velocity can be found using the following equation:

\[ v_f^2 = v_i^2 + 2a\Delta x \]

Rearrange the equation to isolate the unknown:

Take the square root of both sides to isolate \( v_f \).

\[ v_f = \pm \sqrt{(v_i)^2 + 2a\Delta x} \]

3. CALCULATE

Substitute the values into the equation and solve:

\[ v_f = \pm \sqrt{(0 \text{ m/s})^2 + 2(0.500 \text{ m/s}^2)(4.75 \text{ m})} \]

\[ v_f = +2.18 \text{ m/s} \]

4. EVALUATE

Think about the physical situation to determine whether to keep the positive or negative answer from the square root. In this case, the stroller is speeding up because it starts from rest and ends with a speed of 2.18 m/s. An object that is speeding up and has a positive acceleration must have a positive velocity, as shown in Table 3. So, the final velocity must be positive.

The stroller’s velocity after accelerating for 4.75 m is 2.18 m/s to the right.
Final Velocity After Any Displacement

1. Find the velocity after the stroller in Sample Problem E has traveled 6.32 m.

2. A car traveling initially at +7.0 m/s accelerates uniformly at the rate of +0.80 m/s\(^2\) for a distance of 245 m.
   a. What is its velocity at the end of the acceleration?
   b. What is its velocity after it accelerates for 125 m?
   c. What is its velocity after it accelerates for 67 m?

3. A car accelerates uniformly in a straight line from rest at the rate of 2.3 m/s\(^2\).
   a. What is the speed of the car after it has traveled 55 m?
   b. How long does it take the car to travel 55 m?

4. A motorboat accelerates uniformly from a velocity of 6.5 m/s to the west to a velocity of 1.5 m/s to the west. If its acceleration was 2.7 m/s\(^2\) to the east, how far did it travel during the acceleration?

5. An aircraft has a liftoff speed of 33 m/s. What minimum constant acceleration does this require if the aircraft is to be airborne after a take-off run of 240 m?

6. A certain car is capable of accelerating at a uniform rate of 0.85 m/s\(^2\). What is the magnitude of the car’s displacement as it accelerates uniformly from a speed of 83 km/h to one of 94 km/h?

With the four equations presented in this section, it is possible to solve any problem involving one-dimensional motion with uniform acceleration. For your convenience, the equations that are used most often are listed in Table 4. The first column of the table gives the equations in their standard form. For an object initially at rest, \(v_i = 0\). Using this value for \(v_i\) in the equations in the first column will result in the equations in the second column. It is not necessary to memorize the equations in the second column. If \(v_i = 0\) in any problem, you will naturally derive this form of the equation. Referring back to the sample problems in this chapter will guide you through using these equations to solve many problems.

---

**Table 4**

Equations for Constantly Accelerated Straight-Line Motion

<table>
<thead>
<tr>
<th>Form to use when accelerating object has an initial velocity</th>
<th>Form to use when accelerating object starts from rest</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta x = \frac{1}{2}(v_i + v_f)\Delta t)</td>
<td>(\Delta x = \frac{1}{2}v_f\Delta t)</td>
</tr>
<tr>
<td>(v_f = v_i + a\Delta t)</td>
<td>(v_f = a\Delta t)</td>
</tr>
<tr>
<td>(\Delta x = v_i\Delta t + \frac{1}{2}a(\Delta t)^2)</td>
<td>(\Delta x = \frac{1}{2}a(\Delta t)^2)</td>
</tr>
<tr>
<td>(v_f^2 = v_i^2 + 2a\Delta x)</td>
<td>(v_f^2 = 2a\Delta x)</td>
</tr>
</tbody>
</table>
1. Marissa’s car accelerates uniformly at a rate of $+2.60 \text{ m/s}^2$. How long does it take for Marissa’s car to accelerate from a speed of 24.6 m/s to a speed of 26.8 m/s?

2. A bowling ball with a negative initial velocity slows down as it rolls down the lane toward the pins. Is the bowling ball’s acceleration positive or negative as it rolls toward the pins?

3. Nathan accelerates his skateboard uniformly along a straight path from rest to 12.5 m/s in 2.5 s.
   a. What is Nathan’s acceleration?
   b. What is Nathan’s displacement during this time interval?
   c. What is Nathan’s average velocity during this time interval?

4. **Critical Thinking** Two cars are moving in the same direction in parallel lanes along a highway. At some instant, the instantaneous velocity of car A exceeds the instantaneous velocity of car B. Does this mean that car A’s acceleration is greater than car B’s? Explain, and use examples.

5. **Interpreting Graphics** The velocity-versus-time graph for a shuttle bus moving along a straight path is shown in Figure 13.
   a. Identify the time intervals during which the velocity of the shuttle bus is constant.
   b. Identify the time intervals during which the acceleration of the shuttle bus is constant.
   c. Find the value for the average velocity of the shuttle bus during each time interval identified in b.
   d. Find the acceleration of the shuttle bus during each time interval identified in b.
   e. Identify the times at which the velocity of the shuttle bus is zero.
   f. Identify the times at which the acceleration of the shuttle bus is zero.
   g. Explain what the slope of the graph reveals about the acceleration in each time interval.

6. **Interpreting Graphics** Is the shuttle bus in item 5 always moving in the same direction? Explain, and refer to the time intervals shown on the graph.
Falling Objects

SECTION OBJECTIVES

- Relate the motion of a freely falling body to motion with constant acceleration.
- Calculate displacement, velocity, and time at various points in the motion of a freely falling object.
- Compare the motions of different objects in free fall.

FREE FALL

On August 2, 1971, a demonstration was conducted on the moon by astronaut David Scott. He simultaneously released a hammer and a feather from the same height above the moon’s surface. The hammer and the feather both fell straight down and landed on the lunar surface at exactly the same moment. Although the hammer is more massive than the feather, both objects fell at the same rate. That is, they traveled the same displacement in the same amount of time.

Freely falling bodies undergo constant acceleration

In Figure 14, a feather and an apple are released from rest in a vacuum chamber. The two objects fell at exactly the same rate, as indicated by the horizontal alignment of the multiple images.

The amount of time that passed between the first and second images is equal to the amount of time that passed between the fifth and sixth images. The picture, however, shows that the displacement in each time interval did not remain constant. Therefore, the velocity was not constant. The apple and the feather were accelerating.

Compare the displacement between the first and second images to the displacement between the second and third images. As you can see, within each time interval the displacement of the feather increased by the same amount as the displacement of the apple. Because the time intervals are the same, we know that the velocity of each object is increasing by the same amount in each time interval. In other words, the apple and the feather are falling with the same constant acceleration.

If air resistance is disregarded, all objects dropped near the surface of a planet fall with the same constant acceleration. This acceleration is due to gravitational force, and the motion is referred to as free fall. The acceleration due to gravity is denoted with the symbols $a_g$ (generally) or $g$ (on Earth’s surface). The magnitude of $g$ is about 9.81 m/s$^2$, or 32 ft/s$^2$. Unless stated otherwise, this book will use the value 9.81 m/s$^2$ for calculations. This acceleration is directed downward, toward the center of the Earth. In our usual choice of coordinates, the downward direction is negative. Thus, the acceleration of objects in free fall near the surface of the Earth is $a_g = -g = -9.81$ m/s$^2$. Because an object in free fall is acted on only by gravity, $a_g$ is also known as free-fall acceleration.
Acceleration is constant during upward and downward motion

Figure 15 is a strobe photograph of a ball thrown up into the air with an initial upward velocity of +10.5 m/s. The photo on the left shows the ball moving up from its release toward the top of its path, and the photo on the right shows the ball falling back down. Everyday experience shows that when we throw an object up in the air, it will continue to move upward for some time, stop momentarily at the peak, and then change direction and begin to fall. Because the object changes direction, it may seem that the velocity and acceleration are both changing. Actually, objects thrown into the air have a downward acceleration as soon as they are released.

In the photograph on the left, the upward displacement of the ball between each successive image is smaller and smaller until the ball stops and finally begins to move with an increasing downward velocity, as shown on the right. As soon as the ball is released with an initial upward velocity of +10.5 m/s, it has an acceleration of −9.81 m/s². After 1.0 s (Δt = 1.0 s), the ball’s velocity will change by −9.81 m/s to 0.69 m/s upward. After 2.0 s (Δt = 2.0 s), the ball’s velocity will again change by −9.81 m/s, to −9.12 m/s.

The graph in Figure 16 shows the velocity of the ball plotted against time. As you can see, there is an instant when the velocity of the ball is equal to 0 m/s. This happens at the instant when the ball reaches the peak of its upward motion and is about to begin moving downward. Although the velocity is zero at the instant the ball reaches the peak, the acceleration is equal to −9.81 m/s² at every instant regardless of the magnitude or direction of the velocity. It is important to note that the acceleration is −9.81 m/s² even at the peak where the velocity is zero. The straight-line slope of the graph indicates that the acceleration is constant at every moment.
Freely falling objects always have the same downward acceleration

It may seem a little confusing to think of something that is moving upward, like the ball in the example, as having a downward acceleration. Thinking of this motion as motion with a positive velocity and a negative acceleration may help. The downward acceleration is the same when an object is moving up, when it is at rest at the top of its path, and when it is moving down. The only things changing are the position and the magnitude and direction of the velocity.

When an object is thrown up in the air, it has a positive velocity and a negative acceleration. From Table 3 in Section 2, we see that this means the object is slowing down as it rises in the air. From the example of the ball and from everyday experience, we know that this makes sense. The object continues to move upward but with a smaller and smaller speed. In the photograph of the ball, this decrease in speed is shown by the smaller and smaller displacements as the ball moves up to the top of its path.

At the top of its path, the object’s velocity has decreased until it is zero. Although it is impossible to see this because it happens so quickly, the object is actually at rest at the instant it reaches its peak position. Even though the velocity is zero at this instant, the acceleration is still $-9.81 \text{ m/s}^2$.

When the object begins moving down, it has a negative velocity and its acceleration is still negative. From Table 3, we see that a negative acceleration and a negative velocity indicate an object that is speeding up. In fact, this is what happens when objects undergo free-fall acceleration. Objects that are falling toward Earth move faster and faster as they fall. In the photograph of the ball in Figure 15 (on the previous page), this increase in speed is shown by the greater and greater displacements between the images as the ball falls.

Knowing the free-fall acceleration makes it easy to calculate the velocity, time, and displacement of many different motions using the equations for constantly accelerated motion. Because the acceleration is the same throughout the entire motion, you can analyze the motion of a freely falling object during any time interval.

---

**Quick Lab**

**Time Interval of Free Fall**

**MATERIALS LIST**

- 1 meterstick or ruler

**SAFETY CAUTION**

Avoid eye injury; do not swing metersticks.

Your reaction time affects your performance in all kinds of activities—from sports to driving to catching something that you drop. Your reaction time is the time interval between an event and your response to it.

Determine your reaction time by having a friend hold a meterstick vertically between the thumb and index finger of your open hand. The meterstick should be held so that the zero mark is between your fingers with the 1 cm mark above it.

You should not be touching the meterstick, and your catching hand must be resting on a table. Without warning you, your friend should release the meterstick so that it falls between your thumb and your finger. Catch the meterstick as quickly as you can. You can calculate your reaction time from the free-fall acceleration and the distance the meterstick has fallen through your grasp.
Falling Object

**Problem**

Jason hits a volleyball so that it moves with an initial velocity of 6.0 m/s straight upward. If the volleyball starts from 2.0 m above the floor, how long will it be in the air before it strikes the floor?

**Solution**

1. **Define**

   Given:
   
   \( v_i = +6.0 \text{ m/s} \quad a = -g = -9.81 \text{ m/s}^2 \quad \Delta y = -2.0 \text{ m} \)

   Unknown:
   
   \( \Delta t = ? \)

   Diagram:

   Place the origin at the starting point of the ball (\( y_i = 0 \) at \( t_i = 0 \)).

2. **Plan**

   Choose an equation or situation:

   Both \( \Delta t \) and \( v_f \) are unknown. Therefore, first solve for \( v_f \) using the equation that does not require time. Then, the equation for \( v_f \) that does involve time can be used to solve for \( \Delta t \).

   \[
   v_f^2 = v_i^2 + 2a\Delta y
   \]

   \[
   v_f = v_i + a\Delta t
   \]

3. **Calculate**

   Rearrange the equations to isolate the unknown:

   Take the square root of the first equation to isolate \( v_f \). The second equation must be rearranged to solve for \( \Delta t \).

   \[
   v_f = \pm \sqrt{v_i^2 + 2a\Delta y}
   \]

   \[
   \Delta t = \frac{v_f - v_i}{a}
   \]

   Substitute the values into the equations and solve:

   First find the velocity of the ball at the moment that it hits the floor.

   \[
   v_f = \pm \sqrt{(6.0 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(-2.0 \text{ m})}
   \]

   \[
   v_f = \pm \sqrt{36 \text{ m}^2/\text{s}^2 + 39 \text{ m}^2/\text{s}^2} = \pm \sqrt{75 \text{ m}^2/\text{s}^2} = -8.7 \text{ m/s}
   \]

   When you take the square root to find \( v_f \), select the negative answer because the ball will be moving toward the floor, in the negative direction.

   Next, use this value of \( v_f \) in the second equation to solve for \( \Delta t \).

   \[
   \Delta t = \frac{v_f - v_i}{a} = \frac{-8.7 \text{ m/s} - 6.0 \text{ m/s}}{-9.81 \text{ m/s}^2} = -14.7 \text{ m/s}
   \]

   \[
   \Delta t = 1.50 \text{ s}
   \]

4. **Evaluate**

   The solution, 1.50 s, is a reasonable amount of time for the ball to be in the air.
PRACTICE F

Falling Object

1. A robot probe drops a camera off the rim of a 239 m high cliff on Mars, where the free-fall acceleration is \(-3.7 \text{ m/s}^2\).
   a. Find the velocity with which the camera hits the ground.
   b. Find the time required for it to hit the ground.

2. A flowerpot falls from a windowsill 25.0 m above the sidewalk.
   a. How fast is the flowerpot moving when it strikes the ground?
   b. How much time does a passerby on the sidewalk below have to move out of the way before the flowerpot hits the ground?

3. A tennis ball is thrown vertically upward with an initial velocity of \(+8.0 \text{ m/s}\).
   a. What will the ball’s speed be when it returns to its starting point?
   b. How long will the ball take to reach its starting point?

4. Calculate the displacement of the volleyball in Sample Problem F when the volleyball’s final velocity is 1.1 m/s upward.

THE INSIDE STORY ON SKY DIVING

When these sky divers jump from an airplane, they plummet toward the ground. If Earth had no atmosphere, the sky divers would accelerate with the free-fall acceleration, \(g\), equal to 9.81 m/s\(^2\). They would not slow down even after opening their parachutes. Fortunately, Earth does have an atmosphere, and the sky divers do not accelerate indefinitely. Instead, the rate of acceleration decreases as they fall because of air resistance. After a few seconds, the acceleration drops to zero and the speed becomes constant. The constant speed an object reaches when falling through a resisting medium is called terminal velocity.

The terminal velocity of an object depends on the object’s mass, shape, and size. When a sky diver is spread out horizontally to the ground, the sky diver’s terminal velocity is typically about 55 m/s (123 mi/h). If the sky diver curls into a ball, the terminal velocity may increase to close to 90 m/s (200 mi/h). When the sky diver opens the parachute, air resistance increases, and the sky diver decelerates to a new, slower terminal velocity. For a sky diver with an open parachute, the terminal velocity is typically about 5 m/s (11 mi/h).
1. A coin is tossed vertically upward.
   a. What happens to its velocity while it is in the air?
   b. Does its acceleration increase, decrease, or remain constant while it
      is in the air?

2. A pebble is dropped down a well and hits the water 1.5 s later. Using the
   equations for motion with constant acceleration, determine the distance
   from the edge of the well to the water’s surface.

3. A ball is thrown vertically upward. What are its velocity and acceleration
   when it reaches its maximum altitude? What is its acceleration just
   before it hits the ground?

4. Two children are bouncing small rubber balls. One child simply drops a ball.
   At the same time, the second child throws a ball downward so that it has an
   initial speed of 10 m/s. What is the acceleration of each ball while in motion?

5. **Critical Thinking**  A gymnast practices two dismounts from the
   high bar on the uneven parallel bars. During one dismount, she swings
   up off the bar with an initial upward velocity of + 4.0 m/s. In the second,
   she releases from the same height but with an initial downward velocity
   of −3.0 m/s. What is her acceleration in each case? How do the final
   velocities of the gymnast as she reaches the ground differ?

6. **Interpreting Graphics**  Figure 17 is a position-time graph of the
   motion of a basketball thrown straight up. Use the graph to sketch the
   path of the basketball and to sketch a velocity-time graph of the basket-
   ball’s motion.
   a. Is the velocity of the basketball constant?
   b. Is the acceleration of the basketball constant?
   c. What is the initial velocity of the basketball?
Science writers explain science to their readers in a clear and entertaining way. To learn more about science writing as a career, read the interview with Janice VanCleave, author of Janice VanCleave’s A+ Projects in Physics and more than 50 other books about science.

What does a science writer do?
After the topic and basic format of a book are determined, I start researching. My personal library contains about 1,000 science books. For more information, I use online sources to find current printed research. But most important are science consultants in each specific field, such as a NASA astronomer, a research chemist, and physics professors.

After I write the original manuscript, it is edited five different times. I have to make sure any changes do not affect its scientific correctness and that the art represents the text. On average, I write three new books each year between doing reviews on the books written in previous years.

What sort of training do you have?
I taught science for 27 years, and this was the only training I had for my writing career.

Mine is a Cinderella story. A publisher saw an ad for an elementary science enrichment program that I designed and taught. She sent a letter asking if I was interested in writing a science book for young kids. The answer was a big YES!

It didn’t take long to realize that although I had skills to write experiments for my class, I didn’t have a clue about how to write a book. Thankfully, the publisher really wanted the book, so I was given a great deal of personal instruction. I learned by trial and error. In fact, even after writing 50 books, I am still learning how to better write a book.

What is your favorite part of your work?
I love writing because, as in teaching, I learn so much. Now, instead of just writing for my classroom, I am able to share my ideas with students and teachers around the world. A downside is that I have less time with students than when I taught.

What advice do you have for students who are interested in science writing?
Study the market, and know what kinds of books are most wanted. Writing a book may be your smallest problem; getting it published can be a big obstacle. If you are a great writer but not a good salesperson, I suggest hiring an agent. Write about something that you have a passion for, and don’t give up if your work is not accepted. Try, try, and try again.
### Highlights

#### KEY IDEAS

**Section 1 Displacement and Velocity**
- Displacement is a change of position in a certain direction, not the total distance traveled.
- The average velocity of an object during some time interval is equal to the displacement of the object divided by the time interval. Like displacement, velocity has both a magnitude (called speed) and a direction.
- The average velocity is equal to the slope of the straight line connecting the initial and final points on a graph of the position of the object versus time.

**Section 2 Acceleration**
- The average acceleration of an object during a certain time interval is equal to the change in the object’s velocity divided by the time interval. Acceleration has both magnitude and direction.
- The direction of the acceleration is not always the same as the direction of the velocity. The direction of the acceleration depends on the direction of the motion and on whether the velocity is increasing or decreasing.
- The average acceleration is equal to the slope of the straight line connecting the initial and final points on the graph of the velocity of the object versus time.
- The equations in Table 4 are valid whenever acceleration is constant.

**Section 3 Falling Objects**
- An object thrown or dropped in the presence of Earth’s gravity experiences a constant acceleration directed toward the center of Earth. This acceleration is called the free-fall acceleration, or the acceleration due to gravity.
- Free-fall acceleration is the same for all objects, regardless of mass.
- The value for free-fall acceleration on Earth’s surface used in this book is \( a_g = -g = -9.81 \text{ m/s}^2 \). The direction of the free-fall acceleration is considered to be negative because the object accelerates toward Earth.

#### KEY TERMS

frame of reference (p. 40)
displacement (p. 41)
average velocity (p. 43)
instantaneous velocity (p. 46)
acceleration (p. 48)
free fall (p. 60)

### Variable Symbols

<table>
<thead>
<tr>
<th>Quantities</th>
<th>Units</th>
<th>Quantities</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>position</td>
<td>( y ) position</td>
<td>m meters</td>
</tr>
<tr>
<td>( \Delta x )</td>
<td>displacement</td>
<td>( \Delta y ) displacement</td>
<td>m meters</td>
</tr>
<tr>
<td>( v )</td>
<td>velocity</td>
<td>( a ) acceleration</td>
<td>m/s(^2) meters per second (^2)</td>
</tr>
</tbody>
</table>

#### PROBLEM SOLVING

See Appendix D: Equations for a summary of the equations introduced in this chapter. If you need more problem-solving practice, see Appendix I: Additional Problems.
**Review Questions**

1. On the graph below, what is the total distance traveled during the recorded time interval? What is the displacement?

2. On a position-time graph such as the one above, what represents the instantaneous velocity?

3. The position-time graph for a bug crawling along a line is shown in item 4 below. Determine whether the velocity is positive, negative, or zero at each of the times marked on the graph.

4. Use the position-time graph below to answer the following questions:
   a. During which time interval(s) is the velocity negative?
   b. During which time interval(s) is the velocity positive?

5. If the average velocity of a duck is zero in a given time interval, what can you say about the displacement of the duck for that interval?

6. Velocity can be either positive or negative, depending on the direction of the displacement. The time interval, $\Delta t$, is always positive. Why?

**Practice Problems**

For problems 7–11, see Sample Problem A.

7. A school bus takes 0.530 h to reach the school from your house. If the average velocity of the bus is 19.0 km/h to the east, what is the displacement?

8. The Olympic record for the marathon is 2.00 h, 9.00 min, 21.0 s. If the average speed of a runner achieving this record is 5.436 m/s, what is the marathon distance?

9. Two cars are traveling on a desert road, as shown below. After 5.0 s, they are side by side at the next telephone pole. The distance between the poles is 70.0 m. Identify the following quantities:
   a. the displacement of car A after 5.0 s
   b. the displacement of car B after 5.0 s
   c. the average velocity of car A during 5.0 s
   d. the average velocity of car B during 5.0 s

   ![Diagram of two cars](Diagram.png)
10. Sally travels by car from one city to another. She drives for 30.0 min at 80.0 km/h, 12.0 min at 105 km/h, and 45.0 min at 40.0 km/h, and she spends 15.0 min eating lunch and buying gas.
   a. Determine the average speed for the trip.
   b. Determine the total distance traveled.

11. Runner A is initially 6.0 km west of a flagpole and is running with a constant velocity of 9.0 km/h due east. Runner B is initially 5.0 km east of the flagpole and is running with a constant velocity of 8.0 km/h due west. What will be the distance of the two runners from the flagpole when their paths cross? (It is not necessary to convert your answer from kilometers to meters for this problem. You may leave it in kilometers.)

ACCELERATION

Review Questions

12. What would be the acceleration of a turtle that is moving with a constant velocity of 0.25 m/s to the right?

13. Sketch the velocity-time graphs for the following motions.
   a. a city bus that is moving with a constant velocity
   b. a wheelbarrow that is speeding up at a uniform rate of acceleration while moving in the positive direction
   c. a tiger that is speeding up at a uniform rate of acceleration while moving in the negative direction
   d. an iguana that is slowing down at a uniform rate of acceleration while moving in the positive direction
   e. a camel that is slowing down at a uniform rate of acceleration while moving in the negative direction

Conceptual Questions

14. If a car is traveling eastward, can its acceleration be westward? Explain your answer, and use an example in your explanation.

15. The strobe photographs below show a disk moving from left to right under different conditions. The time interval between images is constant. Assuming that the direction to the right is positive, identify the following types of motion in each photograph. (Some may have more than one type of motion.)
   a. the acceleration is positive
   b. the acceleration is negative
   c. the velocity is constant

Practice Problems

For problems 16–17, see Sample Problem B.

16. A car traveling in a straight line has a velocity of +5.0 m/s. After an acceleration of 0.75 m/s², the car’s velocity is +8.0 m/s. In what time interval did the acceleration occur?

17. The velocity-time graph for an object moving along a straight path is shown below. Find the average accelerations during the time intervals 0.0 s to 5.0 s, 5.0 s to 15.0 s, and 0.0 s to 20.0 s.

For problems 18–19, see Sample Problem C.

18. A bus slows down uniformly from 75.0 km/h (21 m/s) to 0 km/h in 21 s. How far does it travel before stopping?
19. A car accelerates uniformly from rest to a speed of 65 km/h (18 m/s) in 12 s. Find the distance the car travels during this time.

For problems 20–23, see Sample Problem D.

20. A car traveling at +7.0 m/s accelerates at the rate of +0.80 m/s² for an interval of 2.0 s. Find v_f.

21. A car accelerates from rest at −3.00 m/s².
   a. What is the velocity at the end of 5.0 s?
   b. What is the displacement after 5.0 s?

22. A car starts from rest and travels for 5.0 s with a uniform acceleration of +1.5 m/s². The driver then applies the brakes, causing a uniform acceleration of −2.0 m/s². If the brakes are applied for 3.0 s, how fast is the car going at the end of the braking period, and how far has it gone from its start?

23. A boy sledding down a hill accelerates at 1.40 m/s². If he started from rest, in what distance would he reach a speed of 7.00 m/s?

For problems 24–25, see Sample Problem E.

24. A sailboat starts from rest and accelerates at a rate of 0.21 m/s² over a distance of 280 m.
   a. Find the magnitude of the boat’s final velocity.
   b. Find the time it takes the boat to travel this distance.

25. An elevator is moving upward at 1.20 m/s when it experiences an acceleration of 0.31 m/s² downward, over a distance of 0.75 m. What will its final velocity be?

FALLING OBJECTS

Conceptual Questions

26. A ball is thrown vertically upward.
   a. What happens to the ball’s velocity while the ball is in the air?
   b. What is its velocity when it reaches its maximum altitude?
   c. What is its acceleration when it reaches its maximum altitude?
   d. What is its acceleration just before it hits the ground?
   e. Does its acceleration increase, decrease, or remain constant?

27. The image at right is a strobe photograph of two falling balls released simultaneously. (This motion does not take place in a vacuum.) The ball on the left side is solid, and the ball on the right side is a hollow table-tennis ball. Analyze the motion of both balls in terms of velocity and acceleration.

28. A juggler throws a bowling pin into the air with an initial velocity v_i. Another juggler drops a pin at the same instant. Compare the accelerations of the two pins while they are in the air.

29. A bouquet is thrown upward.
   a. Will the value for the bouquet’s displacement be the same no matter where you place the origin of the coordinate system?
   b. Will the value for the bouquet’s velocity be the same?
   c. Will the value for the bouquet’s acceleration be the same?

Practice Problems

For problems 30–32, see Sample Problem F.

30. A worker drops a wrench from the top of a tower 80.0 m tall. What is the velocity when the wrench strikes the ground?

31. A peregrine falcon dives at a pigeon. The falcon starts downward from rest with free-fall acceleration. If the pigeon is 76.0 m below the initial position of the falcon, how long does the falcon take to reach the pigeon? Assume that the pigeon remains at rest.

32. A ball is thrown upward from the ground with an initial speed of 25 m/s; at the same instant, a ball is dropped from rest from a building 15 m high. After how long will the balls be at the same height?

MIXED REVIEW

33. If the average speed of an orbiting space shuttle is 27 800 km/h, determine the time required for it to circle Earth. Assume that the shuttle is orbiting about 320.0 km above Earth’s surface, and that Earth’s radius is 6380 km.
34. A ball is thrown directly upward into the air. The graph below shows the vertical position of the ball with respect to time.

a. How much time does the ball take to reach its maximum height?
b. How much time does the ball take to reach one-half its maximum height?
c. Estimate the slope of $\Delta y/\Delta t$ at $t = 0.05$ s, $t = 0.10$ s, $t = 0.15$ s, and $t = 0.20$ s. On your paper, draw a coordinate system with velocity ($v$) on the y-axis and time ($t$) on the x-axis. Plot your velocity estimates against time.
d. From your graph, determine what the acceleration on the ball is.

![Position vs Time Graph]

35. A train travels between stations 1 and 2, as shown below. The engineer of the train is instructed to start from rest at station 1 and accelerate uniformly between points $A$ and $B$, then coast with a uniform velocity between points $B$ and $C$, and finally accelerate uniformly between points $C$ and $D$ until the train stops at station 2. The distances $AB$, $BC$, and $CD$ are all equal, and it takes 5.00 min to travel between the two stations. Assume that the uniform accelerations have the same magnitude, even when they are opposite in direction.

a. How much of this 5.00 min period does the train spend between points $A$ and $B$?
b. How much of this 5.00 min period does the train spend between points $B$ and $C$?
c. How much of this 5.00 min period does the train spend between points $C$ and $D$?

36. Two students are on a balcony 19.6 m above the street. One student throws a ball vertically downward at 14.7 m/s. At the same instant, the other student throws a ball vertically upward at the same speed. The second ball just misses the balcony on the way down.

a. What is the difference in the time the balls spend in the air?
b. What is the velocity of each ball as it strikes the ground?
c. How far apart are the balls 0.800 s after they are thrown?

37. A rocket moves upward, starting from rest with an acceleration of $+29.4 \text{ m/s}^2$ for 3.98 s. It runs out of fuel at the end of the 3.98 s but does not stop. How high does it rise above the ground?

38. Two cars travel westward along a straight highway, one at a constant velocity of 85 km/h, and the other at a constant velocity of 115 km/h.

a. Assuming that both cars start at the same point, how much sooner does the faster car arrive at a destination 16 km away?
b. How far must the cars travel for the faster car to arrive 15 min before the slower car?

39. A small first-aid kit is dropped by a rock climber who is descending steadily at 1.3 m/s. After 2.5 s, what is the velocity of the first-aid kit, and how far is the kit below the climber?

40. A small fish is dropped by a pelican that is rising steadily at 0.50 m/s. After 2.5 s, what is the velocity of the fish?

a. After 2.5 s, what is the velocity of the fish?
b. How far below the pelican is the fish after 2.5 s?

41. A ranger in a national park is driving at 56 km/h when a deer jumps onto the road 65 m ahead of the vehicle. After a reaction time of $t$ s, the ranger applies the brakes to produce an acceleration of $-3.0 \text{ m/s}^2$. What is the maximum reaction time allowed if the ranger is to avoid hitting the deer?
42. A speeder passes a parked police car at 30.0 m/s. The police car starts from rest with a uniform acceleration of 2.44 m/s².
   a. How much time passes before the speeder is overtaken by the police car?
   b. How far does the speeder get before being overtaken by the police car?

43. An ice sled powered by a rocket engine starts from rest on a large frozen lake and accelerates at +13.0 m/s². At $t_1$, the rocket engine is shut down and the sled moves with constant velocity $v$ until $t_2$. The total distance traveled by the sled is $5.30 \times 10^3$ m and the total time is 90.0 s. Find $t_1$, $t_2$, and $v$.
   (See Appendix A for hints on solving quadratic equations.)

44. At the 5800 m mark, the sled in the previous question begins to accelerate at −7.0 m/s². Use your answers from item 43 to answer the following questions.
   a. What is the final position of the sled when it comes to rest?
   b. How long does it take for the sled to come to rest?

45. A tennis ball with a velocity of +10.0 m/s to the right is thrown perpendicularly at a wall. After striking the wall, the ball rebounds in the opposite direction with a velocity of −8.0 m/s to the left. If the ball is in contact with the wall for 0.012 s, what is the average acceleration of the ball while it is in contact with the wall?

46. A parachutist descending at a speed of 10.0 m/s loses a shoe at an altitude of 50.0 m.
   a. When does the shoe reach the ground?
   b. What is the velocity of the shoe just before it hits the ground?

47. A mountain climber stands at the top of a 50.0 m cliff hanging over a calm pool of water. The climber throws two stones vertically 1.0 s apart and observes that they cause a single splash when they hit the water. The first stone has an initial velocity of +2.0 m/s.
   a. How long after release of the first stone will the two stones hit the water?
   b. What is the initial velocity of the second stone when it is thrown?
   c. What will the velocity of each stone be at the instant both stones hit the water?

---

**Graphing Calculator Practice**

**Motion in One Dimension**
At what speed does a falling hailstone travel? Does the speed depend on the distance that the hailstone falls? In this graphing calculator activity, you will have the opportunity to answer these questions. Your calculator will display two graphs: one for displacement (distance fallen) versus time and the other for speed versus time. These two graphs correspond to the following two equations:

\[ Y_1 = 4.9X^2 \]
\[ Y_2 = 9.8X \]

You should be able to use Table 4 of this chapter to correlate these equations with those for an accelerating object that starts from rest.

Visit go.hrw.com and type in the keyword HF6MODX to find this graphing calculator activity. Refer to Appendix B for instructions on downloading the program for this activity.
48. A model rocket is launched straight upward with an initial speed of 50.0 m/s. It accelerates with a constant upward acceleration of 2.00 m/s² until its engines stop at an altitude of 150 m.
   a. What is the maximum height reached by the rocket?
   b. When does the rocket reach maximum height?
   c. How long is the rocket in the air?

49. A professional race-car driver buys a car that can accelerate at +5.9 m/s². The racer decides to race against another driver in a souped-up stock car. Both start from rest, but the stock-car driver leaves 1.0 s before the driver of the race car. The stock car moves with a constant acceleration of +3.6 m/s².
   a. Find the time it takes the race-car driver to overtake the stock-car driver.
   b. Find the distance the two drivers travel before they are side by side.
   c. Find the velocities of both cars at the instant they are side by side.

50. Two cars are traveling along a straight line in the same direction, the lead car at 25 m/s and the other car at 35 m/s. At the moment the cars are 45 m apart, the lead driver applies the brakes, causing the car to have an acceleration of −2.0 m/s².
   a. How long does it take for the lead car to stop?
   b. Assume that the driver of the chasing car applies the brakes at the same time as the driver of the lead car. What must the chasing car’s minimum negative acceleration be to avoid hitting the lead car?
   c. How long does it take the chasing car to stop?

51. One swimmer in a relay race has a 0.50 s lead and is swimming at a constant speed of 4.00 m/s. The swimmer has 20.0 m to swim before reaching the end of the pool. A second swimmer moves in the same direction as the leader. What constant speed must the second swimmer have in order to catch up to the leader at the end of the pool?

Alternative Assessment

1. Can a boat moving eastward accelerate to the west? What happens to the boat’s velocity? Name other examples of objects accelerating in the direction opposite their motion, including one with numerical values. Create diagrams and graphs.

2. The next time you are a passenger in a car, record the numbers displayed on the clock, the odometer, and the speedometer every 15 s for about 5 min. Create different representations of the car’s motion, including maps, charts, and graphs. Exchange your representations with someone who made a different trip, and attempt to reconstruct that trip based on his or her report.

3. Two stones are thrown from a cliff at the same time with the same speed, one upward and one downward. Which stone, if either, hits the ground first? Which, if either, hits with the higher speed? In a group discussion, make your best argument for each possible prediction. Set up numerical examples and solve them to test your prediction.

4. Research typical values for velocities and acceleration of various objects. Include many examples, such as different animals, means of transportation, sports, continental drift, light, subatomic particles, and planets. Organize your findings for display on a poster or some other form.

5. Research Galileo’s work on falling bodies. What did he want to demonstrate? What opinions or theories was he trying to refute? What arguments did he use to persuade others that he was right? Did he depend on experiments, logic, findings of other scientists, or other approaches?

6. The study of various motions in nature requires devices for measuring periods of time. Prepare a presentation on a specific type of clock, such as water clocks, sand clocks, pendulum clocks, wind-up clocks, atomic clocks, or biological clocks. Who invented or discovered the clock? What scale of time does it measure? What are the principles or phenomena behind each clock? Can they be calibrated?
MULTIPLE CHOICE

Use the graphs below to answer questions 1–3.

1. Which graph represents an object moving with a constant positive velocity?
   A. I
   B. II
   C. III
   D. IV

2. Which graph represents an object at rest?
   F. I
   G. II
   H. III
   J. IV

3. Which graph represents an object moving with constant positive acceleration?
   A. I
   B. II
   C. III
   D. IV

4. A bus travels from El Paso, Texas, to Chihuahua, Mexico, in 5.2 h with an average velocity of 73 km/h to the south. What is the bus's displacement?
   F. 73 km to the south
   G. 370 km to the south
   H. 380 km to the south
   J. 14 km/h to the south

Use the following position-time graph of a squirrel running along a clothesline to answer questions 5–6.

5. What is the squirrel’s displacement at time \( t = 3.0 \) s?
   A. \(-6.0 \) m
   B. \(-2.0 \) m
   C. \(+0.8 \) m
   D. \(+2.0 \) m

6. What is the squirrel’s average velocity during the time interval between 0.0 s and 3.0 s?
   F. \(-2.0 \) m/s
   G. \(-0.67 \) m/s
   H. \(0.0 \) m/s
   J. \(+0.53 \) m/s

7. Which of the following statements is true of acceleration?
   A. Acceleration always has the same sign as displacement.
   B. Acceleration always has the same sign as velocity.
   C. The sign of acceleration depends on both the direction of motion and how the velocity is changing.
   D. Acceleration always has a positive sign.

8. A ball initially at rest rolls down a hill and has an acceleration of 3.3 m/s\(^2\). If it accelerates for 7.5 s, how far will it move during this time?
   F. 12 m
   G. 93 m
   H. 120 m
   J. 190 m
9. Which of the following statements is true for a ball thrown vertically upward?
   A. The ball has a negative acceleration on the way up and a positive acceleration on the way down.
   B. The ball has a positive acceleration on the way up and a negative acceleration on the way down.
   C. The ball has zero acceleration on the way up and a positive acceleration on the way down.
   D. The ball has a constant acceleration throughout its flight.

**SHORT RESPONSE**

10. In one or two sentences, explain the difference between displacement and distance traveled.

11. The graph below shows the position of a runner at different times during a run. Use the graph to determine the runner’s displacement and average velocity:
   a. for the time interval from \( t = 0.0 \) min to \( t = 10.0 \) min
   b. for the time interval from \( t = 10.0 \) min to \( t = 20.0 \) min
   c. for the time interval from \( t = 20.0 \) min to \( t = 30.0 \) min
   d. for the entire run

![](image)

12. For an object moving with constant negative acceleration, draw the following:
   a. a graph of position vs. time
   b. a graph of velocity vs. time

For both graphs, assume the object starts with a positive velocity and a positive displacement from the origin.

13. A snowmobile travels in a straight line. The snowmobile’s initial velocity is +3.0 m/s.
   a. If the snowmobile accelerates at a rate of +0.50 m/s\(^2\) for 7.0 s, what is its final velocity?
   b. If the snowmobile accelerates at the rate of −0.60 m/s\(^2\) from its initial velocity of +3.0 m/s, how long will it take to reach a complete stop?

**EXTENDED RESPONSE**

14. A car moving eastward along a straight road increases its speed uniformly from 16 m/s to 32 m/s in 10.0 s.
   a. What is the car’s average acceleration?
   b. What is the car’s average velocity?
   c. How far did the car move while accelerating?

Show all of your work for these calculations.

15. A ball is thrown vertically upward with a speed of 25.0 m/s from a height of 2.0 m.
   a. How long does it take the ball to reach its highest point?
   b. How long is the ball in the air?

Show all of your work for these calculations.

*Test Tip* When filling in your answers on an answer sheet, always check to make sure you are filling in the answer for the right question. If you have to change an answer, be sure to completely erase your previous answer.
In the laboratory, you can use a recording timer to determine the velocity and acceleration of bodies moving in one dimension. A recording timer measures the time it takes an object to move a short distance by making marks at regular time intervals on a strip of paper attached to the moving object. As the paper tape is pulled through the timer, the distance between two dots on the tape is the distance the tape moved during one back-and-forth vibration of the clapper. The time required for one back-and-forth motion of the clapper is called the period of the timer.

In this experiment, you will first calibrate a recording timer by determining an average value for its period. You will then use the recording timer to determine the average velocity and average acceleration of falling bodies of different masses.

**PROCEDURE**

**Preparation**

1. Read the entire lab procedure, and plan the steps you will take.

2. If you are not using a datasheet provided by your teacher, prepare a data table in your lab notebook with six columns and five rows. In the first row, label the first two columns Trial and Mass (kg). The space for the third through sixth columns should be labeled Distance (m). Under this common label, columns 3–6 should be labeled A–B, C–D, E–F, and G–H. In the first column, label the second through fifth rows 1, 2, 3, and 4.

3. If you are not using a datasheet provided by your teacher, prepare a second data table with three columns and five rows in your lab notebook. Label this table Calibration. In the first row, label the columns Trial, Time (s), and Number of Dots. Fill in the first column by labeling the second through fifth rows 1, 2, 3, and 4 for the number of trials.
4. Clamp the recording timer to the ring stand to hold the timer in place. Choose a location that will allow you to pull a long section of paper tape through the timer in a straight line without hitting any obstacles. **Do not plug in the timer until your teacher approves your setup.**

5. Insert a strip of paper tape about 2.0 m long into the timer so that the paper can move freely and will be marked as it moves. Lay the tape flat behind the timer. As shown in **Figure 1**, one student should hold the end of the tape in front of the timer.

6. When your teacher approves your setup, plug the timer into the wall socket.

7. One student should start the timer and the stopwatch at the same time that the other student holding the free end of the tape begins pulling the tape through the timer at a steady pace by walking away from the timer.

   a. After exactly 3.0 s, the first student should turn off the timer and stop the watch, just as the second student with the tape stops walking. Mark the first and last dots on the tape. Tear or cut the dotted strip of tape from the roll and label it with the trial number and the time interval as measured by the stopwatch.

   b. Repeat this procedure three more times. Label all tapes.

8. Count the number of dots for each trial, starting with the second dot. Record this number in your data table.

   a. Compute the period of the timer for each trial by dividing the 3.0 s time interval by the number of dots recorded in the table.

   b. Find the average value for the period of the recording timer. Use this value for all your calculations.

---

**Figure 1**

**Step 4:** Put the ring stand at the edge of the table so the tape can be pulled through parallel to the floor for the calibration step. If your timer will not mount on a ring stand as shown, clamp it to the table instead.

**Step 5:** Thread the tape through the timer and make sure the paper tape is under the carbon disk.
Speed and Acceleration of a Falling Object

9. Set up the apparatus as shown in Figure 2. If the timer cannot be mounted on the stand, clamp the timer to the edge of the table.

10. Cut a length of paper tape that is at least 20 cm longer than the distance between the timer and the floor. Thread the end of the tape through the timer.

11. Fold the end of the paper tape and fasten it with masking tape to make a loop. Hook a 200 g mass through the looped end of the paper tape, as shown.

12. Position the mass at a convenient level near the timer, as shown. Hold the mass in place by holding the tape behind the timer. Make sure the area is clear of people and objects. Simultaneously, start the timer and release the tape so the mass falls to the floor. Stop the timer when the mass hits the floor.

13. Label the tape with the mass used. Label the second and third dots A and B, respectively. Count four dots from B and label the seventh and eighth dots C and D, respectively. Label the twelfth and thirteenth dots E and F, and label the seventeenth and eighteenth dots G and H.

14. Repeat this procedure using different, larger masses, such as 300 g and 400 g masses. Drop each mass from the same level in each trial. Label all tapes, and record all data.

15. On each tape, measure the distance between A and B, between C and D, and so on. Record the distance in meters in your data table.

16. Clean up your work area. Put equipment away safely so that it is ready to be used again.

**ANALYSIS**

1. **Organizing Data**  For each trial with the falling mass, find the magnitude of the average velocity, \( v_{\text{avg}} \). Divide the distance A–B by the average period of the timer. Repeat this calculation for the other marked distances for each trial.

2. **Organizing Data**  Using the results from item 1, calculate the average acceleration. Find the change of speed between the distance A–B and the distance C–D, between the distance C–D and the distance E–F, and so on. (Hint: Remember to use the total time interval for each calculation. For example, for the first calculation, use the time interval from A to D.)
3. **Constructing Graphs** Use your data to plot the following graphs for each trial. On each graph, label the axes and indicate the trial number. Use a graphing calculator, computer, or graph paper.
   
   a. position versus time  
   b. velocity versus time  
   c. acceleration versus time

4. **Organizing Data** Use the values for the average acceleration for all four trials to find the average value.

5. **Evaluating Results** Use the accepted value for the free-fall acceleration given in the text and the average of your results from item 4.
   
   a. Determine the absolute error of your results using the following equation:
      \[
      \text{absolute error} = \left| \text{experimental} - \text{accepted} \right|
      \]
   
   b. Determine the relative error of your results using the following equation:
      \[
      \text{relative error} = \frac{\text{experimental} - \text{accepted}}{\text{accepted}}
      \]

**CONCLUSIONS**

6. **Making Predictions** Based on your results, how long would it take a 1000 kg mass to reach the floor if it were dropped from the same height as the masses in this experiment?

7. **Analyzing Graphs** Calculate the slope of each velocity-time graph from item 3b.

8. **Evaluating Results** Find the average value for the slope of the velocity-time graphs. What is the relationship between this value and the values you found for the average accelerations of the masses?

**EXTENSION**

9. **Designing Experiments** Devise a plan to perform this experiment to study the motion of an object thrown straight up into the air. Make sure you take into account any special safety requirements or equipment you might need to use. If there is time and your teacher approves your plan, perform the experiment. Use your data to plot graphs of the position, velocity, and acceleration versus time.
Without air resistance, any object that is thrown or launched into the air and that is subject to gravitational force will follow a parabolic path. The water droplets in this fountain are one example. The velocity of any object in two-dimensional motion—such as one of these water droplets—can be separated into horizontal and vertical components, as shown in the diagram.

**WHAT TO EXPECT**

In this chapter, you will use vectors to analyze two-dimensional motion and to solve problems in which objects are projected into the air.

**WHY IT MATTERS**

After you know how to analyze two-dimensional motion, you can predict where a falling object will land based on its initial velocity and position.

**CHAPTER PREVIEW**

1 Introduction to Vectors
   - Scalars and Vectors
   - Properties of Vectors

2 Vector Operations
   - Coordinate Systems in Two Dimensions
   - Determining Resultant Magnitude and Direction
   - Resolving Vectors into Components
   - Adding Vectors That Are Not Perpendicular

3 Projectile Motion
   - Two-Dimensional Motion

4 Relative Motion
   - Frames of Reference
   - Relative Velocity
SECTION OBJECTIVES

- Distinguish between a scalar and a vector.
- Add and subtract vectors by using the graphical method.
- Multiply and divide vectors by scalars.

** Scalars and Vectors**

In the chapter “Motion in One Dimension,” our discussion of motion was limited to two directions, forward and backward. Mathematically, we described these directions of motion with a positive or negative sign. That method works only for motion in a straight line. This chapter explains a method of describing the motion of objects that do not travel along a straight line.

Vectors indicate direction; scalars do not

Each of the physical quantities encountered in this book can be categorized as either a scalar quantity or a vector quantity. A scalar is a quantity that has magnitude but no direction. Examples of scalar quantities are speed, volume, and the number of pages in this textbook. A vector is a physical quantity that has both direction and magnitude.

As we look back to the chapter “Motion in One Dimension,” we can see that displacement is an example of a vector quantity. An airline pilot planning a trip must know exactly how far and which way to fly. Velocity is also a vector quantity. If we wish to describe the velocity of a bird, we must specify both its speed (say, 3.5 m/s) and the direction in which the bird is flying (say, northeast). Another example of a vector quantity is acceleration.

Vectors are represented by boldface symbols

In physics, quantities are often represented by symbols, such as \( t \) for time. To help you keep track of which symbols represent vector quantities and which are used to indicate scalar quantities, this book will use **boldface** type to indicate vector quantities. Scalar quantities will be in *italics*. For example, the speed of a bird is written as \( v = 3.5 \text{ m/s} \). But a velocity, which includes a direction, is written as \( \mathbf{v} = 3.5 \text{ m/s to the northeast} \). When writing a vector on your paper, you can distinguish it from a scalar by drawing an arrow above the abbreviation for a quantity, such as \( \mathbf{v} = 3.5 \text{ m/s to the northeast} \).

One way to keep track of vectors and their directions is to use diagrams. In diagrams, vectors are shown as arrows that point in the direction of the vector. The length of a vector arrow in a diagram is proportional to the vector’s magnitude. For example, in **Figure 1** the arrows represent the velocities of the two soccer players running toward the soccer ball.

---

**Figure 1**
The lengths of the vector arrows represent the magnitudes of these two soccer players’ velocities.
A resultant vector represents the sum of two or more vectors

When adding vectors, you must make certain that they have the same units and describe similar quantities. For example, it would be meaningless to add a velocity vector to a displacement vector because they describe different physical quantities. Similarly, it would be meaningless, as well as incorrect, to add two displacement vectors that are not expressed in the same units. For example, you cannot add meters and feet together.

Section 1 of the chapter “Motion in One Dimension” covered vector addition and subtraction in one dimension. Think back to the example of the gecko that ran up a tree from a 20 cm marker to an 80 cm marker. Then the gecko reversed direction and ran back to the 50 cm marker. Because the two parts of this displacement are opposite, they can be added together to give a total displacement of 30 cm. The answer found by adding two vectors in this way is called the resultant.

Vectors can be added graphically

Consider a student walking 1600 m to a friend’s house and then 1600 m to school, as shown in Figure 2. The student’s total displacement during his walk to school is in a direction from his house to the school, as shown by the dotted line. This direct path is the vector sum of the student’s displacement from his house to his friend’s house and his displacement from the friend’s house to school. How can this resultant displacement be found?

One way to find the magnitude and direction of the student’s total displacement is to draw the situation to scale on paper. Use a reasonable scale, such as 50 m on land equals 1 cm on paper. First draw the vector representing the student’s displacement from his house to his friend’s house, giving the proper direction and scaled magnitude. Then draw the vector representing his walk to the school, starting with the tail at the head of the first vector. Again give its scaled magnitude and the right direction. The magnitude of the resultant vector can then be determined by using a ruler. Measure the length of the vector pointing from the tail of the first vector to the head of the second vector. The length of that vector can then be multiplied by 50 (or whatever scale you have chosen) to get the actual magnitude of the student’s total displacement in meters.

The direction of the resultant vector may be determined by using a protractor to measure the angle between the resultant and the first vector or between the resultant and any chosen reference line.
PROPERTIES OF VECTORS

Now consider a case in which two or more vectors act at the same point. When this occurs, it is possible to find a resultant vector that has the same net effect as the combination of the individual vectors. Imagine looking down from the second level of an airport at a toy car moving at 0.80 m/s across a walkway that moves at 1.5 m/s. How can you determine what the car’s resultant velocity will look like from your viewpoint?

Vectors can be moved parallel to themselves in a diagram

Note that the car’s resultant velocity while moving from one side of the walkway to the other will be the combination of two independent motions. Thus, the moving car can be thought of as traveling first at 0.80 m/s across the walkway and then at 1.5 m/s down the walkway. In this way, we can draw a given vector anywhere in the diagram as long as the vector is parallel to its previous alignment (so that it still points in the same direction). Thus, you can draw one vector with its tail starting at the tip of the other as long as the size and direction of each vector do not change. This process is illustrated in Figure 3.

Although both vectors act on the car at the same point, the horizontal vector has been moved up so that its tail begins at the tip of the vertical vector. The resultant vector can then be drawn from the tail of the first vector to the tip of the last vector. This method is known as the triangle (or polygon) method of addition.

Again, the magnitude of the resultant vector can be measured using a ruler, and the angle can be measured with a protractor. In the next section, we will develop a technique for adding vectors that is less time-consuming because it involves a calculator instead of a ruler and protractor.

Vectors can be added in any order

When two or more vectors are added, the sum is independent of the order of the addition. This idea is demonstrated by a runner practicing for a marathon along city streets, as represented in Figure 4. The runner executes the same four displacements in each case, but the order is different. Regardless of which path the runner takes, the runner will have the same total displacement, expressed as \( \mathbf{d} \). Similarly, the vector sum of two or more vectors is the same regardless of the order in which the vectors are added, provided that the magnitude and direction of each vector remain the same.

To subtract a vector, add its opposite

Vector subtraction makes use of the definition of the negative of a vector. The negative of a vector is defined as a vector with the same magnitude as the original vector but opposite in direction. For instance, the negative of the velocity of a car traveling 30 m/s to the west is \(-30 \text{ m/s}\) to the west, or \(+30 \text{ m/s}\) to the east. Thus, adding a vector to its negative vector gives zero.
When subtracting vectors in two dimensions, first draw the negative of the vector to be subtracted. Then add that negative vector to the other vector by using the triangle method of addition.

**Multiplying or dividing vectors by scalars results in vectors**

There are mathematical operations in which vectors can multiply other vectors, but they are not needed in this book. This book does, however, make use of vectors multiplied by scalars, with a vector as the result. For example, if a cab driver obeys a customer who tells him to go twice as fast, that cab’s original velocity vector, \( \mathbf{v}_{\text{cab}} \), is multiplied by the scalar number 2. The result, written as \( 2\mathbf{v}_{\text{cab}} \), is a vector with a magnitude twice that of the original vector and pointing in the same direction.

On the other hand, if another cab driver is told to go twice as fast in the opposite direction, this is the same as multiplying by the scalar number \(-2\). The result is a vector with a magnitude two times the initial velocity but pointing in the opposite direction, written as \(-2\mathbf{v}_{\text{cab}}\).

---

### SECTION REVIEW

1. Which of the following quantities are scalars, and which are vectors?
   - a. the acceleration of a plane as it takes off
   - b. the number of passengers on the plane
   - c. the duration of the flight
   - d. the displacement of the flight
   - e. the amount of fuel required for the flight

2. A roller coaster moves 85 m horizontally, then travels 45 m at an angle of 30.0° above the horizontal. What is its displacement from its starting point? Use graphical techniques.

3. A novice pilot sets a plane’s controls, thinking the plane will fly at \( 2.50 \times 10^2 \) km/h to the north. If the wind blows at 75 km/h toward the southeast, what is the plane’s resultant velocity? Use graphical techniques.

4. While flying over the Grand Canyon, the pilot slows the plane down to one-half the velocity in item 3. If the wind’s velocity is still 75 km/h toward the southeast, what will the plane’s new resultant velocity be? Use graphical techniques.

5. **Critical Thinking** The water used in many fountains is recycled. For instance, a single water particle in a fountain travels through an 85 m system and then returns to the same point. What is the displacement of this water particle during one cycle?
Section 2

Vector Operations

Section Objectives

- Identify appropriate coordinate systems for solving problems with vectors.
- Apply the Pythagorean theorem and tangent function to calculate the magnitude and direction of a resultant vector.
- Resolve vectors into components using the sine and cosine functions.
- Add vectors that are not perpendicular.

Coordinate Systems in Two Dimensions

In the chapter “Motion in One Dimension,” the motion of a gecko climbing a tree was described as motion along the \( y \)-axis. The direction of the displacement of the gecko was denoted by a positive or negative sign. The displacement of the gecko can now be described by an arrow pointing along the \( y \)-axis, as shown in Figure 5. A more versatile system for diagraming the motion of an object, however, employs vectors and the use of both the \( x \)- and \( y \)-axes simultaneously.

The addition of another axis not only helps describe motion in two dimensions but also simplifies analysis of motion in one dimension. For example, two methods can be used to describe the motion of a jet moving at 300 m/s to the northeast. In one approach, the coordinate system can be turned so that the plane is depicted as moving along the \( y \)-axis, as in Figure 6(a). The jet’s motion also can be depicted on a two-dimensional coordinate system whose axes point north and east, as shown in Figure 6(b).

One problem with the first method is that the axis must be turned again if the direction of the plane changes. Another problem is that the first method provides no way to deal with a second airplane that is not traveling in the same direction as the first airplane. Thus, axes are often designated using fixed directions. For example, in Figure 6(b), the positive \( y \)-axis points north and the positive \( x \)-axis points east.

Similarly, when you analyze the motion of objects thrown into the air, orienting the \( y \)-axis parallel to the vertical direction simplifies problem solving.
There are no firm rules for applying coordinate systems to situations involving vectors. As long as you are consistent, the final answer will be correct regardless of the system you choose. Perhaps your best choice for orienting axes is the approach that makes solving the problem easiest for you.

DETERMINING RESULTANT MAGNITUDE AND DIRECTION

In Section 1, the magnitude and direction of a resultant were found graphically. However, this approach is time consuming, and the accuracy of the answer depends on how carefully the diagram is drawn and measured. A simpler method uses the Pythagorean theorem and the tangent function.

Use the Pythagorean theorem to find the magnitude of the resultant

Imagine a tourist climbing a pyramid in Egypt. The tourist knows the height and width of the pyramid and would like to know the distance covered in a climb from the bottom to the top of the pyramid. Assume that the tourist climbs directly up the middle of one face.

As can be seen in Figure 7, the magnitude of the tourist’s vertical displacement, \( \Delta y \), is the height of the pyramid. The magnitude of the horizontal displacement, \( \Delta x \), equals the distance from one edge of the pyramid to the middle, or half the pyramid’s width. Notice that these two vectors are perpendicular and form a right triangle with the displacement, \( d \).

As shown in Figure 8(a), the Pythagorean theorem states that for any right triangle, the square of the hypotenuse—the side opposite the right angle—equals the sum of the squares of the other two sides, or legs.

**PYTHAGOREAN THEOREM FOR RIGHT TRIANGLES**

\[
c^2 = a^2 + b^2
\]

(length of hypotenuse)\(^2\) = (length of one leg)\(^2\) + (length of other leg)\(^2\)

In Figure 8(b), the Pythagorean theorem is applied to find the tourist’s displacement. The square of the displacement is equal to the sum of the square of the horizontal displacement and the square of the vertical displacement. In this way, you can find out the magnitude of the displacement, \( d \).
Use the tangent function to find the direction of the resultant

In order to completely describe the tourist’s displacement, you must also know the direction of the tourist’s motion. Because \( \Delta x, \Delta y, \) and \( d \) form a right triangle, as shown in Figure 9(b), the inverse tangent function can be used to find the angle \( \theta \), which denotes the direction of the tourist’s displacement.

For any right triangle, the tangent of an angle is defined as the ratio of the opposite and adjacent legs with respect to a specified acute angle of a right triangle, as shown in Figure 9(a).

As shown below, the magnitude of the opposite leg divided by the magnitude of the adjacent leg equals the tangent of the angle.

The inverse of the tangent function, which is shown below, gives the angle.

\[
\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)
\]

DEFINITION OF THE TANGENT FUNCTION FOR RIGHT TRIANGLES

\[
\tan \theta = \frac{\text{opposite}}{\text{adjacent}} \quad \text{tangent of angle} = \frac{\text{opposite leg}}{\text{adjacent leg}}
\]

The inverse of the tangent function, which is shown below, gives the angle.

\[
\theta = \tan^{-1}\left(\frac{\text{opp}}{\text{adj}}\right)
\]

SAMPLE PROBLEM A

Finding Resultant Magnitude and Direction

PROBLEM

An archaeologist climbs the Great Pyramid in Giza, Egypt. The pyramid’s height is 136 m and its width is \( 2.30 \times 10^2 \) m. What is the magnitude and the direction of the displacement of the archaeologist after she has climbed from the bottom of the pyramid to the top?

SOLUTION

1. DEFINE

Given: \( \Delta y = 136 \) m \( \Delta x = \frac{1}{2}\) (width) = 115 m

Unknown: \( d = \? \quad \theta = \? \)

Diagram: Choose the archaeologist’s starting position as the origin of the coordinate system.

2. PLAN

Choose an equation or situation:

The Pythagorean theorem can be used to find the magnitude of the archaeologist’s displacement. The direction of the displacement can be found by using the tangent function.

\[
d^2 = \Delta x^2 + \Delta y^2
\]

\[
\tan \theta = \frac{\Delta y}{\Delta x}
\]
Rearrange the equations to isolate the unknowns:

\[ d = \sqrt{\Delta x^2 + \Delta y^2} \]

\[ \theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) \]

Substitute the values into the equations and solve:

\[ d = \sqrt{(115 \text{ m})^2 + (136 \text{ m})^2} \]

\[ d = 178 \text{ m} \]

\[ \theta = \tan^{-1}\left(\frac{136 \text{ m}}{115 \text{ m}}\right) \]

\[ \theta = 49.8^\circ \]

Because \( d \) is the hypotenuse, the archaeologist’s displacement should be less than the sum of the height and half of the width. The angle is expected to be more than 45° because the height is greater than half of the width.

**PRACTICE A**

**Finding Resultant Magnitude and Direction**

1. A truck driver is attempting to deliver some furniture. First, he travels 8 km east, and then he turns around and travels 3 km west. Finally, he turns again and travels 12 km east to his destination.
   a. What distance has the driver traveled?
   b. What is the driver’s total displacement?

2. While following the directions on a treasure map, a pirate walks 45.0 m north and then turns and walks 7.5 m east. What single straight-line displacement could the pirate have taken to reach the treasure?

3. Emily passes a soccer ball 6.0 m directly across the field to Kara. Kara then kicks the ball 14.5 m directly down the field to Luisa. What is the ball’s total displacement as it travels between Emily and Luisa?

4. A hummingbird, 3.4 m above the ground, flies 1.2 m along a straight path. Upon spotting a flower below, the hummingbird drops directly downward 1.4 m to hover in front of the flower. What is the hummingbird’s total displacement?
RESOLVING VECTORS INTO COMPONENTS

In the pyramid example, the horizontal and vertical parts that add up to give the tourist’s actual displacement are called components. The \( x \) component is parallel to the \( x \)-axis. The \( y \) component is parallel to the \( y \)-axis. Any vector can be completely described by a set of perpendicular components.

In this textbook, components of vectors are shown as outlined, open arrows. Components have arrowheads to indicate their direction. Components are scalars (numbers), but they are signed numbers, and the direction is important to determine their sign in a particular coordinate system.

You can often describe an object’s motion more conveniently by breaking a single vector into two components, or resolving the vector. Resolving a vector allows you to analyze the motion in each direction.

This point may be illustrated by examining a scene on the set of a new action movie. For this scene, a biplane travels at 95 km/h at an angle of 20° relative to the ground. Attempting to film the plane from below, a camera team travels in a truck that is directly beneath the plane at all times, as shown in Figure 10.

To find the velocity that the truck must maintain to stay beneath the plane, we must know the horizontal component of the plane’s velocity. Once more, the key to solving the problem is to recognize that a right triangle can be drawn using the plane’s velocity and its \( x \) and \( y \) components. The situation can then be analyzed using trigonometry.

The sine and cosine functions are defined in terms of the lengths of the sides of such right triangles. The sine of an angle is the ratio of the leg opposite that angle to the hypotenuse.

**DEFINITION OF THE SINE FUNCTION FOR RIGHT TRIANGLES**

\[
\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \text{sine of an angle} = \frac{\text{opposite leg}}{\text{hypotenuse}}
\]

In Figure 11, the leg opposite the 20° angle represents the \( y \) component, \( v_y \), which describes the vertical speed of the airplane. The hypotenuse, \( v_{\text{plane}} \), is the resultant vector that describes the airplane’s total velocity.

The cosine of an angle is the ratio between the leg adjacent to that angle and the hypotenuse.

**DEFINITION OF THE COSINE FUNCTION FOR RIGHT TRIANGLES**

\[
\cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \text{cosine of an angle} = \frac{\text{adjacent leg}}{\text{hypotenuse}}
\]

In Figure 11, the adjacent leg represents the \( x \) component, \( v_x \), which describes the airplane’s horizontal speed. This \( x \) component equals the speed that the truck must maintain to stay beneath the plane. Thus, the truck must maintain a speed of \( v_x = (\cos 20°)(95 \text{ km/h}) = 90 \text{ km/h} \).
SAMPLE PROBLEM B

Resolving Vectors

**PROBLEM**
Find the components of the velocity of a helicopter traveling 95 km/h at an angle of 35° to the ground.

**SOLUTION**

1. **DEFINE**

   **Given:** \( v = 95 \text{ km/h} \) \( \theta = 35^\circ \)
   
   **Unknown:** \( v_x = ? \) \( v_y = ? \)
   
   **Diagram:** The most convenient coordinate system is one with the \( x \)-axis directed along the ground and the \( y \)-axis directed vertically.

2. **PLAN**

   **Choose an equation or situation:**
   Because the axes are perpendicular, the sine and cosine functions can be used to find the components.

   \[
   \sin \theta = \frac{v_y}{v} \\
   \cos \theta = \frac{v_x}{v}
   \]

   **Rearrange the equations to isolate the unknowns:**
   \[
   v_y = v \sin \theta \\
   v_x = v \cos \theta
   \]

3. **CALCULATE**

   **Substitute the values into the equations and solve:**
   \[
   v_y = (95 \text{ km/h})(\sin 35^\circ) \\
   v_y = 54 \text{ km/h}
   \]
   
   \[
   v_x = (95 \text{ km/h})(\cos 35^\circ) \\
   v_x = 78 \text{ km/h}
   \]

4. **EVALUATE**

   Because the components of the velocity form a right triangle with the helicopter’s actual velocity, the components must satisfy the Pythagorean theorem.

   \[
   v^2 = v_x^2 + v_y^2 \\
   (95)^2 = (78)^2 + (54)^2 \\
   9025 \approx 9000
   \]

   The slight difference is due to rounding.
Adding Vectors That Are Not Perpendicular

Until this point, the vector-addition problems concerned vectors that are perpendicular to one another. However, many objects move in one direction and then turn at an angle before continuing their motion.

Suppose that a plane initially travels 5 km at an angle of 35° to the ground, then climbs at only 10° relative to the ground for 22 km. How can you determine the magnitude and direction for the vector denoting the total displacement of the plane?

Because the original displacement vectors do not form a right triangle, you can not apply the tangent function or the Pythagorean theorem when adding the original two vectors.

Determining the magnitude and the direction of the resultant can be achieved by resolving each of the plane’s displacement vectors into its x and y components. Then the components along each axis can be added together. As shown in Figure 12, these sums will be the two perpendicular components of the resultant, \( d \). The resultant’s magnitude can then be found by using the Pythagorean theorem, and its direction can be found by using the inverse tangent function.

Module 2
“Vectors” provides an interactive lesson with guided problem-solving practice to teach you how to add different vectors, especially those that are not at right angles.

Resolving Vectors

1. How fast must a truck travel to stay beneath an airplane that is moving 105 km/h at an angle of 25° to the ground?
2. What is the magnitude of the vertical component of the velocity of the plane in item 1?
3. A truck drives up a hill with a 15° incline. If the truck has a constant speed of 22 m/s, what are the horizontal and vertical components of the truck’s velocity?
4. What are the horizontal and vertical components of a cat’s displacement when the cat has climbed 5 m directly up a tree?
SAMPLE PROBLEM C

STRATEGY Adding Vectors Algebraically

PROBLEM A hiker walks 27.0 km from her base camp at 35° south of east. The next day, she walks 41.0 km in a direction 65° north of east and discovers a forest ranger’s tower. Find the magnitude and direction of her resultant displacement between the base camp and the tower.

SOLUTION 1. Select a coordinate system. Then sketch and label each vector.

Given: \( d_1 = 27.0 \text{ km} \) \( \theta_1 = -35° \) \( d_2 = 41.0 \text{ km} \) \( \theta_2 = 65° \)

Unknown: \( d = ? \) \( \theta = ? \)

TIP \( \theta_1 \) is negative, because clockwise movement from the positive x-axis is conventionally considered to be negative.

2. Find the x and y components of all vectors.

Make a separate sketch of the displacements for each day. Use the cosine and sine functions to find the displacement components.

\[
\cos \theta = \frac{\Delta x}{d} \quad \sin \theta = \frac{\Delta y}{d}
\]

(a) For day 1: \( \Delta x_1 = d_1 \cos \theta_1 = (27.0 \text{ km}) \cos (-35°) \) \( = 22 \text{ km} \)
\( \Delta y_1 = d_1 \sin \theta_1 = (27.0 \text{ km}) \sin (-35°) \) \( = -15 \text{ km} \)

(b) For day 2: \( \Delta x_2 = d_2 \cos \theta_2 = (41.0 \text{ km}) \cos 65° \) \( = 17 \text{ km} \)
\( \Delta y_2 = d_2 \sin \theta_2 = (41.0 \text{ km}) \sin 65° \) \( = 37 \text{ km} \)

3. Find the x and y components of the total displacement.

\( \Delta x_{\text{tot}} = \Delta x_1 + \Delta x_2 = 22 \text{ km} + 17 \text{ km} = 39 \text{ km} \)
\( \Delta y_{\text{tot}} = \Delta y_1 + \Delta y_2 = -15 \text{ km} + 37 \text{ km} = 22 \text{ km} \)

4. Use the Pythagorean theorem to find the magnitude of the resultant vector.

\[
d^2 = (\Delta x_{\text{tot}})^2 + (\Delta y_{\text{tot}})^2
\]
\[
d = \sqrt{(\Delta x_{\text{tot}})^2 + (\Delta y_{\text{tot}})^2} = \sqrt{(39 \text{ km})^2 + (22 \text{ km})^2} = 45 \text{ km}
\]

5. Use a suitable trigonometric function to find the angle.

\[
\theta = \tan^{-1} \left( \frac{\Delta y_{\text{tot}}}{\Delta x_{\text{tot}}} \right) = \tan^{-1} \left( \frac{22 \text{ km}}{39 \text{ km}} \right) = 29° \text{ north of east}
\]
**PRACTICE C**

**Adding Vectors Algebraically**

1. A football player runs directly down the field for 35 m before turning to the right at an angle of 25° from his original direction and running an additional 15 m before getting tackled. What is the magnitude and direction of the runner’s total displacement?

2. A plane travels 2.5 km at an angle of 35° to the ground and then changes direction and travels 5.2 km at an angle of 22° to the ground. What is the magnitude and direction of the plane’s total displacement?

3. During a rodeo, a clown runs 8.0 m north, turns 55° north of east, and runs 3.5 m. Then, after waiting for the bull to come near, the clown turns due east and runs 5.0 m to exit the arena. What is the clown’s total displacement?

4. An airplane flying parallel to the ground undergoes two consecutive displacements. The first is 75 km 30.0° west of north, and the second is 155 km 60.0° east of north. What is the total displacement of the airplane?

**SECTION REVIEW**

1. Identify a convenient coordinate system for analyzing each of the following situations:
   a. a dog walking along a sidewalk
   b. an acrobat walking along a high wire
   c. a submarine submerging at an angle of 30° to the horizontal

2. Find the magnitude and direction of the resultant velocity vector for the following perpendicular velocities:
   a. a fish swimming at 3.0 m/s relative to the water across a river that moves at 5.0 m/s
   b. a surfer traveling at 1.0 m/s relative to the water across a wave that is traveling at 6.0 m/s

3. Find the vector components along the directions noted in parentheses.
   a. a car displaced 45° north of east by 10.0 km (north and east)
   b. a duck accelerating away from a hunter at 2.0 m/s² at an angle of 35° to the ground (horizontal and vertical)

4. **Critical Thinking** Why do nonperpendicular vectors need to be resolved into components before you can add the vectors together?
**Projectile Motion**

**TWO-DIMENSIONAL MOTION**

In the last section, quantities such as displacement and velocity were shown to be vectors that can be resolved into components. In this section, these components will be used to understand and predict the motion of objects thrown into the air.

**Use of components avoids vector multiplication**

How can you know the displacement, velocity, and acceleration of a ball at any point in time during its flight? All of the kinematic equations could be rewritten in terms of vector quantities. However, when an object is propelled into the air in a direction other than straight up or down, the velocity, acceleration, and displacement of the object do not all point in the same direction. This makes the vector forms of the equations difficult to solve.

One way to deal with these situations is to avoid using the complicated vector forms of the equations altogether. Instead, apply the technique of resolving vectors into components. Then you can apply the simpler one-dimensional forms of the equations for each component. Finally, you can recombine the components to determine the resultant.

**Components simplify projectile motion**

When a long jumper approaches his jump, he runs along a straight line, which can be called the $x$-axis. When he jumps, as shown in Figure 13, his velocity has both horizontal and vertical components. Movement in this plane can be depicted by using both the $x$- and $y$-axes.

Note that in Figure 14(b), a jumper’s velocity vector is resolved into its two vector components. This way, the jumper’s motion can be analyzed using the kinematic equations applied to one direction at a time.
In this section, we will focus on the form of two-dimensional motion called **projectile motion**. Objects that are thrown or launched into the air and are subject to gravity are called **projectiles**. Some examples of projectiles are softballs, footballs, and arrows when they are projected through the air. Even a long jumper can be considered a projectile.

**Figure 15**

(a) Without air resistance, the soccer ball would travel along a parabola. (b) With air resistance, the soccer ball would travel along a shorter path.

**Projectiles follow parabolic trajectories**

The path of a projectile is a curve called a *parabola*, as shown in **Figure 15(a)**. Many people mistakenly believe that projectiles eventually fall straight down in much the same way that a cartoon character does after running off a cliff. But if an object has an initial horizontal velocity in any given time interval, there will be horizontal motion throughout the flight of the projectile. *Note that for the purposes of samples and exercises in this book, the horizontal velocity of the projectile will be considered constant.* This velocity would not be constant if we accounted for air resistance. With air resistance, a projectile slows down as it collides with air particles, as shown in **Figure 15(b)**.

**Projectile motion is free fall with an initial horizontal velocity**

To understand the motion a projectile undergoes, first examine **Figure 16**. The red ball was dropped at the same instant the yellow ball was launched horizontally. If air resistance is disregarded, both balls hit the ground at the same time. By examining each ball’s position in relation to the horizontal lines and to one another, we see that the two balls fall at the same rate. This may seem impossible because one is given an initial velocity and the other begins from rest. But if the motion is analyzed one component at a time, it makes sense.

First, consider the red ball that falls straight down. It has no motion in the horizontal direction. In the vertical direction, it starts from rest \( v_{y,i} = 0 \text{ m/s} \) and proceeds in free fall. Thus, the kinematic equations from the chapter “Motion in One Dimension” can be applied to analyze the vertical motion of the falling ball, as shown on the next page. Note that on Earth’s surface the acceleration \( a_y \) will equal \(-g\) (\(-9.81 \text{ m/s}^2\)) because the only vertical component of acceleration is free-fall acceleration. Note also that \( \Delta y \) is negative.
Now consider the components of motion of the yellow ball that is launched in Figure 16. This ball undergoes the same horizontal displacement during each time interval. This means that the ball’s horizontal velocity remains constant (if air resistance is assumed to be negligible). Thus, when the kinematic equations are used to analyze the horizontal motion of a projectile, the initial horizontal velocity is equal to the horizontal velocity throughout the projectile’s flight. A projectile’s horizontal motion is described by the following equation.

**VERTICAL MOTION OF A PROJECTILE THAT FALLS FROM REST**

\[
\begin{align*}
v_{y,f} &= a_y \Delta t \\
v_{y,f}^2 &= 2a_y \Delta y \\
\Delta y &= \frac{1}{2}a_y(\Delta t)^2
\end{align*}
\]

Next consider the initial motion of the launched yellow ball in Figure 16. Despite having an initial horizontal velocity, the launched ball has no initial velocity in the vertical direction. Just like the red ball that falls straight down, the launched yellow ball is in free fall. The vertical motion of the launched yellow ball is described by the same free-fall equations. In any time interval, the launched ball undergoes the same vertical displacement as the ball that falls straight down. For this reason, both balls reach the ground at the same time.

To find the velocity of a projectile at any point during its flight, find the vector that has the known components. Specifically, use the Pythagorean theorem to find the magnitude of the velocity, and use the tangent function to find the direction of the velocity.

**HORIZONTAL MOTION OF A PROJECTILE**

\[
\begin{align*}
v_x &= v_{x,i} = \text{constant} \\
\Delta x &= v_x \Delta t
\end{align*}
\]

---

**SAFETY CAUTION**

Perform this experiment away from walls and furniture that can be damaged.

Roll a ball off a table. At the instant the rolling ball leaves the table, drop a second ball from the same height above the floor. Do the two balls hit the floor at the same time? Try varying the speed at which you roll the first ball off the table. Does varying the speed affect whether the two balls strike the ground at the same time? Next roll one of the balls down a slope. Drop the other ball from the base of the slope at the instant the first ball leaves the slope. Which of the balls hits the ground first in this situation?
SAMPLE PROBLEM D

Projectiles Launched Horizontally

PROBLEM
The Royal Gorge Bridge in Colorado rises 321 m above the Arkansas River. Suppose you kick a rock horizontally off the bridge. The magnitude of the rock’s horizontal displacement is 45.0 m. Find the speed at which the rock was kicked.

SOLUTION

1. DEFINE
Given: \( \Delta y = -321 \text{ m} \) \( \Delta x = 45.0 \text{ m} \)
Unknown: \( v_i = v_x = ? \)
Diagram: The initial velocity vector of the rock has only a horizontal component. Choose the coordinate system oriented so that the positive y direction points upward and the positive x direction points to the right.

2. PLAN

Choose an equation or situation:
Because air resistance can be neglected, the rock’s horizontal velocity remains constant.

\[ \Delta x = v_x \Delta t \]
Because there is no initial vertical velocity, the following equation applies.

\[ \Delta y = \frac{1}{2} a_y (\Delta t)^2 \]

Rearrange the equations to isolate the unknowns:
Note that the time interval is the same for the vertical and horizontal displacements, so the second equation can be rearranged to solve for \( \Delta t \).

\[ \Delta t = \sqrt{\frac{2 \Delta y}{a_y}} \]
Next rearrange the first equation for \( v_x \), and substitute the above value of \( \Delta t \) into the new equation.

\[ v_x = \frac{\Delta x}{\Delta t} = \sqrt{\frac{a_y}{2 \Delta y}} \Delta x \]

3. CALCULATE

Substitute the values into the equation and solve:

\[ v_x = \sqrt{\frac{-9.81 \text{ m/s}^2}{2(-321 \text{ m})}} (45.0 \text{ m}) = 5.56 \text{ m/s} \]

4. EVALUATE
To check your work, estimate the value of the time interval for \( \Delta x \) and solve for \( \Delta y \). If \( v_x \) is about 5.5 m/s and \( \Delta x = 45 \text{ m} \), \( \Delta t \approx 8 \text{ s} \). If you use an approximate value of 10 m/s\(^2\) for \( g \), \( \Delta y = -320 \text{ m} \), almost identical to the given value.

TIP: The value for \( v_x \) can be either positive or negative because of the square root. Because the direction was not asked for, use the positive root.
Use components to analyze objects launched at an angle

Let us examine a case in which a projectile is launched at an angle to the horizontal, as shown in Figure 17. The projectile has an initial vertical component of velocity as well as a horizontal component of velocity.

Suppose the initial velocity vector makes an angle $\theta$ with the horizontal. Again, to analyze the motion of such a projectile, you must resolve the initial velocity vector into its components. The sine and cosine functions can be used to find the horizontal and vertical components of the initial velocity.

$$v_{x,i} = v_i \cos \theta \quad \text{and} \quad v_{y,i} = v_i \sin \theta$$

We can substitute these values for $v_{x,i}$ and $v_{y,i}$ into the kinematic equations to obtain a set of equations that can be used to analyze the motion of a projectile launched at an angle.

**PROJECTILES LAUNCHED AT AN ANGLE**

$$v_x = v_{x,i} = v_i \cos \theta = \text{constant}$$

$$\Delta x = (v_i \cos \theta) \Delta t$$

$$v_{y,f} = v_i \sin \theta + a_y \Delta t$$

$$v_{y,f}^2 = v_i^2 (\sin \theta)^2 + 2a_y \Delta y$$

$$\Delta y = (v_i \sin \theta) \Delta t + \frac{1}{2} a_y (\Delta t)^2$$

As we have seen, the velocity of a projectile launched at an angle to the ground has both horizontal and vertical components. The vertical motion is similar to that of an object that is thrown straight up with an initial velocity.
A zookeeper finds an escaped monkey hanging from a light pole. Aiming her tranquilizer gun at the monkey, she kneels 10.0 m from the light pole, which is 5.00 m high. The tip of her gun is 1.00 m above the ground. At the same moment that the monkey drops a banana, the zookeeper shoots. If the dart travels at 50.0 m/s, will the dart hit the monkey, the banana, or neither one?

1. Select a coordinate system.
   The positive $y$-axis points up, and the positive $x$-axis points along the ground toward the pole. Because the dart leaves the gun at a height of 1.00 m, the vertical distance is 4.00 m.

2. Use the inverse tangent function to find the angle that the initial velocity makes with the $x$-axis.
   \[ \theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{4.00 \, \text{m}}{10.0 \, \text{m}}\right) = 21.8^\circ \]

3. Choose a kinematic equation to solve for time.
   Rearrange the equation for motion along the $x$-axis to isolate the unknown, $\Delta t$, which is the time the dart takes to travel the horizontal distance.
   \[ \Delta x = (v_i \cos \theta) \Delta t \]
   \[ \Delta t = \frac{\Delta x}{v_i \cos \theta} = \frac{10.0 \, \text{m}}{(50.0 \, \text{m/s})(\cos 21.8^\circ)} = 0.215 \, \text{s} \]

4. Find out how far each object will fall during this time.
   Use the free-fall kinematic equation in both cases. For the banana, $v_i = 0$. Thus:
   \[ \Delta y_b = \frac{1}{2} a_y (\Delta t)^2 = \frac{1}{2}(-9.81 \, \text{m/s}^2)(0.215 \, \text{s})^2 = -0.227 \, \text{m} \]
   The dart has an initial vertical component of velocity equal to $v_i \sin \theta$, so:
   \[ \Delta y_d = (v_i \sin \theta) \Delta t + \frac{1}{2} a_y (\Delta t)^2 \]
   \[ \Delta y_d = (50.0 \, \text{m/s})(\sin 21.8^\circ)(0.215 \, \text{s}) + \frac{1}{2}(-9.81 \, \text{m/s}^2)(0.215 \, \text{s})^2 \]
   \[ \Delta y_d = 3.99 \, \text{m} - 0.227 \, \text{m} = 3.76 \, \text{m} \]

5. Analyze the results.
   Find the final height of both the banana and the dart.
   \[ y_{\text{banana, f}} = y_{b, i} + \Delta y_b = 5.00 \, \text{m} + (-0.227 \, \text{m}) = 4.77 \, \text{m above the ground} \]
   \[ y_{\text{dart, f}} = y_{d, i} + \Delta y_d = 1.00 \, \text{m} + 3.76 \, \text{m} = 4.76 \, \text{m above the ground} \]

The dart hits the banana. The slight difference is due to rounding.
Projectiles Launched at an Angle

1. In a scene in an action movie, a stuntman jumps from the top of one building to the top of another building 4.0 m away. After a running start, he leaps at a velocity of 5.0 m/s at an angle of 15° with respect to the flat roof. Will he make it to the other roof, which is 2.5 m shorter than the building he jumps from?

2. A golfer hits a golf ball at an angle of 25.0° to the ground. If the golf ball covers a horizontal distance of 301.5 m, what is the ball’s maximum height? (Hint: At the top of its flight, the ball’s vertical velocity component will be zero.)

3. A baseball is thrown at an angle of 25° relative to the ground at a speed of 23.0 m/s. If the ball was caught 42.0 m from the thrower, how long was it in the air? How high did the ball travel before being caught?

4. Salmon often jump waterfalls to reach their breeding grounds. One salmon starts 2.00 m from a waterfall that is 0.55 m tall and jumps at an angle of 32.0°. What must be the salmon’s minimum speed to reach the waterfall?

SECTION REVIEW

1. Which of the following exhibit parabolic motion?
   a. a flat rock skipping across the surface of a lake
   b. a three-point shot in basketball
   c. the space shuttle while orbiting Earth
   d. a ball bouncing across a room
   e. a life preserver dropped from a stationary helicopter

2. During a thunderstorm, a tornado lifts a car to a height of 125 m above the ground. Increasing in strength, the tornado flings the car horizontally with a speed of 90.0 m/s. How long does the car take to reach the ground? How far horizontally does the car travel before hitting the ground?

3. Interpreting Graphics An Alaskan rescue plane drops a package of emergency rations to a stranded party of explorers, as illustrated in Figure 18. The plane is traveling horizontally at 30.0 m/s at a height of 200.0 m above the ground.
   a. What horizontal distance does the package fall before landing?
   b. Find the velocity of the package just before it hits the ground.
Relative Motion

FRAMES OF REFERENCE

If you are moving at 80 km/h north and a car passes you going 90 km/h, to you the faster car seems to be moving north at 10 km/h. Someone standing on the side of the road would measure the velocity of the faster car as 90 km/h toward the north. This simple example demonstrates that velocity measurements depend on the frame of reference of the observer.

Velocity measurements differ in different frames of reference

Observers using different frames of reference may measure different displacements or velocities for an object in motion. That is, two observers moving with respect to each other would generally not agree on some features of the motion.

Consider a stunt dummy that is dropped from an airplane flying horizontally over Earth with a constant velocity. As shown in Figure 19(a), a passenger on the airplane would describe the motion of the dummy as a straight line toward Earth. An observer on the ground would view the trajectory of the dummy as that of a projectile, as shown in Figure 19(b). Relative to the ground, the dummy would have a vertical component of velocity (resulting from free-fall acceleration and equal to the velocity measured by the observer in the airplane) and a horizontal component of velocity given to it by the airplane’s motion. If the airplane continued to move horizontally with the same velocity, the dummy would enter the swimming pool directly beneath the airplane (assuming negligible air resistance).

Figure 19
When viewed from the plane (a), the stunt dummy (represented by the maroon dot) falls straight down. When viewed from a stationary position on the ground (b), the stunt dummy follows a parabolic projectile path.
RELATIVE VELOCITY

The case of the faster car overtaking your car was easy to solve with a minimum of thought and effort, but you will encounter many situations in which a more systematic method of solving such problems is beneficial. To develop this method, write down all the information that is given and that you want to know in the form of velocities with subscripts appended.

\[ v_{sc} = +80 \text{ km/h north} \] (Here the subscript \(se\) means the velocity of the \(slo\)wer car with respect to the \(E\)arth.)

\[ v_{fe} = +90 \text{ km/h north} \] (The subscript \(fe\) means the velocity of the \(fa\)st car with respect to the \(E\)arth.)

We want to know \(v_{fs}\), which is the velocity of the fast car with respect to the slower car. To find this, we write an equation for \(v_{fs}\) in terms of the other velocities, so on the right side of the equation the subscripts start with \(f\) and eventually end with \(s\). Also, each velocity subscript starts with the letter that ended the preceding velocity subscript.

\[ v_{fs} = v_{fe} + v_{es} \]

The boldface notation indicates that velocity is a vector quantity. This approach to adding and monitoring subscripts is similar to vector addition, in which vector arrows are placed head to tail to find a resultant.

We know that \(v_{es} = -v_{se}\) because an observer in the slow car perceives Earth as moving south at a velocity of 80 km/h while a stationary observer on the ground (Earth) views the car as moving north at a velocity of 80 km/h. Thus, this problem can be solved as follows:

\[ v_{fs} = v_{fe} + v_{es} = v_{fe} - v_{se} \]

\[ v_{fs} = (+90 \text{ km/h north}) - (+80 \text{ km/h north}) = +10 \text{ km/h north} \]

When solving relative velocity problems, follow the above technique for writing subscripts. The particular subscripts will vary depending on the problem, but the method for ordering the subscripts does not change. A general form of the relative velocity equation is \(v_{ac} = v_{ab} + v_{bc}\). This general form may help you remember the technique for writing subscripts.

Did you know?

Like velocity, displacement and acceleration depend on the frame in which they are measured. In some cases, it is instructive to visualize gravity as the ground accelerating toward a projectile rather than the projectile accelerating toward the ground.

Advanced Topics

See “Special Relativity and Velocities” in Appendix J: Advanced Topics to learn about how velocities are added in Einstein’s special theory of relativity.

Conceptual Challenge

1. Elevator Acceleration  A boy bounces a rubber ball in an elevator that is going down. If the boy drops the ball as the elevator is slowing down, is the magnitude of the ball’s acceleration relative to the elevator less than or greater than the magnitude of its acceleration relative to the ground?

2. Aircraft Carrier  Why does a plane landing on an aircraft carrier approach the carrier from the stern (rear) instead of from the bow (front)?
SAMPLE PROBLEM F

Relative Velocity

**Problem**
A boat heading north crosses a wide river with a velocity of 10.00 km/h relative to the water. The river has a uniform velocity of 5.00 km/h due east. Determine the boat’s velocity with respect to an observer on shore.

**Solution**

**1. Define**

Given:  
- \( \mathbf{v}_{bw} = 10.00 \text{ km/h due north} \) (velocity of the boat, \( b \), with respect to the water, \( w \))
- \( \mathbf{v}_{we} = 5.00 \text{ km/h due east} \) (velocity of the water, \( w \), with respect to Earth, \( e \))

Unknown:  \( \mathbf{v}_{be} = ? \)

**Diagram:** See the diagram on the right.

**Choose an equation or situation:**

To find \( \mathbf{v}_{be} \), write the equation so that the subscripts on the right start with \( b \) and end with \( e \).

\[
\mathbf{v}_{be} = \mathbf{v}_{bw} + \mathbf{v}_{we}
\]

As in Section 2, we use the Pythagorean theorem to calculate the magnitude of the resultant velocity and the tangent function to find the direction.

\[
(v_{be})^2 = (v_{bw})^2 + (v_{we})^2
\]

\[
\tan \theta = \frac{v_{we}}{v_{bw}}
\]

**Rearrange the equations to isolate the unknowns:**

\[
v_{be} = \sqrt{(v_{bw})^2 + (v_{we})^2}
\]

\[
\theta = \tan^{-1} \left( \frac{v_{we}}{v_{bw}} \right)
\]

**2. Plan**

**3. Calculate**

Substitute the known values into the equations and solve:

\[
v_{be} = \sqrt{(10.00 \text{ km/h})^2 + (5.00 \text{ km/h})^2}
\]

\[
v_{be} = 11.18 \text{ km/h}
\]

\[
\theta = \tan^{-1} \left( \frac{5.00 \text{ km/h}}{10.0 \text{ km/h}} \right)
\]

\[
\theta = 26.6^\circ
\]

**4. Evaluate**
The boat travels at a speed of 11.18 km/h in the direction 26.6° east of north with respect to Earth.
**Relative Velocity**

1. A passenger at the rear of a train traveling at 15 m/s relative to Earth throws a baseball with a speed of 15 m/s in the direction opposite the motion of the train. What is the velocity of the baseball relative to Earth as it leaves the thrower’s hand?

2. A spy runs from the front to the back of an aircraft carrier at a velocity of 3.5 m/s. If the aircraft carrier is moving forward at 18.0 m/s, how fast does the spy appear to be running when viewed by an observer on a nearby stationary submarine?

3. A ferry is crossing a river. If the ferry is headed due north with a speed of 2.5 m/s relative to the water and the river’s velocity is 3.0 m/s to the east, what will the boat’s velocity relative to Earth be? (Hint: Remember to include the direction in describing the velocity.)

4. A pet-store supply truck moves at 25.0 m/s north along a highway. Inside, a dog moves at 1.75 m/s at an angle of 35.0° east of north. What is the velocity of the dog relative to the road?

**SECTION REVIEW**

1. A woman on a 10-speed bicycle travels at 9 m/s relative to the ground as she passes a little boy on a tricycle going in the opposite direction. If the boy is traveling at 1 m/s relative to the ground, how fast does the boy appear to be moving relative to the woman?

2. A girl at an airport rolls a ball north on a moving walkway that moves east. If the ball’s speed with respect to the walkway is 0.15 m/s and the walkway moves at a speed of 1.50 m/s, what is the velocity of the ball relative to the ground?

3. **Critical Thinking** Describe the motion of the following objects if they are observed from the stated frames of reference:
   a. a person standing on a platform viewed from a train traveling north
   b. a train traveling north viewed by a person standing on a platform
   c. a ball dropped by a boy walking at a speed of 1 m/s viewed by the boy
   d. a ball dropped by a boy walking 1 m/s as seen by a nearby viewer who is stationary
How does the body move? This question is just one of the many that kinesiology continually asks. To learn more about kinesiology as a career, read the interview with Lisa Griffin, who teaches in the Department of Kinesiology and Health Education at the University of Texas at Austin.

What training did you receive in order to become a kinesiologist?
I received a B.Sc. degree in human kinetics with a minor in biochemistry and M.Sc. and Ph.D. degrees in neuroscience. Kinesiology typically covers motor control, biomechanics, and exercise physiology. People who work in these branches are known as neuroscientists, biomechanists, and physiologists, respectively.

What makes kinesiology interesting to you?
The field of kinesiology allows me to explore how the central nervous system (CNS) controls human movement. Thus we work with people, and the findings of our work can be used to help others.

What is the nature of your research?
We record force output and single motor unit firing patterns from the muscles of human participants during fatigue and training. We then use these frequency patterns to stimulate their hands artificially with electrical stimulation. We are working toward developing an electrical stimulation system that people with paralysis could use to generate limb movement. This could help many who have spinal cord injuries from accidents or brain damage from stroke.

How does your work address two-dimensional motion and vectors?
I investigate motor unit firing frequencies required to generate force output from muscle over time. Thus we record muscle contraction with strain gauge force transducers, bridge amplifiers, an analog to digital converter, and a computer data acquisition and analysis program. For example, the muscles of the thumb produce force in both x and y directions. We record the x and y forces on two different channels, and then we calculate the resultant force online so that we can view the net output during contraction.

What are your most and least favorite things about your work?
My favorite thing is coming up with new ideas and working with students who are excited about their work. The thing I would most like to change is the amount of time it takes to get the results of the experiments after you think of the ideas.

What advice would you offer to students who are interested in this field?
Do not underestimate the depth of the questions that can be addressed with human participants.
KEY IDEAS

Section 1 Introduction to Vectors
• A scalar is a quantity completely specified by only a number with appropriate units, whereas a vector is a quantity that has magnitude and direction.
• Vectors can be added graphically using the triangle method of addition, in which the tail of one vector is placed at the head of the other. The resultant is the vector drawn from the tail of the first vector to the head of the last vector.

Section 2 Vector Operations
• The Pythagorean theorem and the inverse tangent function can be used to find the magnitude and direction of a resultant vector.
• Any vector can be resolved into its component vectors by using the sine and cosine functions.

Section 3 Projectile Motion
• Neglecting air resistance, a projectile has a constant horizontal velocity and a constant downward free-fall acceleration.
• In the absence of air resistance, projectiles follow a parabolic path.

Section 4 Relative Motion
• If the frame of reference is denoted with subscripts (\( \mathbf{v}_{ab} \) is the velocity of object or frame \( a \) with respect to object or frame \( b \)), then the velocity of an object with respect to a different frame of reference can be found by adding the known velocities so that the subscript starts with the letter that ends the preceding velocity subscript: \( \mathbf{v}_{ac} = \mathbf{v}_{ab} + \mathbf{v}_{bc} \).
• If the order of the subscripts is reversed, there is a change in sign; for example, \( \mathbf{v}_{cd} = -\mathbf{v}_{dc} \).

Variable Symbols

<table>
<thead>
<tr>
<th>Quantities</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbf{d} ) (vector)</td>
<td>displacement</td>
</tr>
<tr>
<td>( \mathbf{v} ) (vector)</td>
<td>velocity</td>
</tr>
<tr>
<td>( \mathbf{a} ) (vector)</td>
<td>acceleration</td>
</tr>
<tr>
<td>( \Delta x ) (scalar)</td>
<td>horizontal component</td>
</tr>
<tr>
<td>( \Delta y ) (scalar)</td>
<td>vertical component</td>
</tr>
</tbody>
</table>

KEY TERMS

scalar (p. 82)
vector (p. 82)
resultant (p. 83)
components of a vector (p. 90)
projectile motion (p. 96)

Problem Solving
See Appendix D: Equations for a summary of the equations introduced in this chapter. If you need more problem-solving practice, see Appendix I: Additional Problems.
VECTORS AND THE GRAPHICAL METHOD

Review Questions

1. The magnitude of a vector is a scalar. Explain this statement.

2. If two vectors have unequal magnitudes, can their sum be zero? Explain.

3. What is the relationship between instantaneous speed and instantaneous velocity?

4. What is another way of saying $-30 \text{ m/s}$ west?

5. Is it possible to add a vector quantity to a scalar quantity? Explain.

6. Vector $\vec{A}$ is 3.00 units in length and points along the positive $x$-axis. Vector $\vec{B}$ is 4.00 units in length and points along the negative $y$-axis. Use graphical methods to find the magnitude and direction of the following vectors:
   a. $\vec{A} + \vec{B}$
   b. $\vec{A} - \vec{B}$
   c. $\vec{A} + 2\vec{B}$
   d. $\vec{B} - \vec{A}$

7. Each of the displacement vectors $\vec{A}$ and $\vec{B}$ shown in the figure below has a magnitude of 3.00 m. Graphically find the following:
   a. $\vec{A} + \vec{B}$
   b. $\vec{A} - \vec{B}$
   c. $\vec{B} - \vec{A}$
   d. $\vec{A} - 2\vec{B}$

8. A dog searching for a bone walks 3.50 m south, then 8.20 m at an angle of 30.0° north of east, and finally 15.0 m west. Use graphical techniques to find the dog’s resultant displacement vector.

9. A man lost in a maze makes three consecutive displacements so that at the end of the walk he is back where he started, as shown below. The first displacement is 8.00 m westward, and the second is 13.0 m northward. Use the graphical method to find the third displacement.

- Conceptual Questions

10. If $\vec{B}$ is added to $\vec{A}$, under what conditions does the resultant have the magnitude equal to $\vec{A} + \vec{B}$?

11. Give an example of a moving object that has a velocity vector and an acceleration vector in the same direction and an example of one that has velocity and acceleration vectors in opposite directions.

12. A student accurately uses the method for combining vectors. The two vectors she combines have magnitudes of 55 and 25 units. The answer that she gets is either 85, 20, or 55. Pick the correct answer, and explain why it is the only one of the three that can be correct.

13. If a set of vectors laid head to tail forms a closed polygon, the resultant is zero. Is this statement true? Explain your reasoning.
VECTOR OPERATIONS

Review Questions

14. Can a vector have a component equal to zero and still have a nonzero magnitude?

15. Can a vector have a component greater than its magnitude?

16. Explain the difference between vector addition and vector resolution.

17. How would you add two vectors that are not perpendicular or parallel?

Conceptual Questions

18. If \( \mathbf{A} + \mathbf{B} \) equals 0, what can you say about the components of the two vectors?

19. Under what circumstances would a vector have components that are equal in magnitude?

20. The vector sum of three vectors gives a resultant equal to zero. What can you say about the vectors?

Practice Problems

For problems 21–23, see Sample Problem A.

21. A girl delivering newspapers travels three blocks west, four blocks north, and then six blocks east.
   a. What is her resultant displacement?
   b. What is the total distance she travels?

22. A quarterback takes the ball from the line of scrimmage, runs backward for 10.0 yards, and then runs sideways parallel to the line of scrimmage for 15.0 yards. At this point, he throws a 50.0-yard forward pass straight down the field. What is the magnitude of the football’s resultant displacement?

23. A shopper pushes a cart 40.0 m south down one aisle and then turns 90.0° and moves 15.0 m. He then makes another 90.0° turn and moves 20.0 m. Find the shopper’s total displacement. (There could be more than one correct answer.)

For problems 24–25, see Sample Problem B.

24. A submarine dives 110.0 m at an angle of 10.0° below the horizontal. What are the two components?

25. A person walks 25.0° north of east for 3.10 km. How far would another person walk due north and due east to arrive at the same location?

For problem 26, see Sample Problem C.

26. A person walks the path shown below. The total trip consists of four straight-line paths. At the end of the walk, what is the person’s resultant displacement measured from the starting point?

PROJECTILE MOTION

Review Questions

27. A dart is fired horizontally from a dart gun, and another dart is dropped simultaneously from the same height. If air resistance can be neglected, which dart hits the ground first?

28. If a rock is dropped from the top of a sailboat’s mast, will it hit the deck at the same point whether the boat is at rest or in motion at constant velocity?

29. Does a ball dropped out of the window of a moving car take longer to reach the ground than one dropped at the same height from a car at rest?

30. A rock is dropped at the same instant that a ball at the same elevation is thrown horizontally. Which will have the greater speed when it reaches ground level?

Practice Problems

For problems 31–33, see Sample Problem D.

31. The fastest recorded pitch in Major League Baseball was thrown by Nolan Ryan in 1974. If this pitch were thrown horizontally, the ball would fall 0.809 m (2.65 ft) by the time it reached home plate, 18.3 m (60 ft) away. How fast was Ryan’s pitch?
32. A person standing at the edge of a seaside cliff kicks a stone over the edge with a speed of 18 m/s. The cliff is 52 m above the water’s surface, as shown at right. How long does it take for the stone to fall to the water? With what speed does it strike the water?

33. A spy in a speed boat is being chased down a river by government officials in a faster craft. Just as the officials’ boat pulls up next to the spy’s boat, both boats reach the edge of a 5.0 m waterfall. If the spy’s speed is 15 m/s and the officials’ speed is 26 m/s, how far apart will the two vessels be when they land below the waterfall?

For problems 34–37, see Sample Problem E.

34. A shell is fired from the ground with an initial speed of $1.70 \times 10^3$ m/s (approximately five times the speed of sound) at an initial angle of 55.0° to the horizontal. Neglecting air resistance, find
   a. the shell’s horizontal range
   b. the amount of time the shell is in motion

35. A place kicker must kick a football from a point 36.0 m (about 40.0 yd) from the goal. As a result of the kick, the ball must clear the crossbar, which is 3.05 m high. When kicked, the ball leaves the ground with a speed of 20.0 m/s at an angle of 53° to the horizontal.
   a. By how much does the ball clear or fall short of clearing the crossbar?
   b. Does the ball approach the crossbar while still rising or while falling?

36. When a water gun is fired while being held horizontally at a height of 1.00 m above ground level, the water travels a horizontal distance of 5.00 m. A child, who is holding the same gun in a horizontal position, is also sliding down a 45.0° incline at a constant speed of 2.00 m/s. If the child fires the gun when it is 1.00 m above the ground and the water takes 0.329 s to reach the ground, how far will the water travel horizontally?

37. A ship maneuvers to within $2.50 \times 10^3$ m of an island’s $1.80 \times 10^3$ m high mountain peak and fires a projectile at an enemy ship $6.10 \times 10^2$ m on the other side of the peak, as illustrated below. If the ship shoots the projectile with an initial velocity of $2.50 \times 10^2$ m/s at an angle of 75.0°, how close to the enemy ship does the projectile land? How close (vertically) does the projectile come to the peak?

RELATIVE MOTION

Review Questions

38. Explain the statement “All motion is relative.”

39. What is a frame of reference?

40. When we describe motion, what is a common frame of reference?

41. A small airplane is flying at 50 m/s toward the east. A wind of 20 m/s toward the east suddenly begins to blow and gives the plane a velocity of 70 m/s east.
   a. Which vector is the resultant vector?
   b. What is the magnitude of the wind velocity?

42. A ball is thrown upward in the air by a passenger on a train that is moving with constant velocity.
   a. Describe the path of the ball as seen by the passenger. Describe the path as seen by a stationary observer outside the train.
   b. How would these observations change if the train were accelerating along the track?

Practice Problems

For problems 43–46, see Sample Problem E.

43. A river flows due east at 1.50 m/s. A boat crosses the river from the south shore to the north shore by maintaining a constant velocity of 10.0 m/s due north relative to the water.
   a. What is the velocity of the boat as viewed by an observer on shore?
b. If the river is 325 m wide, how far downstream is the boat when it reaches the north shore?

44. The pilot of an aircraft wishes to fly due west in a 50.0 km/h wind blowing toward the south. The speed of the aircraft in the absence of a wind is 205 km/h.
   a. In what direction should the aircraft head?
   b. What should its speed relative to the ground be?

45. A hunter wishes to cross a river that is 1.5 km wide and that flows with a speed of 5.0 km/h. The hunter uses a small powerboat that moves at a maximum speed of 12 km/h with respect to the water. What is the minimum time necessary for crossing?

46. A swimmer can swim in still water at a speed of 9.50 m/s. He intends to swim directly across a river that has a downstream current of 3.75 m/s.
   a. What must the swimmer’s direction be?
   b. What is his velocity relative to the bank?

**MIXED REVIEW**

47. A ball player hits a home run, and the baseball just clears a wall 21.0 m high located 130.0 m from home plate. The ball is hit at an angle of 35.0° to the horizontal, and air resistance is negligible. Assume the ball is hit at a height of 1.0 m above the ground.
   a. What is the initial speed of the ball?
   b. How much time does it take for the ball to reach the wall?
   c. Find the components of the velocity and the speed of the ball when it reaches the wall.

48. A daredevil jumps a canyon 12 m wide. To do so, he drives a car up a 15° incline.
   a. What minimum speed must he achieve to clear the canyon?
   b. If the daredevil jumps at this minimum speed, what will his speed be when he reaches the other side?

49. A 2.00 m tall basketball player attempts a goal 10.00 m from the basket (3.05 m high). If he shoots the ball at a 45.0° angle, at what initial speed must he throw the basketball so that it goes through the hoop without striking the backboard?

50. An escalator is 20.0 m long. If a person stands on the escalator, it takes 50.0 s to ride to the top.
   a. If a person walks up the moving escalator with a speed of 0.500 m/s relative to the escalator, how long does it take the person to get to the top?
   b. If a person walks down the “up” escalator with the same relative speed as in item (a), how long does it take to reach the bottom?

51. A ball is projected horizontally from the edge of a table that is 1.00 m high, and it strikes the floor at a point 1.20 m from the base of the table.
   a. What is the initial speed of the ball?
   b. How high is the ball above the floor when its velocity vector makes a 45.0° angle with the horizontal?

52. How long does it take an automobile traveling 60.0 km/h to become even with a car that is traveling in another lane at 40.0 km/h if the cars’ front bumpers are initially 125 m apart?

53. The eye of a hurricane passes over Grand Bahama Island. It is moving in a direction 60.0° north of west with a speed of 41.0 km/h. Exactly three hours later, the course of the hurricane shifts due north, and its speed slows to 25.0 km/h, as shown below. How far from Grand Bahama is the hurricane 4.50 h after it passes over the island?

54. A boat moves through a river at 7.5 m/s relative to the water, regardless of the boat’s direction. If the water in the river is flowing at 1.5 m/s, how long does it take the boat to make a round trip consisting of a 250 m displacement downstream followed by a 250 m displacement upstream?
55. A car is parked on a cliff overlooking the ocean on an incline that makes an angle of 24.0° below the horizontal. The negligent driver leaves the car in neutral, and the emergency brakes are defective. The car rolls from rest down the incline with a constant acceleration of 4.00 m/s² and travels 50.0 m to the edge of the cliff. The cliff is 30.0 m above the ocean.

a. What is the car’s position relative to the base of the cliff when the car lands in the ocean?

b. How long is the car in the air?

56. A golf ball with an initial angle of 34° lands exactly 240 m down the range on a level course.

a. Neglecting air friction, what initial speed would achieve this result?

b. Using the speed determined in item (a), find the maximum height reached by the ball.

57. A car travels due east with a speed of 50.0 km/h. Rain is falling vertically with respect to Earth. The traces of the rain on the side windows of the car make an angle of 60.0° with the vertical. Find the velocity of the rain with respect to the following:

a. the car

b. Earth

58. A shopper in a department store can walk up a stationary (stalled) escalator in 30.0 s. If the normally functioning escalator can carry the standing shopper to the next floor in 20.0 s, how long would it take the shopper to walk up the moving escalator? Assume the same walking effort for the shopper whether the escalator is stalled or moving.

Graphing Calculator Practice

Two Dimensional Motion
Recall the following equation from your studies of projectiles launched at an angle.

\[ \Delta y = (v_i \sin \theta) \Delta t + \frac{1}{2} a_y (\Delta t)^2 \]

Consider a baseball that is thrown straight up in the air. The equation for projectile motion can be entered as \( Y_1 \) on a graphing calculator.

\[ Y_1 = VX - 4.9X^2 \]

Given the initial velocity (V), your graphing calculator can calculate the height (Y1) of the baseball versus the time interval (X) that the ball remains in the air. Why is the factor \( \sin \theta \) missing from the equation for \( Y_1 \)?

In this activity, you will determine the maximum height and flight time of a baseball thrown vertically at various initial velocities.

Visit go.hrw.com and type in the keyword HF6TDMX to find this graphing calculator activity. Refer to Appendix B for instructions on downloading the program for this activity.
59. If a person can jump a horizontal distance of 3.0 m on Earth, how far could the person jump on the moon, where the free-fall acceleration is \( g/6 \) and \( g = 9.81 \text{ m/s}^2 \)? How far could the person jump on Mars, where the acceleration due to gravity is \( 0.38g \)?

60. A science student riding on a flatcar of a train moving at a constant speed of 10.0 m/s throws a ball toward the caboose along a path that the student judges as making an initial angle of 60.0° with the horizontal. The teacher, who is standing on the ground nearby, observes the ball rising vertically. How high does the ball rise?

61. A football is thrown directly toward a receiver with an initial speed of 18.0 m/s at an angle of 35.0° above the horizontal. At that instant, the receiver is 18.0 m from the quarterback. In what direction and with what constant speed should the receiver run to catch the football at the level at which it was thrown?

62. A rocket is launched at an angle of 53° above the horizontal with an initial speed of 75 m/s, as shown below. It moves for 25 s along its initial line of motion with an acceleration of 25 m/s². At this time, its engines fail and the rocket proceeds to move as a free body.

   a. What is the rocket’s maximum altitude?
   b. What is the rocket’s total time of flight?
   c. What is the rocket’s horizontal range?

---

Alternative Assessment

1. Work in cooperative groups to analyze a game of chess in terms of displacement vectors. Make a model chessboard, and draw arrows showing all the possible moves for each piece as vectors made of horizontal and vertical components. Then have two members of your group play the game while the others keep track of each piece’s moves. Be prepared to demonstrate how vector addition can be used to explain where a piece would be after several moves.

2. Use a garden hose to investigate the laws of projectile motion. Design experiments to investigate how the angle of the hose affects the range of the water stream. (Assume that the initial speed of water is constant and is determined by the pressure indicated by the faucet’s setting.) What quantities will you measure, and how will you measure them? What variables do you need to control? What is the shape of the water stream? How can you reach the maximum range? How can you reach the highest point? Present your results to the rest of the class and discuss the conclusions.

3. You are helping NASA engineers design a basketball court for a colony on the moon. How do you anticipate the ball’s motion compared with its motion on Earth? What changes will there be for the players—how they move and how they throw the ball? What changes would you recommend for the size of the court, the basket height, and other regulations in order to adapt the sport to the moon’s low gravity? Create a presentation or a report presenting your suggestions, and include the physics concepts behind your recommendations.

4. There is conflicting testimony in a court case. A police officer claims that his radar monitor indicated that a car was traveling at 176 km/h (110 mi/h). The driver argues that the radar must have recorded the relative velocity because he was only going 88 km/h (55 mi/h). Is it possible that both are telling the truth? Could one be lying? Prepare scripts for expert witnesses, for both the prosecution and the defense, that use physics to justify their positions before the jury. Create visual aids to be used as evidence to support the different arguments.
MULTIPLE CHOICE

1. Vector A has a magnitude of 30 units. Vector B is perpendicular to vector A and has a magnitude of 40 units. What would the magnitude of the resultant vector $A + B$ be?
   A. 10 units
   B. 50 units
   C. 70 units
   D. zero

2. What term represents the magnitude of a velocity vector?
   F. acceleration
   G. momentum
   H. speed
   J. velocity

Use the diagram below to answer questions 3–4.

3. What is the direction of the resultant vector $A + B$?
   A. 15° above the x-axis
   B. 75° above the x-axis
   C. 15° below the x-axis
   D. 75° below the x-axis

4. What is the direction of the resultant vector $A - B$?
   F. 15° above the x-axis
   G. 75° above the x-axis
   H. 15° below the x-axis
   J. 75° below the x-axis

Use the passage below to answer questions 5–6.

A motorboat heads due east at 5.0 m/s across a river that flows toward the south at a speed of 5.0 m/s.

5. What is the resultant velocity relative to an observer on the shore?
   A. 3.2 m/s to the southeast
   B. 5.0 m/s to the southeast
   C. 7.1 m/s to the southeast
   D. 10.0 m/s to the southeast

6. If the river is 125 m wide, how long does the boat take to cross the river?
   F. 39 s
   G. 25 s
   H. 17 s
   J. 12 s

7. The pilot of a plane measures an air velocity of 165 km/h south relative to the plane. An observer on the ground sees the plane pass overhead at a velocity of 145 km/h toward the north. What is the velocity of the wind that is affecting the plane relative to the observer?
   A. 20 km/h to the north
   B. 20 km/h to the south
   C. 165 km/h to the north
   D. 310 km/h to the south

8. A golfer takes two putts to sink his ball in the hole once he is on the green. The first putt displaces the ball 6.00 m east, and the second putt displaces the ball 5.40 m south. What displacement would put the ball in the hole in one putt?
   F. 11.40 m southeast
   G. 8.07 m at 48.0° south of east
   H. 3.32 m at 42.0° south of east
   J. 8.07 m at 42.0° south of east
Use the information below to answer questions 9–12.

A girl riding a bicycle at 2.0 m/s throws a tennis ball horizontally forward at a speed of 1.0 m/s from a height of 1.5 m. At the same moment, a boy standing on the sidewalk drops a tennis ball straight down from a height of 1.5 m.

9. What is the initial speed of the girl’s ball relative to the boy?
   A. 1.0 m/s
   B. 1.5 m/s
   C. 2.0 m/s
   D. 3.0 m/s

10. If air resistance is disregarded, which ball will hit the ground first?
    F. the boy’s ball
    G. the girl’s ball
    H. neither
    J. The answer cannot be determined from the given information.

11. If air resistance is disregarded, which ball will have a greater speed (relative to the ground) when it hits the ground?
    A. the boy’s ball
    B. the girl’s ball
    C. neither
    D. The answer cannot be determined from the given information.

12. What is the speed of the girl’s ball when it hits the ground?
    F. 1.0 m/s
    G. 3.0 m/s
    H. 6.2 m/s
    J. 8.4 m/s

**EXTENDED RESPONSE**

16. A human cannonball is shot out of a cannon at 45.0° to the horizontal with an initial speed of 25.0 m/s. A net is positioned at a horizontal distance of 50.0 m from the cannon. At what height above the cannon should the net be placed in order to catch the human cannonball? Show your work.

**Read the following passage to answer question 17.**

Three airline executives are discussing ideas for developing flights that are more energy efficient.

Executive A: Because the Earth rotates from west to east, we could operate “static flights”—a helicopter or airship could begin by rising straight up from New York City and then descend straight down four hours later when San Francisco arrives below.

Executive B: This approach could work for one-way flights, but the return trip would take 20 hours.

Executive C: That approach will never work. Think about it. When you throw a ball straight up in the air, it comes straight back down to the same point.

Executive A: The ball returns to the same point because Earth’s motion is not significant during such a short time.

17. In a paragraph, state which of the executives is correct, and explain why.

**SHORT RESPONSE**

13. If one of the components of one vector along the direction of another vector is zero, what can you conclude about these two vectors?

14. A roller coaster travels 41.1 m at an angle of 40.0° above the horizontal. How far does it move horizontally and vertically?

15. A ball is thrown straight upward and returns to the thrower’s hand after 3.00 s in the air. A second ball is thrown at an angle of 30.0° with the horizontal. At what speed must the second ball be thrown to reach the same height as the one thrown vertically?
When a ball rolls down an inclined plane, then rolls off the edge of a table, the ball becomes a projectile with some positive horizontal velocity and an initial vertical velocity of zero. However, the length of time that the projectile stays in the air depends not on the horizontal velocity but on the height of the table above the ground. The horizontal velocity determines how far the projectile travels during the time it is in the air.

In this lab, you will roll a ball down an inclined plane, off the edge of a table, and onto a piece of carbon paper on the floor. You will design your own experiment by deciding the details of the setup and the procedure, including how many trials to perform, the angles of the inclined plane, and how high up the plane you will release the ball. Your experiment should include trials with the plane inclined to different angles and multiple trials at different heights along the plane at each angle of inclination. Your procedure should include steps to measure the height from which the ball is released, the length of the ball’s travel along the plane, and the horizontal displacement of the ball after it leaves the table.

**PROCEDURE**

1. Study the materials provided, and read the Analysis and Conclusions questions. Design an experiment using the provided materials to meet the goals stated above and to allow you to answer the questions.

2. Write out your lab procedure, including a detailed description of the measurements to take during each step and the number of trials to perform. Create a data table to record your measurements for each trial. You may use Figure 1 as a guide to one possible setup. Your setup should include a box to catch the ball at the end of each trial.

3. Ask your teacher to approve your procedure.

4. Follow all steps of your procedure.

5. Clean up your work area as directed by your teacher.
ANALYSIS

1. Organizing Data Find the time interval for the ball’s motion from the edge of the table to the floor using the equation for the vertical motion of a projectile. In those equations, \( \Delta y \) is the vertical displacement of the ball after it leaves the table. The result is the time interval for each trial.

2. Organizing Data Using the time interval from item 1 and the value for Displacement \( \Delta x \), calculate the average horizontal velocity for each trial during the ball’s motion from the edge of the table to the floor.

3. Constructing Graphs Plot a graph of average horizontal velocity versus height of release. You may use graph paper, a computer, or a graphing calculator.

4. Constructing Graphs Plot a graph of average horizontal velocity versus length of travel along the plane. You may use graph paper, a computer, or a graphing calculator.

CONCLUSIONS

5. Drawing Conclusions What is the relationship between the height of the inclined plane and the horizontal velocity of the ball? Explain.

6. Drawing Conclusions What is the relationship between the length of the inclined plane and the horizontal velocity of the ball? Explain.

7. Evaluating Methods Why might using the vertical displacement to calculate the time interval be more reliable than using a stopwatch for each trial?

8. Applying Conclusions In which trials would the total velocity of the ball when it hits the ground be the greatest?

EXTENSION

9. Designing Experiments Design an experiment to test the assumption that the time the ball is in the air is independent of the horizontal velocity of the ball. If you have time and your teacher approves your plan, carry out the experiment.

Figure 1
- Use tape to cover the sharp edges of the aluminum sheet before taping it to the end of the plane. The aluminum keeps the ball from bouncing as it rolls onto the table.
- Use a washer hanging from a string to find the zero-displacement point directly under the edge of the table.
- Use a box lined with a soft cloth to catch the ball after it lands.
At General Motors’ Milford Proving Grounds in Michigan, technicians place a crash-test dummy behind the steering wheel of a new car. When the car crashes, the dummy continues moving forward and hits the dashboard. The dashboard then exerts a force on the dummy that accelerates the dummy backward, as shown in the illustration. Sensors in the dummy record the forces and accelerations involved in the collision.

**WHAT TO EXPECT**

In this chapter, you will learn to analyze interactions by identifying the forces involved. Then, you can predict and understand many types of motion.

**WHY IT MATTERS**

Forces play an important role in engineering. For example, technicians study the accelerations and forces involved in car crashes in order to design safer cars and more-effective restraint systems.

**CHAPTER PREVIEW**

1 **Changes in Motion**
   - Force
   - Force Diagrams

2 **Newton’s First Law**
   - Inertia
   - Equilibrium

3 **Newton’s Second and Third Laws**
   - Newton’s Second Law
   - Newton’s Third Law

4 **Everyday Forces**
   - Weight
   - The Normal Force
   - The Force of Friction

For advanced project ideas from *Scientific American*, visit go.hrw.com and type in the keyword HF6SAB.
SECTION OBJECTIVES

- Describe how force affects the motion of an object.
- Interpret and construct free-body diagrams.

FORCE

You exert a force on a ball when you throw or kick the ball, and you exert a force on a chair when you sit in the chair. Forces describe the interactions between an object and its environment.

Forces can cause accelerations

In many situations, a force exerted on an object can change the object’s velocity with respect to time. Some examples of these situations are shown in Figure 1. A force can cause a stationary object to move, as when you throw a ball. Force also causes moving objects to stop, as when you catch a ball. A force can also cause a moving object to change direction, such as when a baseball collides with a bat and flies off in another direction. Notice that in each of these cases, the force is responsible for a change in velocity with respect to time—an acceleration.

The SI unit of force is the newton

The SI unit of force is the newton, named after Sir Isaac Newton (1642–1727), whose work contributed much to the modern understanding of force and motion. The newton (N) is defined as the amount of force that, when acting on a 1 kg mass, produces an acceleration of 1 m/s². Therefore, 1 N = 1 kg \times 1 \text{ m/s}^2.

The weight of an object is a measure of the magnitude of the gravitational force exerted on the object. It is the result of the interaction of an object’s mass with the gravitational field of another object, such as Earth. Many of the
terms and units you use every day to talk about weight are really units of force that can be converted to newtons. For example, a $\frac{1}{4}$ lb stick of margarine has a weight equivalent to a force of about 1 N, as shown in the following conversions:

$$1 \text{ lb} = 4.448 \text{ N}$$
$$1 \text{ N} = 0.225 \text{ lb}$$

**Forces can act through contact or at a distance**

If you pull on a spring, the spring stretches. If you pull on a wagon, the wagon moves. When a football is caught, its motion is stopped. These pushes and pulls are examples of contact forces, which are so named because they result from physical contact between two objects. Contact forces are usually easy to identify when you analyze a situation.

Another class of forces—called field forces—does not involve physical contact between two objects. One example of this kind of force is gravitational force. Whenever an object falls to Earth, the object is accelerated by Earth’s gravity. In other words, Earth exerts a force on the object even when Earth is not in immediate physical contact with the object.

Another common example of a field force is the attraction or repulsion between electric charges. You can observe this force by rubbing a balloon against your hair and then observing how little pieces of paper appear to jump up and cling to the balloon’s surface, as shown in Figure 2. The paper is pulled by the balloon’s electric field.

The theory of fields was developed as a tool to explain how objects could exert force on each other without touching. According to this theory, masses create gravitational fields in the space around them. An object falls to Earth because of the interaction between the object’s mass and Earth’s gravitational field. Similarly, charged objects create electromagnetic fields.

The distinction between contact forces and field forces is useful when dealing with forces that we observe at the macroscopic level. (Macroscopic refers to the realm of phenomena that are visible to the naked eye.) As we will see later, all macroscopic contact forces are actually due to microscopic field forces. For instance, contact forces in a collision are due to electric fields between atoms and molecules. In fact, every force can be categorized as one of four fundamental field forces.

---

**Table 1**

<table>
<thead>
<tr>
<th>System</th>
<th>Mass</th>
<th>Acceleration</th>
<th>Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>SI</td>
<td>kg</td>
<td>m/s(^2)</td>
<td>N = kg•m/s(^2)</td>
</tr>
<tr>
<td>cgs</td>
<td>g</td>
<td>cm/s(^2)</td>
<td>dyne = g•cm/s(^2)</td>
</tr>
<tr>
<td>Avoirdupois</td>
<td>slug</td>
<td>ft/s(^2)</td>
<td>lb = slug•ft/s(^2)</td>
</tr>
</tbody>
</table>

The symbol for the pound, lb, comes from *libra*, the Latin word for “pound,” a unit of measure that has been used since medieval times to measure weight.
When you push a toy car, it accelerates. If you push the car harder, the acceleration will be greater. In other words, the acceleration of the car depends on the force's magnitude. The direction in which the car moves depends on the direction of the force. For example, if you push the toy car from the front, the car will move in a different direction than if you push it from behind.

**Force is a vector**

Because the effect of a force depends on both magnitude and direction, force is a vector quantity. Diagrams that show force vectors as arrows, such as Figure 3(a), are called force diagrams. In this book, the arrows used to represent forces are blue. The tail of an arrow is attached to the object on which the force is acting. A force vector points in the direction of the force, and its length is proportional to the magnitude of the force.

At this point, we will disregard the size and shape of objects and assume that all forces act at the center of an object. In force diagrams, all forces are drawn as if they act at that point, no matter where the force is applied.

**A free-body diagram helps analyze a situation**

After engineers analyzing a test-car crash have identified all of the forces involved, they isolate the car from the other objects in its environment. One of their goals is to determine which forces affect the car and its passengers. Figure 3(b) is a free-body diagram. This diagram represents the same collision that the force diagram (a) does but shows only the car and the forces acting on the car. The forces exerted by the car on other objects are not included in the free-body diagram because they do not affect the motion of the car.

A free-body diagram is used to analyze only the forces affecting the motion of a single object. Free-body diagrams are constructed and analyzed just like other vector diagrams. In Sample Problem A, you will learn to draw free-body diagrams for some situations described in this book. In Section 2, you will learn to use free-body diagrams to find component and resultant forces.

**Figure 3**

(a) In a force diagram, vector arrows represent all the forces acting in a situation. (b) A free-body diagram shows only the forces acting on the object of interest—in this case, the car.
SAMPLE PROBLEM A

STRATEGY Drawing Free-Body Diagrams

PROBLEM

The photograph at right shows a person pulling a sled. Draw a free-body diagram for this sled. The magnitudes of the forces acting on the sled are 60 N by the string, 130 N by the Earth (gravitational force), and 90 N upward by the ground.

SOLUTION

1. Identify the forces acting on the object and the directions of the forces.
   - The string exerts 60 N on the sled in the direction that the string pulls.
   - The Earth exerts a downward force of 130 N on the sled.
   - The ground exerts an upward force of 90 N on the sled.

   **TIP**
   In a free-body diagram, only include forces acting on the object. Do not include forces that the object exerts on other objects. In this problem, the forces are given, but later in the chapter, you will need to identify the forces when drawing a free-body diagram.

2. Draw a diagram to represent the isolated object.
   It is often helpful to draw a very simple shape with some distinguishing characteristics that will help you visualize the object, as shown in (a). Free-body diagrams are often drawn using simple squares, circles, or even points to represent the object.

3. Draw and label vector arrows for all external forces acting on the object.
   A free-body diagram of the sled will show all the forces acting on the sled as if the forces are acting on the center of the sled. First, draw and label an arrow that represents the force exerted by the string attached to the sled. The arrow should point in the same direction as the force that the string exerts on the sled, as in (b).

   **TIP**
   When you draw an arrow representing a force, it is important to label the arrow with either the magnitude of the force or a name that will distinguish it from the other forces acting on the object. Also, be sure that the length of the arrow approximately represents the magnitude of the force.

   Next, draw and label the gravitational force, which is directed toward the center of Earth, as shown in (c). Finally, draw and label the upward force exerted by the ground, as shown in (d). Diagram (d) is the completed free-body diagram of the sled being pulled.
1. A truck pulls a trailer on a flat stretch of road. The forces acting on the trailer are the force due to gravity (250 000 N downward), the force exerted by the road (250 000 N upward), and the force exerted by the cable connecting the trailer to the truck (20 000 N to the right). The forces acting on the truck are the force due to gravity (80 000 N downward), the force exerted by the road (80 000 N upward), the force exerted by the cable (20 000 N to the left), and the force causing the truck to move forward (26 400 N to the right).
   a. Draw and label a free-body diagram of the trailer.
   b. Draw and label a free-body diagram of the truck.


---

**SECTION REVIEW**

1. List three examples of each of the following:
   a. a force causing an object to start moving
   b. a force causing an object to stop moving
   c. a force causing an object to change its direction of motion

2. Give two examples of field forces described in this section and two examples of contact forces you observe in everyday life. Explain why you think that these are forces.

3. What is the SI unit of force? What is this unit equivalent to in terms of fundamental units?

4. Why is force a vector quantity?

5. Draw a free-body diagram of a football being kicked. Assume that the only forces acting on the ball are the force due to gravity and the force exerted by the kicker.

6. **Interpreting Graphics** Study the force diagram shown in Figure 3(a). Redraw the diagram, and label each vector arrow with a description of the force. In each description, include the object exerting the force and the object on which the force is acting.
Newton’s First Law

INERTIA

A hovercraft, such as the one in Figure 4, glides along the surface of the water on a cushion of air. A common misconception is that an object on which no force is acting will always be at rest. This situation is not always the case. If the hovercraft shown in Figure 4 is moving at a constant velocity, then there is no net force acting on it. To see why this is the case, consider how a block will slide on different surfaces.

First, imagine a block on a deep, thick carpet. If you apply a force by pushing the block, the block will begin sliding, but soon after you remove the force, the block will come to rest. Next, imagine pushing the same block across a smooth, waxed floor. When you push with the same force, the block will slide much farther before coming to rest. In fact, a block sliding on a perfectly smooth surface would slide forever in the absence of an applied force.

In the 1630s, Galileo concluded correctly that it is an object’s nature to maintain its state of motion or rest. Note that an object on which no force is acting is not necessarily at rest; the object could also be moving with a constant velocity. This concept was further developed by Newton in 1687 and has come to be known as Newton’s first law of motion.

NEWTON’S FIRST LAW

An object at rest remains at rest, and an object in motion continues in motion with constant velocity (that is, constant speed in a straight line) unless the object experiences a net external force.

Inertia is the tendency of an object not to accelerate. Newton’s first law is often referred to as the law of inertia because it states that in the absence of a net force, a body will preserve its state of motion. In other words, Newton’s first law says that when the net external force on an object is zero, the object’s acceleration (or the change in the object’s velocity) is zero.

SECTION OBJECTIVES

- Explain the relationship between the motion of an object and the net external force acting on the object.
- Determine the net external force on an object.
- Calculate the force required to bring an object into equilibrium.

Figure 4
A hovercraft floats on a cushion of air above the water. Air provides less resistance to motion than water does.
The sum of forces acting on an object is the net force

Consider a car traveling at a constant velocity. Newton’s first law tells us that the net external force on the car must be equal to zero. However, Figure 5 shows that many forces act on a car in motion. The vector $F_{\text{forward}}$ represents the forward force of the road on the tires. The vector $F_{\text{resistance}}$, which acts in the opposite direction, is due partly to friction between the road surface and tires and is due partly to air resistance. The vector $F_{\text{gravity}}$ represents the downward gravitational force on the car, and the vector $F_{\text{ground-on-car}}$ represents the upward force that the road exerts on the car.

To understand how a car under the influence of so many forces can maintain a constant velocity, you must understand the distinction between external force and net external force. An external force is a single force that acts on an object as a result of the interaction between the object and its environment. All four forces in Figure 5 are external forces acting on the car. The net force is the vector sum of all forces acting on an object.

When all external forces acting on an object are known, the net force can be found by using the methods for finding resultant vectors. The net force is equivalent to the one force that would produce the same effect on the object that all of the external forces combined would. Although four forces are acting on the car in Figure 5, the car will maintain its constant velocity as long as the vector sum of these forces is equal to zero.

Mass is a measure of inertia

Imagine a basketball and a bowling ball at rest side by side on the ground. Newton’s first law states that both balls remain at rest as long as no net external force acts on them. Now, imagine supplying a net force by pushing each ball. If the two are pushed with equal force, the basketball will accelerate much more than the bowling ball. The bowling ball experiences a smaller acceleration because it has more inertia than the basketball does.

As the example of the bowling ball and the basketball shows, the inertia of an object is proportional to the object’s mass. The greater the mass of a body, the less the body accelerates under an applied force. Similarly, a light object undergoes a larger acceleration than does a heavy object under the same force. Therefore, mass, which is a measure of the amount of matter in an object, is also a measure of the inertia of an object.

**Quick Lab**

**Inertia**

**MATERIALS LIST**

- skateboard or cart
- toy balls with various masses

**SAFETY CAUTION**

Perform this experiment away from walls and furniture that can be damaged.

Perform several trials, placing the ball in different positions, such as in the middle of the skateboard and near the front of the skateboard. Repeat all trials using balls with different masses, and compare the results. Perform this experiment at different speeds, and compare the results.

the ball when the skateboard hits the wall.
**SAMPLE PROBLEM B**

**STRATEGY** Determining Net Force

**PROBLEM**
Derek leaves his physics book on top of a drafting table that is inclined at a 35° angle. The free-body diagram at right shows the forces acting on the book. Find the net force acting on the book.

**SOLUTION**

1. Define the problem, and identify the variables.
   
   **Given:**
   - \( F_{\text{gravity-on-book}} = F_g = 22 \text{ N} \)
   - \( F_{\text{friction}} = F_f = 11 \text{ N} \)
   - \( F_{\text{table-on-book}} = F_t = 18 \text{ N} \)

   **Unknown:** \( F_{\text{net}} = ? \)

2. Select a coordinate system, and apply it to the free-body diagram.
   Choose the \( x \)-axis parallel to and the \( y \)-axis perpendicular to the incline of the table, as shown in (a). This coordinate system is the most convenient because only one force needs to be resolved into \( x \) and \( y \) components.

   *Tip*: To simplify the problem, always choose the coordinate system in which as many forces as possible lie on the \( x \)- and \( y \)-axes.

3. Find the \( x \) and \( y \) components of all vectors.
   Draw a sketch, as shown in (b), to help find the components of the vector \( F_g \). The angle \( \theta \) is equal to \( 180° - 90° - 35° = 55° \).

   \[
   \cos \theta = \frac{F_{g,x}}{F_g} \quad \sin \theta = \frac{F_{g,y}}{F_g}
   \]

   \[
   F_{g,x} = F_g \cos \theta \quad F_{g,y} = F_g \sin \theta
   \]

   \[
   F_{g,x} = (22 \text{ N})(\cos 55°) = 13 \text{ N} \quad F_{g,y} = (22 \text{ N})(\sin 55°) = 18 \text{ N}
   \]

   Add both components to the free-body diagram, as shown in (c).

4. Find the net force in both the \( x \) and \( y \) directions.
   Diagram (d) shows another free-body diagram of the book, now with forces acting only along the \( x \)- and \( y \)-axes.

   For the \( x \) direction:
   \[
   \Sigma F_x = F_{g,x} - F_f
   \]
   \[
   \Sigma F_x = 13 \text{ N} - 11 \text{ N} = 2 \text{ N}
   \]

   For the \( y \) direction:
   \[
   \Sigma F_y = F_t - F_{g,y}
   \]
   \[
   \Sigma F_y = 18 \text{ N} - 18 \text{ N} = 0 \text{ N}
   \]

5. Find the net force.
   Add the net forces in the \( x \) and \( y \) directions together as vectors to find the total net force. In this case, \( F_{\text{net}} = 2 \text{ N} \) in the \(+x\) direction, as shown in (e). Thus, the book accelerates down the incline.
The purpose of a seat belt is to prevent serious injury by holding a passenger firmly in place in the event of a collision. A seat belt may also lock when a car rapidly slows down or turns a corner. While inertia causes passengers in a car to continue moving forward as the car slows down, inertia also causes seat belts to lock into place.

The illustration shows how one type of shoulder harness operates. Under normal conditions, the ratchet turns freely to allow the harness to wind or unwind along the pulley. In a collision, the car undergoes a large acceleration and rapidly comes to rest. Because of its inertia, the large mass under the seat continues to slide forward along the tracks, in the direction indicated by the arrow. The pin connection between the mass and the rod causes the rod to pivot and lock the ratchet wheel in place. At this point, the harness no longer unwinds, and the seat belt holds the passenger firmly in place.

TIP

If there is a net force in both the $x$ and $y$ directions, use vector addition to find the total net force.

**Determining Net Force**

1. A man is pulling on his dog with a force of 70.0 N directed at an angle of $+30.0^\circ$ to the horizontal. Find the $x$ and $y$ components of this force.

2. A gust of wind blows an apple from a tree. As the apple falls, the gravitational force on the apple is 2.25 N downward, and the force of the wind on the apple is 1.05 N to the right. Find the magnitude and direction of the net force on the apple.

3. The wind exerts a force of 452 N north on a sailboat, while the water exerts a force of 325 N west on the sailboat. Find the magnitude and direction of the net force on the sailboat.

When the car suddenly slows down, inertia causes the large mass under the seat to continue moving, which activates the lock on the safety belt.

**THE INSIDE STORY ON SEAT BELTS**

The purpose of a seat belt is to prevent serious injury by holding a passenger firmly in place in the event of a collision. A seat belt may also lock when a car rapidly slows down or turns a corner. While inertia causes passengers in a car to continue moving forward as the car slows down, inertia also causes seat belts to lock into place.

The illustration shows how one type of shoulder harness operates. Under normal conditions, the ratchet turns freely to allow the harness to wind or unwind along the pulley. In a collision, the car undergoes a large acceleration and rapidly comes to rest. Because of its inertia, the large mass under the seat continues to slide forward along the tracks, in the direction indicated by the arrow. The pin connection between the mass and the rod causes the rod to pivot and lock the ratchet wheel in place. At this point, the harness no longer unwinds, and the seat belt holds the passenger firmly in place.
EQUILIBRIUM

Objects that are either at rest or moving with constant velocity are said to be in equilibrium. Newton’s first law describes objects in equilibrium, whether they are at rest or moving with a constant velocity. Newton’s first law states one condition that must be true for equilibrium: the net force acting on a body in equilibrium must be equal to zero.

The net force on the fishing bob in Figure 6(a) is equal to zero because the bob is at rest. Imagine that a fish bites the bait, as shown in Figure 6(b). Because a net force is acting on the line, the bob accelerates toward the hooked fish.

Now, consider a different scenario. Suppose that at the instant the fish begins pulling on the line, the person reacts by applying a force to the bob that is equal and opposite to the force exerted by the fish. In this case, the net force on the bob remains zero, as shown in Figure 6(c), and the bob remains at rest. In this example, the bob is at rest while in equilibrium, but an object can also be in equilibrium while moving at a constant velocity.

An object is in equilibrium when the vector sum of the forces acting on the object is equal to zero. To determine whether a body is in equilibrium, find the net force, as shown in Sample Problem B. If the net force is zero, the body is in equilibrium. If there is a net force, a second force equal and opposite to this net force will put the body in equilibrium.

SECTION REVIEW

1. If a car is traveling westward with a constant velocity of 20 m/s, what is the net force acting on the car?

2. If a car is accelerating downhill under a net force of 3674 N, what additional force would cause the car to have a constant velocity?

3. The sensor in the torso of a crash-test dummy records the magnitude and direction of the net force acting on the dummy. If the dummy is thrown forward with a force of 130.0 N while simultaneously being hit from the side with a force of 4500.0 N, what force will the sensor report?

4. What force will the seat belt have to exert on the dummy in item 3 to hold the dummy in the seat?

5. Critical Thinking Can an object be in equilibrium if only one force acts on the object?
SECTION 3

Newton’s Second and Third Laws

SECTION OBJECTIVES

- Describe an object’s acceleration in terms of its mass and the net force acting on it.
- Predict the direction and magnitude of the acceleration caused by a known net force.
- Identify action-reaction pairs.

NEWTON’S SECOND LAW

From Newton’s first law, we know that an object with no net force acting on it is in a state of equilibrium. We also know that an object experiencing a net force undergoes a change in its velocity. But exactly how much does a known force affect the motion of an object?

Force is proportional to mass and acceleration

Imagine pushing a stalled car through a level intersection, as shown in Figure 7. Because a net force causes an object to accelerate, the speed of the car will increase. When you push the car by yourself, however, the acceleration will be so small that it will take a long time for you to notice an increase in the car’s speed. If you get several friends to help you, the net force on the car is much greater, and the car will soon be moving so fast that you will have to run to keep up with it. This change happens because the acceleration of an object is directly proportional to the net force acting on the object. (Note that this is an idealized example that disregards any friction forces that would hinder the motion. In reality, the car accelerates initially. However, when the force exerted by the pushers equals the frictional force, the net force becomes zero, and the car moves at a constant velocity.)

Experience reveals that the mass of an object also affects the object’s acceleration. A lightweight car accelerates more than a heavy truck if the same force is applied to both. Thus, it requires less force to accelerate a low-mass object than it does to accelerate a high-mass object at the same rate.
Newton’s second law relates force, mass, and acceleration

The relationships between mass, force, and acceleration are quantified in Newton’s second law.

**NEWTON’S SECOND LAW**

The acceleration of an object is directly proportional to the net force acting on the object and inversely proportional to the object’s mass.

According to Newton’s second law, if equal forces are applied to two objects of different masses, the object with greater mass will experience a smaller acceleration, and the object with less mass will experience a greater acceleration.

In equation form, we can state Newton’s law as follows:

\[ \Sigma F = ma \]

In this equation, \( a \) is the acceleration of the object and \( m \) is the object’s mass. Note that \( \Sigma \) is the Greek symbol \( \sigma \), which represents the sum of the quantities that come after it. In this case, \( \Sigma F \) represents the vector sum of all external forces acting on the object, or the net force.

**SAMPLE PROBLEM C**

**Newton’s Second Law**

**PROBLEM**

Roberto and Laura are studying across from each other at a wide table. Laura slides a 2.2 kg book toward Roberto. If the net force acting on the book is 1.6 N to the right, what is the book’s acceleration?

**SOLUTION**

Given: \( m = 2.2 \text{ kg} \)

\( F_{\text{net}} = \Sigma F = 1.6 \text{ N to the right} \)

Unknown: \( a = ? \)

Use Newton’s second law, and solve for \( a \).

\[ \Sigma F = ma, \text{ so } a = \frac{\Sigma F}{m} \]

\[ a = \frac{1.6 \text{ N}}{2.2 \text{ kg}} = 0.73 \text{ m/s}^2 \]

**If more than one force is acting on an object, you must find the net force as shown in Sample Problem B before applying Newton’s second law. The acceleration will be in the direction of the net force.**

\[ a = 0.73 \text{ m/s}^2 \text{ to the right} \]
Gravity and Rocks
The force due to gravity is twice as great on a 2 kg rock as it is on a 1 kg rock. Why doesn’t the 2 kg rock have a greater free-fall acceleration?

Leaking Truck
A truck loaded with sand accelerates at 0.5 m/s\(^2\) on the highway. If the driving force on the truck remains constant, what happens to the truck’s acceleration if sand leaks at a constant rate from a hole in the truck bed?

Newton’s Second Law

1. The net force on the propeller of a 3.2 kg model airplane is 7.0 N forward. What is the acceleration of the airplane?
2. The net force on a golf cart is 390 N north. If the cart has a total mass of 270 kg, what are the magnitude and direction of the cart’s acceleration?
3. A car has a mass of 1.50 \(\times\) 10\(^3\) kg. If the force acting on the car is 6.75 \(\times\) 10\(^3\) N to the east, what is the car’s acceleration?
4. A soccer ball kicked with a force of 13.5 N accelerates at 6.5 m/s\(^2\) to the right. What is the mass of the ball?
5. A 2.0 kg otter starts from rest at the top of a muddy incline 85 cm long and slides down to the bottom in 0.50 s. What net force acts on the otter along the incline?

For some problems, it may be easier to use the equation for Newton’s second law twice: once for all of the forces acting in the x direction (\(\Sigma F_x = ma_x\)) and once for all of the forces acting in the y direction (\(\Sigma F_y = ma_y\)). If the net force in both directions is zero, then \(a = 0\), which corresponds to the equilibrium situation in which \(v\) is either constant or zero.

Conceptual Challenge

1. Gravity and Rocks
The force due to gravity is twice as great on a 2 kg rock as it is on a 1 kg rock. Why doesn’t the 2 kg rock have a greater free-fall acceleration?

2. Leaking Truck
A truck loaded with sand accelerates at 0.5 m/s\(^2\) on the highway. If the driving force on the truck remains constant, what happens to the truck’s acceleration if sand leaks at a constant rate from a hole in the truck bed?

Newton’s Third Law
A force is exerted on an object when that object interacts with another object in its environment. Consider a moving car colliding with a concrete barrier. The car exerts a force on the barrier at the moment of collision. Furthermore, the barrier exerts a force on the car so that the car rapidly slows down after coming into contact with the barrier. Similarly, when your hand applies a force to a door to push it open, the door simultaneously exerts a force back on your hand.

Forces always exist in pairs
From examples like those discussed in the previous paragraph, Newton recognized that a single isolated force cannot exist. Instead, forces always exist in pairs. The car exerts a force on the barrier, and at the same time, the barrier exerts a force on the car. Newton described this type of situation with his third law of motion.
An alternative statement of this law is that for every action, there is an equal and opposite reaction. When two objects interact with one another, the forces that the objects exert on each other are called an action-reaction pair. The force that object 1 exerts on object 2 is sometimes called the action force, while the force that object 2 exerts on object 1 is called the reaction force. The action force is equal in magnitude and opposite in direction to the reaction force. The terms action and reaction sometimes cause confusion because they are used a little differently in physics than they are in everyday speech. In everyday speech, the word reaction is used to refer to something that happens after and in response to an event. In physics, however, the reaction force occurs at exactly the same time as the action force.

Because the action and reaction forces coexist, either force can be called the action or the reaction. For example, you could call the force that the car exerts on the barrier the action and the force that the barrier exerts on the car the reaction. Likewise, you could choose to call the force that the barrier exerts on the car the action and the force that the car exerts on the barrier the reaction.

**Action and reaction forces each act on different objects**

One important thing to remember about action-reaction pairs is that each force acts on a different object. Consider the task of driving a nail into wood, as illustrated in **Figure 8**. To accelerate the nail and drive it into the wood, the hammer exerts a force on the nail. According to Newton’s third law, the nail exerts a force on the hammer that is equal to the magnitude of the force that the hammer exerts on the nail.

The concept of action-reaction pairs is a common source of confusion because some people assume incorrectly that the equal and opposite forces balance one another and make any change in the state of motion impossible. If the force that the nail exerts on the hammer is equal to the force the hammer exerts on the nail, why doesn’t the nail remain at rest?

The motion of the nail is affected only by the forces acting on the nail. To determine whether the nail will accelerate, draw a free-body diagram to isolate the forces acting on the nail, as shown in **Figure 9**. The force of the nail on the hammer is not included in the diagram because it does not act on the nail. According to the diagram, the nail will be driven into the wood because there is a net force acting on the nail. Thus, action-reaction pairs do not imply that the net force on either object is zero. The action-reaction forces are equal and opposite, but either object may still have a net force acting on it.
**Field forces also exist in pairs**

Newton’s third law also applies to field forces. For example, consider the gravitational force exerted by Earth on an object. During calibration at the crash-test site, engineers calibrate the sensors in the heads of crash-test dummies by removing the heads and dropping them from a known height.

The force that Earth exerts on a dummy’s head is \( \mathbf{F}_g \). Let’s call this force the action. What is the reaction? Because \( \mathbf{F}_g \) is the force exerted on the falling head by Earth, the reaction to \( \mathbf{F}_g \) is the force exerted on Earth by the falling head.

According to Newton’s third law, the force of the dummy on Earth is equal to the force of Earth on the dummy. Thus, as a falling object accelerates toward Earth, Earth also accelerates toward the object.

The thought that Earth accelerates toward the dummy’s head may seem to contradict our experience. One way to make sense of this idea is to refer to Newton’s second law. The mass of Earth is much greater than that of the dummy’s head. Therefore, while the dummy’s head undergoes a large acceleration due to the force of Earth, the acceleration of Earth due to this reaction force is negligibly small because of Earth’s enormous mass.

**SECTION REVIEW**

1. A 6.0 kg object undergoes an acceleration of 2.0 m/s².
   a. What is the magnitude of the net force acting on the object?
   b. If this same force is applied to a 4.0 kg object, what acceleration is produced?

2. A child causes a wagon to accelerate by pulling it with a horizontal force. Newton’s third law says that the wagon exerts an equal and opposite force on the child. How can the wagon accelerate? (Hint: Draw a free-body diagram for each object.)

3. Identify the action-reaction pairs in the following situations:
   a. A person takes a step.
   b. A snowball hits someone in the back.
   c. A baseball player catches a ball.
   d. A gust of wind strikes a window.

4. The forces acting on a sailboat are 390 N north and 180 N east. If the boat (including crew) has a mass of 270 kg, what are the magnitude and direction of the boat’s acceleration?

5. **Critical Thinking** If a small sports car collides head-on with a massive truck, which vehicle experiences the greater impact force? Which vehicle experiences the greater acceleration? Explain your answers.
Everyday Forces

WEIGHT

How do you know that a bowling ball weighs more than a tennis ball? If you imagine holding one ball in each hand, you can imagine the downward forces acting on your hands. Because the bowling ball has more mass than the tennis ball does, gravitational force pulls more strongly on the bowling ball. Thus, the bowling ball pushes your hand down with more force than the tennis ball does.

The gravitational force exerted on the ball by Earth, \( F_g \), is a vector quantity, directed toward the center of Earth. The magnitude of this force, \( F_g \), is a scalar quantity called **weight**. The weight of an object can be calculated using the equation \( F_g = ma \), where \( a \) is the magnitude of the acceleration due to gravity, or free-fall acceleration. On the surface of Earth, \( a = g \), and \( F_g = mg \). In this book, \( g = 9.81 \text{ m/s}^2 \) unless otherwise specified.

Weight, unlike mass, is not an inherent property of an object. Because it is equal to the magnitude of the force due to gravity, weight depends on location. For example, if the astronaut in Figure 10 weighs 800 N (180 lb) on Earth, he would weigh only about 130 N (30 lb) on the moon. As you will see in the chapter “Circular Motion and Gravitation,” the value of \( a \) on the surface of a planet depends on the planet’s mass and radius. On the moon, \( a \) is about 1.6 m/s\(^2\)—much smaller than 9.81 m/s\(^2\).

Even on Earth, an object’s weight may vary with location. Objects weigh less at higher altitudes than they do at sea level because the value of \( a \) decreases as distance from the surface of Earth increases. The value of \( a \) also varies slightly with changes in latitude.

THE NORMAL FORCE

Imagine a television set at rest on a table. We know that the gravitational force is acting on the television. How can we use Newton’s laws to explain why the television does not continue to fall toward the center of Earth?

An analysis of the forces acting on the television will reveal the forces that are in equilibrium. First, we know that the gravitational force of Earth, \( F_g \), is acting downward. Because the television is in equilibrium, we know that another force, equal in magnitude to \( F_g \) but in the opposite direction, must be acting on it. This force is the force exerted on the television by the table. This force is called the **normal force**, \( F_n \).
The word *normal* is used because the direction of the contact force is perpendicular to the table surface and one meaning of the word *normal* is “perpendicular.” Figure 11 shows the forces acting on the television.

The normal force is always perpendicular to the contact surface but is not always opposite in direction to the force due to gravity. Figure 12 shows a free-body diagram of a refrigerator on a loading ramp. The normal force is perpendicular to the ramp, not directly opposite the force due to gravity. In the absence of other forces, the normal force, \( F_n \), is equal and opposite to the component of \( F_g \) that is perpendicular to the contact surface. The magnitude of the normal force can be calculated as \( F_n = mg \cos \theta \). The angle \( \theta \) is the angle between the normal force and a vertical line and is also the angle between the contact surface and a horizontal line.

**THE FORCE OF FRICTION**

Consider a jug of juice at rest (in equilibrium) on a table, as in Figure 13(a). We know from Newton’s first law that the net force acting on the jug is zero. Newton’s second law tells us that any additional unbalanced force applied to the jug will cause the jug to accelerate and to remain in motion unless acted on by another force. But experience tells us that the jug will not move at all if we apply a very small horizontal force. Even when we apply a force large enough to move the jug, the jug will stop moving almost as soon as we remove this applied force.

**Friction opposes the applied force**

When the jug is at rest, the only forces acting on it are the force due to gravity and the normal force exerted by the table. These forces are equal and opposite, so the jug is in equilibrium. When you push the jug with a small horizontal force \( F \), as shown in Figure 13(b), the table exerts an equal force in the opposite direction. As a result, the jug remains in equilibrium and therefore also remains at rest. The resistive force that keeps the jug from moving is called the force of static friction, abbreviated as \( F_s \).
As long as the jug does not move, the force of static friction is always equal to and opposite in direction to the component of the applied force that is parallel to the surface \( (F_s = -F_{\text{applied}}) \). As the applied force increases, the force of static friction also increases; if the applied force decreases, the force of static friction also decreases. When the applied force is as great as it can be without causing the jug to move, the force of static friction reaches its maximum value, \( F_{s,\text{max}} \).

**Kinetic friction is less than static friction**

When the applied force on the jug exceeds \( F_{s,\text{max}} \), the jug begins to move with an acceleration to the left, as shown in Figure 13(c). A frictional force is still acting on the jug as it moves, but that force is actually less than \( F_{s,\text{max}} \). The retarding frictional force on an object in motion is called the force of kinetic friction \( (F_k) \). The magnitude of the net force acting on the object is equal to the difference between the applied force and the force of kinetic friction \( (F_{\text{applied}} - F_k) \).

At the microscopic level, frictional forces arise from complex interactions between contacting surfaces. Most surfaces, even those that seem very smooth to the touch, are actually quite rough at the microscopic level, as illustrated in Figure 14. Notice that the surfaces are in contact at only a few points. When two surfaces are stationary with respect to each other, the surfaces stick together somewhat at the contact points. This *adhesion* is caused by electrostatic forces between molecules of the two surfaces.

In free-body diagrams, the force of friction is always parallel to the surface of contact. The force of kinetic friction is always opposite the direction of motion. To determine the direction of the force of static friction, use the principle of equilibrium. For an object in equilibrium, the frictional force must point in the direction that results in a net force of zero.

**The force of friction is proportional to the normal force**

It is easier to push a chair across the floor at a constant speed than to push a heavy desk across the floor at the same speed. Experimental observations show that the magnitude of the force of friction is approximately proportional to the magnitude of the normal force that a surface exerts on an object. Because the desk is heavier than the chair, the desk also experiences a greater normal force and therefore greater friction.

**Friction can be calculated approximately**

Keep in mind that the force of friction is really a macroscopic effect caused by a complex combination of forces at a microscopic level. However, we can approximately calculate the force of friction with certain assumptions. The relationship between normal force and the force of friction is one factor that affects friction. For instance, it is easier to slide a light textbook across a desk than it is to slide a heavier textbook. The relationship between the normal force and the force of friction provides a good approximation for the friction between dry, flat surfaces that are at rest or sliding past one another.
The force of friction also depends on the composition and qualities of the surfaces in contact. For example, it is easier to push a desk across a tile floor than across a floor covered with carpet. Although the normal force on the desk is the same in both cases, the force of friction between the desk and the carpet is higher than the force of friction between the desk and the tile. The quantity that expresses the dependence of frictional forces on the particular surfaces in contact is called the **coefficient of friction**. The coefficient of friction between a waxed snowboard and the snow will affect the acceleration of the snowboarder shown in **Figure 15**. The coefficient of friction is represented by the symbol \( \mu \), the lowercase Greek letter \( \text{mu} \).

**The coefficient of friction is a ratio of forces**

The coefficient of friction is defined as the ratio of the force of friction to the normal force between two surfaces. The **coefficient of kinetic friction** is the ratio of the force of kinetic friction to the normal force.

\[
\mu_k = \frac{F_k}{F_n}
\]

The **coefficient of static friction** is the ratio of the maximum value of the force of static friction to the normal force.

\[
\mu_s = \frac{F_{s,\text{max}}}{F_n}
\]

If the value of \( \mu \) and the normal force on the object are known, then the magnitude of the force of friction can be calculated directly.

\[
F_f = \mu F_n
\]

**Table 2** shows some experimental values of \( \mu_s \) and \( \mu_k \) for different materials. Because kinetic friction is less than or equal to the maximum static friction, the coefficient of kinetic friction is always less than or equal to the coefficient of static friction.

<table>
<thead>
<tr>
<th></th>
<th>( \mu_s )</th>
<th>( \mu_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>steel on steel</td>
<td>0.74</td>
<td>0.57</td>
</tr>
<tr>
<td>aluminum on steel</td>
<td>0.61</td>
<td>0.47</td>
</tr>
<tr>
<td>rubber on dry concrete</td>
<td>1.0</td>
<td>0.8</td>
</tr>
<tr>
<td>rubber on wet concrete</td>
<td>—</td>
<td>0.5</td>
</tr>
<tr>
<td>wood on wood</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>glass on glass</td>
<td>0.9</td>
<td>0.4</td>
</tr>
<tr>
<td>waxed wood on wet snow</td>
<td>0.14</td>
<td>0.1</td>
</tr>
<tr>
<td>waxed wood on dry snow</td>
<td>—</td>
<td>0.04</td>
</tr>
<tr>
<td>metal on metal (lubricated)</td>
<td>0.15</td>
<td>0.06</td>
</tr>
<tr>
<td>ice on ice</td>
<td>0.1</td>
<td>0.03</td>
</tr>
<tr>
<td>Teflon on Teflon</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>synovial joints in humans</td>
<td>0.01</td>
<td>0.003</td>
</tr>
</tbody>
</table>
SAMPLE PROBLEM D

Coefficients of Friction

PROBLEM

A 24 kg crate initially at rest on a horizontal floor requires a 75 N horizontal force to set it in motion. Find the coefficient of static friction between the crate and the floor.

SOLUTION

Given: \( F_{s,max} = F_{applied} = 75 \text{ N} \quad m = 24 \text{ kg} \)

Unknown: \( \mu_s = ? \)

Use the equation for the coefficient of static friction.

Because the crate is on a horizontal surface, the magnitude of the normal force \( (F_n) \) equals the crate’s weight \( (mg) \).

\[
\mu_s = \frac{F_{s,max}}{F_n} = \frac{F_{s,max}}{mg} \\
\mu_s = \frac{75 \text{ N}}{24 \text{ kg} \times 9.81 \text{ m/s}^2} \\
\mu_s = 0.32
\]

PRACTICE D

Coefficients of Friction

1. Once the crate in Sample Problem D is in motion, a horizontal force of 53 N keeps the crate moving with a constant velocity. Find \( \mu_k \), the coefficient of kinetic friction, between the crate and the floor.

2. A 25 kg chair initially at rest on a horizontal floor requires a 165 N horizontal force to set it in motion. Once the chair is in motion, a 127 N horizontal force keeps it moving at a constant velocity.
   a. Find the coefficient of static friction between the chair and the floor.
   b. Find the coefficient of kinetic friction between the chair and the floor.

3. A museum curator moves artifacts into place on various different display surfaces. Use the values in Table 2 to find \( F_{s,max} \) and \( F_k \) for the following situations:
   a. moving a 145 kg aluminum sculpture across a horizontal steel platform
   b. pulling a 15 kg steel sword across a horizontal steel shield
   c. pushing a 250 kg wood bed on a horizontal wood floor
   d. sliding a 0.55 kg glass amulet on a horizontal glass display case
SAMPLE PROBLEM E

Overcoming Friction

**PROBLEM**

A student attaches a rope to a 20.0 kg box of books. He pulls with a force of 90.0 N at an angle of 30.0° with the horizontal. The coefficient of kinetic friction between the box and the sidewalk is 0.500. Find the acceleration of the box.

**SOLUTION**

1. **DEFINE**

   Given: \( m = 20.0 \text{ kg} \) \( \mu_k = 0.500 \)
   
   \( F_{\text{applied}} = 90.0 \text{ N at } \theta = 30.0^\circ \)

   Unknown: \( a = ? \)

   **Diagram:**

   \( F_n \)
   
   \( F_k \)
   
   \( F_{\text{applied}} \)
   
   \( F_g \)

2. **PLAN**

   Choose a convenient coordinate system, and find the \( x \) and \( y \) components of all forces.

   The diagram at right shows the most convenient coordinate system, because the only force to resolve into components is \( F_{\text{applied}} \).

   \( F_{\text{applied},y} = (90.0 \text{ N})(\sin 30.0^\circ) = 45.0 \text{ N (upward)} \)
   
   \( F_{\text{applied},x} = (90.0 \text{ N})(\cos 30.0^\circ) = 77.9 \text{ N (to the right)} \)

3. **CALCULATE**

   **Choose an equation or situation:**

   **A.** Find the normal force, \( F_n \), by applying the condition of equilibrium in the vertical direction: \( \Sigma F_y = 0 \).

   **B.** Calculate the force of kinetic friction on the box: \( F_k = \mu_k F_n \).

   **C.** Apply Newton’s second law along the horizontal direction to find the acceleration of the box: \( \Sigma F_x = ma_x \)

   **Substitute the values into the equations and solve:**

   **A.** To apply the condition of equilibrium in the vertical direction, you need to account for all of the forces in the \( y \) direction: \( F_g, F_n \), and \( F_{\text{applied},y} \). You know \( F_{\text{applied},y} \) and can use the box’s mass to find \( F_g \):

   \( F_{\text{applied},y} = 45.0 \text{ N} \)
   
   \( F_g = (20.0 \text{ kg})(9.81 \text{ m/s}^2) = 196 \text{ N} \)
Next, apply the equilibrium condition, $\Sigma F_y = 0$, and solve for $F_n$:

$$\Sigma F_y = F_n + F_{\text{applied}, y} - F_g = 0$$

$$F_n + 45.0 \text{ N} - 196 \text{ N} = 0$$

$$F_n = 151 \text{ N}$$

B. Use the normal force to find the force of kinetic friction.

$$F_k = \mu_k F_n = (0.500)(151 \text{ N}) = 75.5 \text{ N}$$

C. Use Newton’s second law to determine the horizontal acceleration.

$$\Sigma F_x = F_{\text{applied}, x} - F_k = ma_x$$

$$a_x = \frac{F_{\text{applied}, x} - F_k}{m} = \frac{77.9 \text{ N} - 75.5 \text{ N}}{20.0 \text{ kg}} = \frac{2.4 \text{ N}}{20.0 \text{ kg}} = 2.4 \text{ kg} \cdot \text{m/s}^2$$

$$a = 0.12 \text{ m/s}^2$$ to the right

The normal force is not equal in magnitude to the weight because the $y$ component of the student’s pull on the rope helps support the box.

### Practice E

**Overcoming Friction**

1. A student pulls on a rope attached to a box of books and moves the box down the hall. The student pulls with a force of 185 N at an angle of 25.0° above the horizontal. The box has a mass of 35.0 kg, and $\mu_k$ between the box and the floor is 0.27. Find the acceleration of the box.

2. The student in item 1 moves the box up a ramp inclined at 12° with the horizontal. If the box starts from rest at the bottom of the ramp and is pulled at an angle of 25.0° with respect to the incline and with the same 185 N force, what is the acceleration up the ramp? Assume that $\mu_k = 0.27$.

3. A 75 kg box slides down a 25.0° ramp with an acceleration of 3.60 m/s².
   a. Find $\mu_k$ between the box and the ramp.
   b. What acceleration would a 175 kg box have on this ramp?

4. A box of books weighing 325 N moves at a constant velocity across the floor when the box is pushed with a force of 425 N exerted downward at an angle of 35.2° below the horizontal. Find $\mu_k$ between the box and the floor.
Air resistance is a form of friction

Another type of friction, the retarding force produced by air resistance, is important in the analysis of motion. Whenever an object moves through a fluid medium, such as air or water, the fluid provides a resistance to the object’s motion.

For example, the force of air resistance, $F_R$, on a moving car acts in the direction opposite the direction of the car’s motion. At low speeds, the magnitude of $F_R$ is roughly proportional to the car’s speed. At higher speeds, $F_R$ is roughly proportional to the square of the car’s speed. When the magnitude of $F_R$ equals the magnitude of the force moving the car forward, the net force is zero and the car moves at a constant speed.

A similar situation occurs when an object falls through air. As a free-falling body accelerates, its velocity increases. As the velocity increases, the resistance of the air to the object’s motion also constantly increases. When the upward force of air resistance balances the downward gravitational force, the net force on the object is zero and the object continues to move downward with a constant maximum speed, called the terminal speed.
There are four fundamental forces

At the microscopic level, friction results from interactions between the protons and electrons in atoms and molecules. Magnetic force also results from atomic phenomena. These forces are classified as electromagnetic forces. The electromagnetic force is one of four fundamental forces in nature. The other three fundamental forces are gravitational force, the strong nuclear force, and the weak nuclear force. All four fundamental forces are field forces.

The strong and weak nuclear forces have very small ranges, so their effects are not directly observable. The electromagnetic and gravitational forces act over long ranges. Thus, any force you can observe at the macroscopic level is either due to gravitational or electromagnetic forces.

The strong nuclear force is the strongest of all four fundamental forces. Gravity is the weakest. Although the force due to gravity holds the planets, stars, and galaxies together, its effect on subatomic particles is negligible. This explains why electric and magnetic effects can easily overcome gravity. For example, a bar magnet has the ability to lift another magnet off a desk.

SECTION REVIEW

1. Draw a free-body diagram for each of the following objects:
   a. a projectile accelerating downward in the presence of air resistance
   b. a crate being pushed across a flat surface at a constant speed

2. A bag of sugar has a mass of 2.26 kg.
   a. What is its weight in newtons on the moon, where the acceleration due to gravity is one-sixth that on Earth?
   b. What is its weight on Jupiter, where the acceleration due to gravity is 2.64 times that on Earth?

3. A 2.0 kg block on an incline at a 60.0° angle is held in equilibrium by a horizontal force.
   a. Determine the magnitude of this horizontal force. (Disregard friction.)
   b. Determine the magnitude of the normal force on the block.

4. A 55 kg ice skater is at rest on a flat skating rink. A 198 N horizontal force is needed to set the skater in motion. However, after the skater is in motion, a horizontal force of 175 N keeps the skater moving at a constant velocity. Find the coefficients of static and kinetic friction between the skates and the ice.

5. Critical Thinking The force of air resistance acting on a certain falling object is roughly proportional to the square of the object’s velocity and is directed upward. If the object falls fast enough, will the force of air resistance eventually exceed the weight of the object and cause the object to move upward? Explain.
CHAPTER 4

Highlights

KEY TERMS
force (p. 120)
inertia (p. 125)
net force (p. 126)
equilibrium (p. 129)
weight (p. 135)
normal force (p. 135)
static friction (p. 136)
kinetic friction (p. 137)
coefficient of friction (p. 138)

KEY IDEAS

Section 1  Changes in Motion
• Force is a vector quantity that causes acceleration (when unbalanced).
• Force can act either through the physical contact of two objects (contact force) or at a distance (field force).
• A free-body diagram shows only the forces that act on one object. These forces are the only ones that affect the motion of that object.

Section 2  Newton’s First Law
• The tendency of an object not to accelerate is called inertia. Mass is the physical quantity used to measure inertia.
• The net force acting on an object is the vector sum of all external forces acting on the object. An object is in a state of equilibrium when the net force acting on the object is zero.

Section 3  Newton’s Second and Third Laws
• The net force acting on an object is equal to the product of the object’s mass and the object’s acceleration.
• When two bodies exert force on each other, the forces are equal in magnitude and opposite in direction. These forces are called an action-reaction pair. Forces always exist in such pairs.

Section 4  Everyday Forces
• The weight of an object is the magnitude of the gravitational force on the object and is equal to the object’s mass times the acceleration due to gravity.
• A normal force is a force that acts on an object in a direction perpendicular to the surface of contact.
• Friction is a resistive force that acts in a direction opposite to the direction of the relative motion of two contacting surfaces. The force of friction between two surfaces is proportional to the normal force.

Variable Symbols

<table>
<thead>
<tr>
<th>Quantities</th>
<th>Units</th>
<th>Conversions</th>
</tr>
</thead>
<tbody>
<tr>
<td>F (vector)</td>
<td>force</td>
<td>N newtons</td>
</tr>
<tr>
<td>F (scalar)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>μ</td>
<td>coefficient of friction</td>
<td>(no units)</td>
</tr>
</tbody>
</table>
**FORCES AND NEWTON’S FIRST LAW**

**Review Questions**

1. Is it possible for an object to be in motion if no net force is acting on it? Explain.

2. If an object is at rest, can we conclude that no external forces are acting on it?

3. An object thrown into the air stops at the highest point in its path. Is it in equilibrium at this point? Explain.

4. What physical quantity is a measure of the amount of inertia an object has?

**Conceptual Questions**

5. A beach ball is left in the bed of a pickup truck. Describe what happens to the ball when the truck accelerates forward.

6. A large crate is placed on the bed of a truck but is not tied down.
   - a. As the truck accelerates forward, the crate slides across the bed until it hits the tailgate. Explain what causes this.
   - b. If the driver slammed on the brakes, what could happen to the crate?

**Practice Problems**

*For problems 7–9, see Sample Problem A.*

7. Earth exerts a downward gravitational force of 8.9 N on a cake that is resting on a plate. The plate exerts a force of 11.0 N upward on the cake, and a knife exerts a downward force of 2.1 N on the cake. Draw a free-body diagram of the cake.

8. A chair is pushed forward with a force of 185 N. The gravitational force of Earth on the chair is 155 N downward, and the floor exerts a force of 155 N upward on the chair. Draw a free-body diagram showing the forces acting on the chair.

9. Draw a free-body diagram representing each of the following objects:
   - a. a ball falling in the presence of air resistance
   - b. a helicopter lifting off a landing pad
   - c. an athlete running along a horizontal track

*For problems 10–12, see Sample Problem B.*

10. Four forces act on a hot-air balloon, shown from the side in the figure below. Find the magnitude and direction of the resultant force on the balloon.

11. Two lifeguards pull on ropes attached to a raft. If they pull in the same direction, the raft experiences a net force of 334 N to the right. If they pull in opposite directions, the raft experiences a net force of 106 N to the left.
   - a. Draw a free-body diagram representing the raft for each situation.
   - b. Find the force exerted by each lifeguard on the raft for each situation. (Disregard any other forces acting on the raft.)

12. A dog pulls on a pillow with a force of 5 N at an angle of 37° above the horizontal. Find the x and y components of this force.
NEWTON’S SECOND AND THIRD LAWS

Review Questions

13. The force that attracts Earth to an object is equal to and opposite the force that Earth exerts on the object. Explain why Earth’s acceleration is not equal to and opposite the object’s acceleration.


15. An astronaut on the moon has a 110 kg crate and a 230 kg crate. How do the forces required to lift the crates straight up on the moon compare with the forces required to lift them on Earth? (Assume that the astronaut lifts with constant velocity in both cases.)

16. Draw a force diagram to identify all the action-reaction pairs that exist for a horse pulling a cart.

Conceptual Questions

17. A space explorer is moving through space far from any planet or star and notices a large rock, taken as a specimen from an alien planet, floating around the cabin of the ship. Should the explorer push it gently or kick it toward the storage compartment? Why?

18. Explain why a rope climber must pull downward on the rope in order to move upward. Discuss the force exerted by the climber’s arms in relation to the weight of the climber during the various stages of each “step” up the rope.

19. An 1850 kg car is moving to the right at a constant speed of 1.44 m/s.
   a. What is the net force on the car?
   b. What would be the net force on the car if it were moving to the left?

Practice Problems

For problems 20–22, see Sample Problem C.

20. What acceleration will you give to a 24.3 kg box if you push it horizontally with a net force of 85.5 N?

21. What net force is required to give a 25 kg suitcase an acceleration of 2.2 m/s² to the right?

22. Two forces are applied to a car in an effort to accelerate it, as shown below.
   a. What is the resultant of these two forces?
   b. If the car has a mass of 3200 kg, what acceleration does it have? (Disregard friction.)

WEIGHT, FRICTION, AND NORMAL FORCE

Review Questions

23. Explain the relationship between mass and weight.

24. A 0.150 kg baseball is thrown upward with an initial speed of 20.0 m/s.
   a. What is the force on the ball when it reaches half of its maximum height? (Disregard air resistance.)
   b. What is the force on the ball when it reaches its peak?

25. Draw free-body diagrams showing the weight and normal forces on a laundry basket in each of the following situations:
   a. at rest on a horizontal surface
   b. at rest on a ramp inclined 12° above the horizontal
   c. at rest on a ramp inclined 25° above the horizontal
   d. at rest on a ramp inclined 45° above the horizontal

26. If the basket in item 25 has a mass of 5.5 kg, find the magnitude of the normal force for the situations described in (a) through (d).
27. A teapot is initially at rest on a horizontal tabletop, then one end of the table is lifted slightly. Does the normal force increase or decrease? Does the force of static friction increase or decrease?

28. Which is usually greater, the maximum force of static friction or the force of kinetic friction?

29. A 5.4 kg bag of groceries is in equilibrium on an incline of angle $\theta = 15^\circ$. Find the magnitude of the normal force on the bag.

### Conceptual Questions

30. Imagine an astronaut in space at the midpoint between two stars of equal mass. If all other objects are infinitely far away, what is the weight of the astronaut? Explain your answer.

31. A ball is held in a person's hand.
   a. Identify all the external forces acting on the ball and the reaction force to each.
   b. If the ball is dropped, what force is exerted on it while it is falling? Identify the reaction force in this case. (Disregard air resistance.)

32. Explain why pushing downward on a book as you push it across a table increases the force of friction between the table and the book.

33. Analyze the motion of a rock dropped in water in terms of its speed and acceleration. Assume that a resistive force acting on the rock increases as the speed increases.

34. A sky diver falls through the air. As the speed of the sky diver increases, what happens to the sky diver’s acceleration? What is the acceleration when the sky diver reaches terminal speed?

### Practice Problems

**For problems 35–37, see Sample Problem D.**

35. A 95 kg clock initially at rest on a horizontal floor requires a 650 N horizontal force to set it in motion. After the clock is in motion, a horizontal force of 560 N keeps it moving with a constant velocity. Find $\mu_s$ and $\mu_k$ between the clock and the floor.

36. A box slides down a 30.0° ramp with an acceleration of 1.20 m/s². Determine the coefficient of kinetic friction between the box and the ramp.

37. A 4.00 kg block is pushed along the ceiling with a constant applied force of 85.0 N that acts at an angle of 55.0° with the horizontal, as in the figure. The block accelerates to the right at 6.00 m/s². Determine the coefficient of kinetic friction between the block and the ceiling.

**For problems 38–39, see Sample Problem E.**

38. A clerk moves a box of cans down an aisle by pulling on a strap attached to the box. The clerk pulls with a force of 185.0 N at an angle of 25.0° with the horizontal. The box has a mass of 35.0 kg, and the coefficient of kinetic friction between box and floor is 0.450. Find the acceleration of the box.

39. A 925 N crate is being pulled across a level floor by a force $\mathbf{F}$ of 325 N at an angle of 25° above the horizontal. The coefficient of kinetic friction between the crate and floor is 0.25. Find the magnitude of the acceleration of the crate.

### MIXED REVIEW

40. A block with a mass of 6.0 kg is held in equilibrium on an incline of angle $\theta = 30.0^\circ$ by a horizontal force $\mathbf{F}$, as shown in the figure. Find the magnitudes of the normal force on the block and of $\mathbf{F}$. (Ignore friction.)

41. A 2.0 kg mass starts from rest and slides down an inclined plane $8.0 \times 10^{-1}$ m long in 0.50 s. What net force is acting on the mass along the incline?

42. A 2.26 kg book is dropped from a height of 1.5 m.
   a. What is its acceleration?
   b. What is its weight in newtons?
43. A 5.0 kg bucket of water is raised from a well by a rope. If the upward acceleration of the bucket is 3.0 m/s², find the force exerted by the rope on the bucket of water.

44. A 3.46 kg briefcase is sitting at rest on a level floor.
   a. What is the briefcase’s acceleration?
   b. What is its weight in newtons?

45. A boat moves through the water with two forces acting on it. One is a $2.10 \times 10^3$ N forward push by the motor, and the other is a $1.80 \times 10^3$ N resistive force due to the water.
   a. What is the acceleration of the 1200 kg boat?
   b. If it starts from rest, how far will it move in 12 s?
   c. What will its speed be at the end of this time interval?

46. A girl on a sled coasts down a hill. Her speed is 7.0 m/s when she reaches level ground at the bottom. The coefficient of kinetic friction between the sled’s runners and the hard, icy snow is 0.050, and the girl and sled together weigh 645 N. How far does the sled travel on the level ground before coming to rest?

47. A box of books weighing 319 N is shoved across the floor by a force of 485 N exerted downward at an angle of 35° below the horizontal.
   a. If $\mu_k$ between the box and the floor is 0.57, how long does it take to move the box 4.00 m, starting from rest?
   b. If $\mu_k$ between the box and the floor is 0.75, how long does it take to move the box 4.00 m, starting from rest?

48. A 3.00 kg block starts from rest at the top of a 30.0° incline and accelerates uniformly down the incline, moving 2.00 m in 1.50 s.
   a. Find the magnitude of the acceleration of the block.
   b. Find the coefficient of kinetic friction between the block and the incline.
   c. Find the magnitude of the frictional force acting on the block.
   d. Find the speed of the block after it has slid a distance of 2.00 m.

49. A hockey puck is hit on a frozen lake and starts moving with a speed of 12.0 m/s. Exactly 5.0 s later, its speed is 6.0 m/s. What is the puck’s average acceleration? What is the coefficient of kinetic friction between the puck and the ice?

50. The parachute on a race car that weighs 8820 N opens at the end of a quarter-mile run when the car is traveling 35 m/s. What net retarding force must be supplied by the parachute to stop the car in a distance of 1100 m?

51. A 1250 kg car is pulling a 325 kg trailer. Together, the car and trailer have an acceleration of 2.15 m/s² directly forward.
   a. Determine the net force on the car.
   b. Determine the net force on the trailer.

52. The coefficient of static friction between the 3.00 kg crate and the 35.0° incline shown here is 0.300. What is the magnitude of the minimum force, $F$, that must be applied to the crate perpendicularly to the incline to prevent the crate from sliding down the incline?

53. The graph below shows a plot of the speed of a person’s body during a chin-up. All motion is vertical and the mass of the person (excluding the arms) is 64.0 kg. Find the magnitude of the net force exerted on the body at 0.50 s intervals.

54. A machine in an ice factory is capable of exerting $3.00 \times 10^2$ N of force to pull a large block of ice up a slope. The block weighs $1.22 \times 10^4$ N. Assuming there is no friction, what is the maximum angle that the slope can make with the horizontal if the machine is to be able to complete the task?
1. Predict what will happen in the following test of the laws of motion. You and a partner face each other, each holding a bathroom scale. Place the scales back to back, and slowly begin pushing on them. Record the measurements of both scales at the same time. Perform the experiment. Which of Newton’s laws have you verified?

2. Research how the work of scientists Antoine Lavoisier, Isaac Newton, and Albert Einstein related to the study of mass. Which of these scientists might have said the following?
   a. The mass of a body is a measure of the quantity of matter in the body.
   b. The mass of a body is the body’s resistance to a change in motion.
   c. The mass of a body depends on the body’s velocity.

   To what extent are these statements compatible or contradictory? Present your findings to the class for review and discussion.

3. Imagine an airplane with a series of special instruments anchored to its walls: a pendulum, a 100 kg mass on a spring balance, and a sealed half-full aquarium. What will happen to each instrument when the plane takes off, makes turns, slows down, lands, etc.? If possible, test your predictions by simulating airplane motion in elevators, car rides, and other situations. Use instruments similar to those described above, and also observe your body sensations. Write a report comparing your predictions with your experiences.

---

**Graphing Calculator Practice**

**Static Friction**

The force of static friction depends on two factors: the coefficient of static friction for the two surfaces in contact, and the normal force between the two surfaces. The relationship can be represented on a graphing calculator by the following equation:

\[ Y_1 = SX \]

Given a value for the coefficient of static friction (S), the graphing calculator can calculate and graph the force of static friction (Y₁) as a function of normal force (X).

In this activity, you will use a graphing calculator program to compare the force of static friction of wood boxes on a wood surface with that of steel boxes on a steel surface.

Visit [go.hrw.com](http://go.hrw.com) and type in the keyword **HF6FORX** to find this graphing calculator activity. Refer to Appendix B for instructions on downloading the program for this activity.
MULTIPLE CHOICE

Use the passage below to answer questions 1–2.

Two blocks of masses $m_1$ and $m_2$ are placed in contact with each other on a smooth, horizontal surface. Block $m_1$ is on the left of block $m_2$. A constant horizontal force $F$ to the right is applied to $m_1$.

1. What is the acceleration of the two blocks?
   A. $a = \frac{F}{m_1}$
   B. $a = \frac{F}{m_2}$
   C. $a = \frac{F}{m_1 + m_2}$
   D. $a = \frac{F}{(m_1)(m_2)}$

2. What is the horizontal force acting on $m_2$?
   F. $m_1a$
   G. $m_2a$
   H. $(m_1 + m_2)a$
   J. $m_1m_2a$

3. A crate is pulled to the right (positive x-axis) with a force of 82.0 N, to the left with a force of 115 N, upward with a force of 565 N, and downward with a force of 236 N. Find the magnitude and direction of the net force on the crate.
   A. 3.30 N at 96° counterclockwise from the positive x-axis
   B. 3.30 N at 6° counterclockwise from the positive x-axis
   C. $3.30 \times 10^2$ N at 96° counterclockwise from the positive x-axis
   D. $3.30 \times 10^2$ N at 6° counterclockwise from the positive x-axis

4. A ball with a mass of $m$ is thrown into the air, as shown in the figure below. What is the force exerted on Earth by the ball?
   F. $m_{ball}g$, directed down
   G. $m_{ball}g$, directed up
   H. $m_{Earth}g$, directed down
   J. $m_{Earth}g$, directed up

5. A freight train has a mass of $1.5 \times 10^7$ kg. If the locomotive can exert a constant pull of $7.5 \times 10^5$ N, how long would it take to increase the speed of the train from rest to 85 km/h? (Disregard friction.)
   A. $4.7 \times 10^2$ s
   B. $4.7$ s
   C. $5.0 \times 10^{-2}$ s
   D. $5.0 \times 10^4$ s

Use the passage below to answer questions 6–7.

A truck driver slams on the brakes and skids to a stop through a displacement $\Delta x$.

6. If the truck’s mass doubles, find the truck’s skidding distance in terms of $\Delta x$. (Hint: Increasing the mass increases the normal force.)
   F. $\Delta x/4$
   G. $\Delta x$
   H. $2\Delta x$
   J. $4\Delta x$
7. If the truck’s initial velocity were halved, what would be the truck’s skidding distance?
   A. $\Delta x/4$
   B. $\Delta x$
   C. $2\Delta x$
   D. $4\Delta x$

   Use the graph below to answer questions 8–9. The graph shows the relationship between the applied force and the force of friction.

   8. What is the relationship between the forces at point A?
      F. $F_s = F_{applied}$
      G. $F_k = F_{applied}$
      H. $F_s < F_{applied}$
      J. $F_k > F_{applied}$

   9. What is the relationship between the forces at point B?
      A. $F_{s, max} = F_k$
      B. $F_k > F_{s, max}$
      C. $F_k > F_{applied}$
      D. $F_k < F_{applied}$

   **SHORT RESPONSE**

   Base your answers to questions 10–12 on the information below.

   A 3.00 kg ball is dropped from rest from the roof of a building 176.4 m high. While the ball is falling, a horizontal wind exerts a constant force of 12.0 N on the ball.

   10. How long does the ball take to hit the ground?

   11. How far from the building does the ball hit the ground?

   12. When the ball hits the ground, what is its speed?

   **Base your answers to questions 13–15 on the information below.**

   A crate rests on the horizontal bed of a pickup truck. For each situation described below, indicate the motion of the crate relative to the ground, the motion of the crate relative to the truck, and whether the crate will hit the front wall of the truck bed, the back wall, or neither. Disregard friction.

   13. Starting at rest, the truck accelerates to the right.
   14. The crate is at rest relative to the truck while the truck moves with a constant velocity to the right.
   15. The truck in item 14 slows down.

   **EXTENDED RESPONSE**

   16. A student pulls a rope attached to a 10.0 kg wooden sled and moves the sled across dry snow. The student pulls with a force of 15.0 N at an angle of 45.0°. If $\mu_k$ between the sled and the snow is 0.040, what is the sled’s acceleration? Show your work.

   17. You can keep a 3 kg book from dropping by pushing it horizontally against a wall. Draw force diagrams, and identify all the forces involved. How do they combine to result in a zero net force? Will the force you must supply to hold the book up be different for different types of walls? Design a series of experiments to test your answer. Identify exactly which measurements will be necessary and what equipment you will need.

   **Test Tip** For a question involving experimental data, determine the constants, variables, and control before answering the question.
Newton’s second law states that any net external force applied to a mass causes the mass to accelerate according to the equation $F = ma$. Because of frictional forces, experience does not always seem to support this. For example, when you are driving a car, you must apply a constant force to keep the car moving with a constant velocity. In the absence of friction, the car would continue to move with a constant velocity after the force was removed. The continued application of force would cause the car to accelerate.

In this lab, you will study the motion of a dynamics cart pulled by the weight of masses falling from a table to the floor. The cart is set up so that any applied force will cause it to move with a constant velocity. In the first part of the experiment, the total mass will remain constant while the force acting on the cart will be different for each trial. In the second part, the force acting on the cart will remain constant, but the total mass will change for each trial.

**PROCEDURE**

**Preparation**

1. Read the entire lab procedure, and plan the steps you will take.

2. If you are not using a datasheet provided by your teacher, prepare a data table in your lab notebook with six columns and six rows. In the first row, label the first through sixth columns Trial, Total Mass (kg), Accelerating Mass (kg), Accelerating Force (N), Time Interval (s), and Distance (m). In the first column, label the second through sixth rows 1, 2, 3, 4, and 5.

3. Choose a location where the cart will be able to move a considerable distance without any obstacles and where you will be able to clamp the pulley to a table edge.
Apparatus Setup

4. Set up the apparatus as shown in Figure 1. Clamp the pulley to the edge of the table so that it is level with the top of the cart. Clamp the recording timer to a ring stand or to the edge of the table to hold it in place. If the timer is clamped to the table, leave 0.5 m between the timer and the initial position of the cart. Insert the carbon disk into the timer, and thread the tape through the guides under the disk. When your teacher approves your setup, plug the timer into a wall outlet.

5. If you have not used the recording timer before, refer to the lab in the chapter “Motion in One Dimension” for instructions. Calibrate the recording timer with the stopwatch or use the previously determined value for the timer’s period.

6. Record the value for the timer’s period at the top of the data table.

7. Fasten the timing tape to one end of the cart.

Constant Mass with Varying Force

8. Carefully measure the mass of the cart assembly on the platform balance, making sure that the cart does not roll or fall off the balance. Then load the cart with masses equal to 0.60 kg. Lightly tape the masses to the cart to hold them in place. Add these masses to the mass of the cart and record the total.

9. Attach one end of the cord to a small mass hanger and the other end of the cord to the cart. Pass the cord over the pulley and fasten a small mass to the end to offset the frictional force on the cart. You have chosen the correct mass if the cart moves forward with a constant velocity when you give it a push. This counterweight should stay on the string throughout the entire experiment. Add the mass of the counterweight to the mass of the cart and masses, and record the sum as Total Mass in your data table.

10. For the first trial, remove a 0.10 kg mass from the cart, and securely fasten it to the end of the string along with the counterweight. Record 0.10 kg as the Accelerating Mass in the data table.

11. Hold the cart by holding the tape behind the timer. Make sure the area under the falling mass is clear of obstacles. Start the timer and release the tape simultaneously.
12. Carefully stop the cart when the 0.10 kg mass hits the floor, and then stop the timer. **Do not let the cart fall off the table.**

13. Remove the tape and label it with the trial number.

14. Use a meterstick to measure the distance the weights fell. Record the Distance in your data table.

15. On the tape, measure this distance starting from the first clear dot. Mark the end of this distance. Count the number of dots between the first dot and this mark.

16. Calculate and record the Time Interval represented by the number of dots. Fasten a new timing tape to the end of the cart.

17. Replace the 0.10 kg mass in the cart. Remove 0.20 kg from the cart and attach it securely to the end of the cord. Repeat the procedure, label the tape, and record the results in your data table as Trial 2.

18. Leave the 0.20 kg mass on the end of the cord and attach the 0.10 kg mass from the cart securely to the end of the cord. Repeat the procedure, label the tape, and record the results in your data table as Trial 3.

**Constant Force with Varying Mass**

19. For the two trials in this part of the experiment, keep 0.30 kg and the counterweight on the string. Be sure to include this mass when recording the total mass for these three trials.

20. Add 0.50 kg to the cart. Tape the mass to the cart to keep it in place. Run the experiment and record the total mass, accelerating mass, accelerating force, distance, and time under **Trial 4** in your data table.

21. Tape 1.00 kg to the cart and repeat the procedure. Record the data under **Trial 5** in your data table.

22. Clean up your work area. Put equipment away as instructed.

**ANALYSIS**

1. **Analyzing Data**  Calculate the Accelerating Force for each trial. Use Newton’s second law equation, \( F = ma \), where \( m = \) Accelerating Mass and \( a = a_g \). Enter these values in your data table.

2. **Organizing Data**  Use your values for the distance and time to find the acceleration of the cart for each trial, using the equation \( \Delta x = \frac{1}{2}at^2 \) for constantly accelerated motion.

3. **Constructing Graphs**  Using the data from Trials 1–3, plot a graph of the acceleration of the cart versus the accelerating force. Use a graphing calculator, computer, or graph paper.
4. **Analyzing Graphs** Based on your graph from item 3, what is the relationship between the acceleration of the cart and the accelerating force? Explain how your graph supports your answer.

5. **Constructing Graphs** Using the data from *Trials* 3–5, plot a graph of the total mass versus the acceleration. Use a graphing calculator, computer, or graph paper.

6. **Interpreting Graphs** Based on your graph from item 5, what is the relationship between the total mass and the acceleration? Explain how your graph supports your answer.

**CONCLUSIONS**

7. **Evaluating Methods** Why does the mass in *Trials* 1–3 remain constant even though masses are removed from the cart during the trials?

8. **Evaluating Methods** Do the carts move with the same velocity and acceleration as the accelerating masses that are dropped? If not, why not?

9. **Drawing Conclusions** Do your data support Newton’s second law? Use your data and your analysis of your graphs to support your conclusions.

10. **Applying Conclusions** A team of automobile safety engineers developed a new type of car and performed some test crashes to find out whether the car is safe. The engineers tested the new car by involving it in a series of different types of accidents. For each test, the engineers applied a known force to the car and measured the acceleration of the car after the crash. The graph in Figure 2 shows the acceleration of the car plotted against the applied force. Compare this with the data you collected and the graphs you made for this experiment to answer the following questions.

    a. Based on the graph, what is the relationship between the acceleration of the new car and the force of the collision?
    
    b. Does this graph support Newton’s second law? Use your analysis of this graph to support your conclusions.
    
    c. Do the data from the crash tests meet your expectations based on this lab? Explain what you think may have happened to affect the results. If you were on the engineering team, how would you find out whether your results were in error?

**EXTENSION**

11. **Designing Experiments** How would your results be affected if you used the mass of the cart and its contents instead of the total mass? Predict what would happen if you performed *Trials* 1–3 again, keeping the mass of the cart and its contents constant while varying the accelerating mass. If there is time and your teacher approves your plan, go into the lab and try it. Plot your data using a graphing calculator, computer, or graph paper.
1543 – Andries van Wesel, better known as Andreas Vesalius, completes his Seven Books on the Structure of the Human Body. It is the first work on anatomy to be based on the dissection of human bodies.

1544 – Nicholas Copernicus' On the Revolutions of the Heavenly Bodies is published. It is the first work on astronomy to provide an analytical basis for the motion of the planets, including Earth, around the sun.

1543 – Galileo Galilei is appointed professor of mathematics at the University of Padua. While there, he performs experiments on the motions of bodies.

1556 – Akbar becomes ruler of the Moghul Empire in North India, Pakistan, and Afghanistan. By ensuring religious tolerance, he establishes greater unity in India, making it one of the world's great powers.

1564 – English writers Christopher Marlowe and William Shakespeare are born.

1588 – Queen Elizabeth I of England sends the English fleet to repel the invasion by the Spanish Armada. The success of the English navy marks the beginning of Great Britain's status as a major naval power.

1605 – The first part of Miguel de Cervantes's Don Quixote is published.
1669 – Danish geologist Niclaus Steno correctly determines the structure of crystals and identifies fossils as organic remains.

1608 – The first telescopes are constructed in the Netherlands. Using these instruments as models, Galileo constructs his first telescope the following year.

1609

\[ T^2 \propto a^3 \]

*New Astronomy*, by Johannes Kepler, is published. In it, Kepler demonstrates that the orbit of Mars is elliptical rather than circular.

1637 – René Descartes’s *Discourse on Method* is published. According to Descartes’s philosophy of rationalism, the laws of nature can be deduced by reason.

1644 – The Ch’ing, or Manchu, Dynasty is established in China. China becomes the most prosperous nation in the world, then declines until the Ch’ing Dynasty is replaced by the Chinese Republic in 1911.

1655 – The first paintings of Dutch artist Jan Vermeer are produced around this time. Vermeer’s paintings portray middle- and working-class people in everyday situations.

1678

\[ c = f\lambda \]

Christiaan Huygens completes the bulk of his *Treatise on Light*, in which he presents his model of secondary wavelets, known today as Huygens’ principle. The completed book is published 12 years later.

1687

\[ F = ma \]

Issac Newton’s masterpiece, *Mathematical Principles of Natural Philosophy*, is published. In this extensive work, Newton systematically presents a unified model of mechanics.

1608

Physics and Its World 1540–1690 157
This whimsical piece of art is called an *audiokinetic sculpture*. Balls are raised to a high point on the curved blue track. As the balls move down the track, they turn levers, spin rotors, and bounce off elastic membranes. The energy that each ball has—whether associated with the ball’s motion, the ball’s position above the ground, or the ball’s loss of mechanical energy due to friction—varies in a way that keeps the total energy of the system constant.

**WHAT TO EXPECT**

In this chapter, you will learn about work and different types of energy that are relevant to mechanics. Kinetic energy, which is associated with motion, and potential energy, which is related to an object’s position, are two forms of energy that you will study.

**WHY IT MATTERS**

Work, energy, and power are related to one another. Everyday machines such as motors are usually described by the amount of work that they are capable of doing or by the amount of power that they produce.

**CHAPTER PREVIEW**

1. **Work**
   - Definition of Work
2. **Energy**
   - Kinetic Energy
   - Potential Energy
3. **Conservation of Energy**
   - Conserved Quantities
   - Mechanical Energy
4. **Power**
   - Rate of Energy Transfer
Work

DEFINITION OF WORK

Many of the terms you have encountered so far in this book have meanings in physics that are similar to their meanings in everyday life. In its everyday sense, the term work means to do something that takes physical or mental effort. But in physics, work has a distinctly different meaning. Consider the following situations:

• A student holds a heavy chair at arm's length for several minutes.
• A student carries a bucket of water along a horizontal path while walking at constant velocity.

It might surprise you to know that as the term work is used in physics, there is no work done on the chair or the bucket, even though effort is required in both cases. We will return to these examples later.

Work is done on an object when a force causes a displacement of the object

Imagine that your car, like the car shown in Figure 1, has run out of gas and you have to push it down the road to the gas station. If you push the car with a constant horizontal force, the work you do on the car is equal to the magnitude of the force, \( F \), times the magnitude of the displacement of the car. Using the symbol \( d \) instead of \( \Delta x \) for displacement, we define work for a constant force as:

\[ W = Fd \]

Work is not done on an object unless the object is moved with the action of a force. The application of a force alone does not constitute work. For this reason, no work is done on the chair when a student holds the chair at arm's length. Even though the student exerts a force to support the chair, the chair does not move. The student’s tired arms suggest that work is being done, which is indeed true. The quivering muscles in the student's arms go through many small displacements and do work within the student’s body. However, work is not done on the chair.

Work is done only when components of a force are parallel to a displacement

When the force on an object and the object’s displacement are in different directions, only the component of the force that is parallel to the object’s displacement does work. Components of the force perpendicular to a displacement do not do work.
For example, imagine pushing a crate along the ground. If the force you exert is horizontal, all of your effort moves the crate. If your force is at an angle, only the horizontal component of your applied force causes a displacement and contributes to the work. If the angle between the force and the direction of the displacement is $\theta$, as in Figure 2, work can be expressed as follows:

$$W = Fd\cos \theta$$

If $\theta = 0^\circ$, then $\cos 0^\circ = 1$ and $W = Fd$, which is the definition of work given earlier. If $\theta = 90^\circ$, however, then $\cos 90^\circ = 0$ and $W = 0$. So, no work is done on a bucket of water being carried by a student walking horizontally. The upward force exerted by the student to support the bucket is perpendicular to the displacement of the bucket, which results in no work done on the bucket.

Finally, if many constant forces are acting on an object, you can find the net work done on the object by first finding the net force on the object.

**NET WORK DONE BY A CONSTANT NET FORCE**

$$W_{net} = F_{net}d\cos \theta$$

Net work = net force $\times$ displacement $\times$ cosine of the angle between them

Work has dimensions of force times length. In the SI system, work has a unit of newtons times meters ($N \cdot m$), or joules (J). To give you an idea of how large a joule is, consider that the work done in lifting an apple from your waist to the top of your head is about 1 J.

**SAMPLE PROBLEM A**

**Work**

**Problem**

How much work is done on a vacuum cleaner pulled 3.0 m by a force of 50.0 N at an angle of 30.0° above the horizontal?

**Solution**

Given: $F = 50.0\ N$ \quad $\theta = 30.0^\circ$ \quad $d = 3.0\ m$

Unknown: $W = ?$

Use the equation for net work done by a constant force:

$$W = Fd\cos \theta$$

Only the horizontal component of the applied force is doing work on the vacuum cleaner.

$$W = (50.0\ N)(3.0\ m)(\cos 30.0^\circ)$$

$$W = 130\ J$$
The sign of work is important

Work is a scalar quantity and can be positive or negative, as shown in Figure 3. Work is positive when the component of force is in the same direction as the displacement. For example, when you lift a box, the work done by the force you exert on the box is positive because that force is upward, in the same direction as the displacement. Work is negative when the force is in the direction opposite to the displacement.

**Figure 3**
Depending on the angle, an applied force can either cause a moving car to slow down (left), which results in negative work done on the car, or speed up (right), which results in positive work done on the car.
opposite the displacement. For example, the force of kinetic friction between a sliding box and the floor is opposite to the displacement of the box, so the work done by the force of friction on the box is negative. If you are very careful in applying the equation for work, your answer will have the correct sign: \( \cos \theta \) is negative for angles greater than 90° but less than 270°.

If the work done on an object results only in a change in the object’s speed, the sign of the net work on the object tells you whether the object’s speed is increasing or decreasing. If the net work is positive, the object speeds up and work is done on the object. If the net work is negative, the object slows down and work is done by the object on something else.

SECTION REVIEW

1. For each of the following cases, indicate whether the work done on the second object in each example will have a positive or a negative value.
   a. The road exerts a friction force on a speeding car skidding to a stop.
   b. A rope exerts a force on a bucket as the bucket is raised up a well.
   c. Air exerts a force on a parachute as the parachutist falls to Earth.

2. If a neighbor pushes a lawnmower four times as far as you do but exerts only half the force, which one of you does more work and by how much?

3. A worker pushes a 1.50 \( \times \) 10³ N crate with a horizontal force of 345 N a distance of 24.0 m. Assume the coefficient of kinetic friction between the crate and the floor is 0.220.
   a. How much work is done by the worker on the crate?
   b. How much work is done by the floor on the crate?
   c. What is the net work done on the crate?

4. A 0.075 kg ball in a kinetic sculpture moves at a constant speed along a motorized vertical conveyor belt. The ball rises 1.32 m above the ground. A constant frictional force of 0.350 N acts in the direction opposite the conveyor belt’s motion. What is the net work done on the ball?

5. Critical Thinking For each of the following statements, identify whether the everyday or the scientific meaning of work is intended.
   a. Jack had to work against time as the deadline neared.
   b. Jill had to work on her homework before she went to bed.
   c. Jack did work carrying the pail of water up the hill.

6. Critical Thinking Determine whether work is being done in each of the following examples:
   a. a train engine pulling a loaded boxcar initially at rest
   b. a tug of war that is evenly matched
   c. a crane lifting a car
KINETIC ENERGY

Kinetic energy is energy associated with an object in motion. Figure 4 shows a cart of mass \( m \) moving to the right on a frictionless air track under the action of a constant net force, \( \mathbf{F} \), acting to the right. Because the force is constant, we know from Newton’s second law that the cart moves with a constant acceleration, \( \mathbf{a} \). While the force is applied, the cart accelerates from an initial velocity \( v_i \) to a final velocity \( v_f \). If the cart is displaced a distance of \( \Delta x \), the work done by \( \mathbf{F} \) during this displacement is

\[
W_{\text{net}} = F \Delta x = ma \Delta x
\]

When you studied one-dimensional motion, you learned that the following relationship holds when an object undergoes constant acceleration:

\[
v_f^2 = v_i^2 + 2a \Delta x
\]

\[
a \Delta x = \frac{v_f^2 - v_i^2}{2}
\]

Substituting this result into the equation \( W_{\text{net}} = ma \Delta x \) gives

\[
W_{\text{net}} = m \left( \frac{v_f^2 - v_i^2}{2} \right)
\]

\[
W_{\text{net}} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2
\]

Kinetic energy depends on speed and mass

The quantity \( \frac{1}{2} m v^2 \) has a special name in physics: kinetic energy. The kinetic energy of an object with mass \( m \) and speed \( v \), when treated as a particle, is given by the expression shown on the next page.

**Figure 4**

The work done on an object by a constant force equals the object’s mass times its acceleration times its displacement.
Kinetic energy is a scalar quantity, and the SI unit for kinetic energy (and all other forms of energy) is the joule. Recall that a joule is also used as the basic unit for work.

Kinetic energy depends on both an object’s speed and its mass. If a bowling ball and a volleyball are traveling at the same speed, which do you think has more kinetic energy? You may think that because they are moving with identical speeds they have exactly the same kinetic energy. However, the bowling ball has more kinetic energy than the volleyball traveling at the same speed because the bowling ball has more mass than the volleyball.

\[ KE = \frac{1}{2} m v^2 \]

kinetic energy = \( \frac{1}{2} \times \text{mass} \times (\text{speed})^2 \)

### SAMPLE PROBLEM B

**Kinetic Energy**

**Problem**

A 7.00 kg bowling ball moves at 3.00 m/s. How fast must a 2.45 g table-tennis ball move in order to have the same kinetic energy as the bowling ball? Is this speed reasonable for a table-tennis ball in play?

**Solution**

Given:

The subscripts \( b \) and \( t \) indicate the bowling ball and the table-tennis ball, respectively.

\( m_b = 7.00 \text{ kg} \quad m_t = 2.45 \text{ g} \quad v_b = 3.00 \text{ m/s} \)

Unknown: \( v_t = ? \)

First, calculate the kinetic energy of the bowling ball.

\[ KE_b = \frac{1}{2} m_b v_b^2 = \frac{1}{2} (7.00 \text{ kg})(3.00 \text{ m/s})^2 = 31.5 \text{ J} \]

Then, solve for the speed of the table-tennis ball having the same kinetic energy as the bowling ball.

\[ KE_t = \frac{1}{2} m_t v_t^2 = KE_b = 31.5 \text{ J} \]

\[ v_t = \sqrt{\frac{2KE_b}{m_t}} = \sqrt{\frac{(2)(31.5 \text{ J})}{2.45 \times 10^{-3} \text{ kg}}} \]

\[ v_t = 1.60 \times 10^2 \text{ m/s} \]

This speed would be very fast for a table-tennis ball.
The net work done on a body equals its change in kinetic energy

The equation \( W_{\text{net}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \) derived at the beginning of this section says that the net work done by a net force acting on an object is equal to the change in the kinetic energy of the object. This important relationship, known as the work–kinetic energy theorem, is often written as follows:

**WORK–KINETIC ENERGY THEOREM**

\[ W_{\text{net}} = \Delta KE \]

net work = change in kinetic energy

When you use this theorem, you must include all the forces that do work on the object in calculating the net work done. From this theorem, we see that the speed of the object increases if the net work done on it is positive, because the final kinetic energy is greater than the initial kinetic energy. The object’s speed decreases if the net work is negative, because the final kinetic energy is less than the initial kinetic energy.

The work–kinetic energy theorem allows us to think of kinetic energy as the work that an object can do while the object changes speed or as the amount of energy stored in the motion of an object. For example, the moving hammer in the ring-the-bell game in Figure 5 has kinetic energy and can therefore do work on the puck. The puck can do work against gravity by moving up and striking the bell. When the bell is struck, part of the energy is converted into sound.
Work–Kinetic Energy Theorem

**PROBLEM**
On a frozen pond, a person kicks a 10.0 kg sled, giving it an initial speed of 2.2 m/s. How far does the sled move if the coefficient of kinetic friction between the sled and the ice is 0.10?

**SOLUTION**

1. **DEFINE**
   - Given: \( m = 10.0 \text{ kg} \), \( v_i = 2.2 \text{ m/s} \), \( v_f = 0 \text{ m/s} \), \( \mu_k = 0.10 \)
   - Unknown: \( d = ? \)
   - Diagram:

2. **PLAN**
   **Choose an equation or situation:**
   This problem can be solved using the definition of work and the work–kinetic energy theorem.

   \[
   W_{\text{net}} = F_{\text{net}}d \cos \theta
   \]

   The net work done on the sled is provided by the force of kinetic friction.

   \[
   W_{\text{net}} = F_kd \cos \theta = \mu_kmgd \cos \theta
   \]

   The force of kinetic friction is in the direction opposite \( d \), so \( \theta = 180^\circ \).

   Because the sled comes to rest, the final kinetic energy is zero.

   \[
   W_{\text{net}} = \Delta KE = KE_f - KE_i = -\frac{1}{2}mv_i^2
   \]

   Use the work-kinetic energy theorem, and solve for \( d \).

   \[
   -\frac{1}{2}mv_i^2 = \mu_kmgd \cos \theta
   \]

   \[
   d = \frac{-v_i^2}{2\mu_kg \cos \theta}
   \]

3. **CALCULATE**
   **Substitute values into the equation:**

   \[
   d = \frac{-(2.2 \text{ m/s})^2}{2(0.10)(9.81 \text{ m/s}^2)(\cos 180^\circ)}
   \]

   \[
   d = 2.5 \text{ m}
   \]

4. **EVALUATE**
   According to Newton’s second law, the acceleration of the sled is about \(-1 \text{ m/s}^2\) and the time it takes the sled to stop is about 2 s. Thus, the distance the sled traveled in the given amount of time should be less than the distance it would have traveled in the absence of friction.

   \[
   2.5 \text{ m} < (2.2 \text{ m/s})(2 \text{ s}) = 4.4 \text{ m}
   \]
Work–Kinetic Energy Theorem

1. A student wearing frictionless in-line skates on a horizontal surface is pushed by a friend with a constant force of 45 N. How far must the student be pushed, starting from rest, so that her final kinetic energy is 352 J?

2. A $2.0 \times 10^3$ kg car accelerates from rest under the actions of two forces. One is a forward force of 1140 N provided by traction between the wheels and the road. The other is a 950 N resistive force due to various frictional forces. Use the work–kinetic energy theorem to determine how far the car must travel for its speed to reach 2.0 m/s.

3. A $2.1 \times 10^3$ kg car starts from rest at the top of a driveway that is sloped at an angle of 20.0° with the horizontal. An average friction force of $4.0 \times 10^3$ N impedes the car’s motion so that the car’s speed at the bottom of the driveway is 3.8 m/s. What is the length of the driveway?

4. A 75 kg bobsled is pushed along a horizontal surface by two athletes. After the bobsled is pushed a distance of 4.5 m starting from rest, its speed is 6.0 m/s. Find the magnitude of the net force on the bobsled.

The Inside Story on the Energy in Food

The food that you eat provides your body with energy. Your body needs this energy to move your muscles, to maintain a steady internal temperature, and to carry out many other bodily processes. The energy in food is stored as a kind of potential energy in the chemical bonds within sugars and other organic molecules.

When you digest food, some of this energy is released. The energy is then stored again in sugar molecules, usually as glucose. When cells in your body need energy to carry out cellular processes, the cells break down the glucose molecules through a process called cellular respiration. The primary product of cellular respiration is a high-energy molecule called adenosine triphosphate (ATP), which has a significant role in many chemical reactions in cells.

Nutritionists and food scientists use units of Calories to quantify the energy in food. A standard calorie (cal) is defined as the amount of energy required to increase the temperature of 1 mL of water by 1°C, which equals 4.186 joules (J). A food Calorie is actually 1 kilocalorie, or 4186 J.

People who are trying to lose weight often monitor the number of Calories that they eat each day. These people count Calories because the body stores unused energy as fat. Most food labels show the number of Calories in each serving of food. The amount of energy that your body needs each day depends on many factors, including your age, your weight, and the amount of exercise that you get. A typically healthy and active person requires about 1500 to 2000 Calories per day.
POTENTIAL ENERGY

Consider the balanced boulder shown in Figure 6. As long as the boulder remains balanced, it has no kinetic energy. If it becomes unbalanced, it will fall vertically to the desert floor and will gain kinetic energy as it falls. A similar example is an arrow ready to be released on a bent bow. Once the arrow is in flight, it will have kinetic energy.

Potential energy is stored energy

As we have seen, an object in motion has kinetic energy. But a system can have other forms of energy. The examples above describe a form of energy that is due to the position of an object in relation to other objects or to a reference point. Potential energy is associated with an object that has the potential to move because of its position relative to some other location. Unlike kinetic energy, potential energy depends not only on the properties of an object but also on the object’s interaction with its environment.

Gravitational potential energy depends on height from a zero level

You learned earlier how gravitational forces influence the motion of a projectile. If an object is thrown up in the air, the force of gravity will eventually cause the object to fall back down, provided that the object was not thrown too hard. Similarly, the force of gravity will cause the unbalanced boulder in the previous example to fall. The energy associated with an object due to the object’s position relative to a gravitational source is called gravitational potential energy.

Imagine an egg falling off a table. As it falls, it gains kinetic energy. But where does the egg’s kinetic energy come from? It comes from the gravitational potential energy that is associated with the egg’s initial position on the table relative to the floor. Gravitational potential energy can be determined using the following equation:

\[ P_{Eg} = mgh \]

Gravitational potential energy = mass \times free-fall acceleration \times height

The SI unit for gravitational potential energy, like for kinetic energy, is the joule. Note that the definition for gravitational potential energy in this chapter is valid only when the free-fall acceleration is constant over the entire height, such as at any point near the Earth’s surface. Furthermore, gravitational potential energy depends on both the height and the free-fall acceleration, neither of which is a property of an object.
Suppose you drop a volleyball from a second-floor roof and it lands on the first-floor roof of an adjacent building (see Figure 7). If the height is measured from the ground, the gravitational potential energy is not zero because the ball is still above the ground. But if the height is measured from the first-floor roof, the potential energy is zero when the ball lands on the roof. Gravitational potential energy is a result of an object’s position, so it must be measured relative to some zero level. The zero level is the vertical coordinate at which gravitational potential energy is defined to be zero. This zero level is arbitrary, and it is chosen to make a specific problem easier to solve. In many cases, the statement of the problem suggests what to use as a zero level.

**Elastic potential energy depends on distance compressed or stretched**

Imagine you are playing with a spring on a tabletop. You push a block into the spring, compressing the spring, and then release the block. The block slides across the tabletop. The kinetic energy of the block came from the stored energy in the compressed spring. This potential energy is called **elastic potential energy**. Elastic potential energy is stored in any compressed or stretched object, such as a spring or the stretched strings of a tennis racket or guitar.

The length of a spring when no external forces are acting on it is called the **relaxed length** of the spring. When an external force compresses or stretches the spring, elastic potential energy is stored in the spring. The amount of energy depends on the distance the spring is compressed or stretched from its relaxed length, as shown in Figure 8. Elastic potential energy can be determined using the following equation:

\[
PE_{\text{elastic}} = \frac{1}{2}kx^2
\]

The symbol \( k \) is called the **spring constant**, or force constant. For a flexible spring, the spring constant is small, whereas for a stiff spring, the spring constant is large. Spring constants have units of newtons divided by meters (N/m).
**SAMPLE PROBLEM D**

### Potential Energy

**PROBLEM**

A 70.0 kg stuntman is attached to a bungee cord with an unstretched length of 15.0 m. He jumps off a bridge spanning a river from a height of 50.0 m. When he finally stops, the cord has a stretched length of 44.0 m. Treat the stuntman as a point mass, and disregard the weight of the bungee cord. Assuming the spring constant of the bungee cord is 71.8 N/m, what is the total potential energy relative to the water when the man stops falling?

**SOLUTION**

1. **DEFINE**

   Given:
   
   \[ m = 70.0 \text{ kg} \quad k = 71.8 \text{ N/m} \quad g = 9.81 \text{ m/s}^2 \]
   
   \[ h = 50.0 \text{ m} - 44.0 \text{ m} = 6.0 \text{ m} \]
   
   \[ x = 44.0 \text{ m} - 15.0 \text{ m} = 29.0 \text{ m} \]
   
   \[ PE = 0 \text{ J at river level} \]

   Unknown:
   
   \[ PE_{\text{total}} = ? \]

   Diagram:
   
   ![Diagram showing a bungee cord stretched from 15.0 m to 44.0 m with 50.0 m Stretched length and 44.0 m Relaxed length.]

2. **PLAN**

   Choose an equation or situation:

   The zero level for gravitational potential energy is chosen to be at the surface of the water. The total potential energy is the sum of the gravitational and elastic potential energy.

   \[ PE_{\text{total}} = PE_g + PE_{\text{elastic}} \]
   
   \[ PE_g = mgh \]
   
   \[ PE_{\text{elastic}} = \frac{1}{2}kx^2 \]

3. **CALCULATE**

   Substitute the values into the equations and solve:

   \[ PE_g = (70.0 \text{ kg})(9.81 \text{ m/s}^2)(6.0 \text{ m}) = 4.1 \times 10^3 \text{ J} \]
   
   \[ PE_{\text{elastic}} = \frac{1}{2}(71.8 \text{ N/m})(29.0 \text{ m})^2 = 3.02 \times 10^4 \text{ J} \]
   
   \[ PE_{\text{total}} = 4.1 \times 10^3 \text{ J} + 3.02 \times 10^4 \text{ J} \]
   
   \[ PE_{\text{total}} = 3.43 \times 10^4 \text{ J} \]

4. **EVALUATE**

   One way to evaluate the answer is to make an order-of-magnitude estimate. The gravitational potential energy is on the order of \( 10^2 \text{ kg} \times 10 \text{ m/s}^2 \times 10 \text{ m} = 10^4 \text{ J} \). The elastic potential energy is on the order of \( 1 \times 10^2 \text{ N/m} \times 10^2 \text{ m}^2 = 10^4 \text{ J} \). Thus, the total potential energy should be on the order of \( 2 \times 10^4 \text{ J} \). This number is close to the actual answer.
Potential Energy

1. A spring with a force constant of 5.2 N/m has a relaxed length of 2.45 m. When a mass is attached to the end of the spring and allowed to come to rest, the vertical length of the spring is 3.57 m. Calculate the elastic potential energy stored in the spring.

2. The staples inside a stapler are kept in place by a spring with a relaxed length of 0.115 m. If the spring constant is 51.0 N/m, how much elastic potential energy is stored in the spring when its length is 0.150 m?

3. A 40.0 kg child is in a swing that is attached to ropes 2.00 m long. Find the gravitational potential energy associated with the child relative to the child’s lowest position under the following conditions:
   a. when the ropes are horizontal
   b. when the ropes make a 30.0° angle with the vertical
   c. at the bottom of the circular arc

SECTION REVIEW

1. A pinball bangs against a bumper, giving the ball a speed of 42 cm/s. If the ball has a mass of 50.0 g, what is the ball’s kinetic energy in joules?

2. A student slides a 0.75 kg textbook across a table, and it comes to rest after traveling 1.2 m. Given that the coefficient of kinetic friction between the book and the table is 0.34, use the work–kinetic energy theorem to find the book’s initial speed.

3. A spoon is raised 21.0 cm above a table. If the spoon and its contents have a mass of 30.0 g, what is the gravitational potential energy associated with the spoon at that height relative to the surface of the table?

4. Critical Thinking What forms of energy are involved in the following situations?
   a. a bicycle coasting along a level road
   b. heating water
   c. throwing a football
   d. winding the mainspring of a clock

5. Critical Thinking How do the forms of energy in item 4 differ from one another? Be sure to discuss mechanical versus nonmechanical energy, kinetic versus potential energy, and gravitational versus elastic potential energy.
SECTION 3

CONSERVED QUANTITIES

When we say that something is conserved, we mean that it remains constant. If we have a certain amount of a conserved quantity at some instant of time, we will have the same amount of that quantity at a later time. This does not mean that the quantity cannot change form during that time, but if we consider all the forms that the quantity can take, we will find that we always have the same amount.

For example, the amount of money you now have is not a conserved quantity because it is likely to change over time. For the moment, however, let us assume that you do not spend the money you have, so your money is conserved. This means that if you have a dollar in your pocket, you will always have that same amount, although it may change form. One day it may be in the form of a bill. The next day you may have a hundred pennies, and the next day you may have an assortment of dimes and nickels. But when you total the change, you always have the equivalent of a dollar. It would be nice if money were like this, but of course it isn’t. Because money is often acquired and spent, it is not a conserved quantity.

An example of a conserved quantity that you are already familiar with is mass. For instance, imagine that a light bulb is dropped on the floor and shatters into many pieces. No matter how the bulb shatters, the total mass of all of the pieces together is the same as the mass of the intact light bulb because mass is conserved.

MECHANICAL ENERGY

We have seen examples of objects that have either kinetic or potential energy. The description of the motion of many objects, however, often involves a combination of kinetic and potential energy as well as different forms of potential energy. Situations involving a combination of these different forms of energy can often be analyzed simply. For example, consider the motion of the different parts of a pendulum clock. The pendulum swings back and forth. At the highest point of its swing, there is only gravitational potential energy associated with its position. At other points in its swing, the pendulum is in motion, so it has kinetic energy as well. Elastic potential energy is also present in the many springs that are part of the inner workings of the clock. The motion of the pendulum in a clock is shown in Figure 9.
Analyzing situations involving kinetic, gravitational potential, and elastic potential energy is relatively simple. Unfortunately, analyzing situations involving other forms of energy—such as chemical potential energy—is not as easy.

We can ignore these other forms of energy if their influence is negligible or if they are not relevant to the situation being analyzed. In most situations that we are concerned with, these forms of energy are not involved in the motion of objects. In ignoring these other forms of energy, we will find it useful to define a quantity called mechanical energy. The mechanical energy is the sum of kinetic energy and all forms of potential energy associated with an object or group of objects.

\[ ME = KE + \Sigma PE \]

All energy, such as nuclear, chemical, internal, and electrical, that is not mechanical energy is classified as nonmechanical energy. Do not be confused by the term mechanical energy. It is not a unique form of energy. It is merely a way of classifying energy, as shown in Figure 10. As you learn about new forms of energy in this book, you will be able to add them to this chart.

Figure 10
Energy can be classified in a number of ways.

Figure 11
The total mechanical energy, potential energy plus kinetic energy, is conserved as the egg falls.

**Mechanical energy is often conserved**

Imagine a 75 g egg located on a countertop 1.0 m above the ground, as shown in Figure 11. The egg is knocked off the edge and falls to the ground. Because the acceleration of the egg is constant as it falls, you can use the kinematic formulas to determine the speed of the egg and the distance the egg has fallen at any subsequent time. The distance fallen can then be subtracted from the initial height to find the height of the egg above the ground at any subsequent time. For example, after 0.10 s, the egg has a speed of 0.98 m/s and has fallen a distance of 0.05 m, corresponding to a height above the ground of 0.95 m. Once the egg’s speed and its height above the ground are known as a function of time, you can use what you have learned in this chapter to calculate both the kinetic energy of the egg and the gravitational potential energy associated with the position of the egg at any subsequent time. Adding the kinetic and potential energy gives the total mechanical energy at each position.
In the absence of friction, the total mechanical energy remains the same. This principle is called *conservation of mechanical energy*. Although the amount of mechanical energy is constant, mechanical energy itself can change form. For instance, consider the forms of energy for the falling egg, as shown in Table 1. As the egg falls, the potential energy is continuously converted into kinetic energy. If the egg were thrown up in the air, kinetic energy would be converted into gravitational potential energy. In either case, mechanical energy is conserved. The conservation of mechanical energy can be written symbolically as follows:

\[
ME_i = ME_f
\]

(initial mechanical energy = final mechanical energy)

(in the absence of friction)

The mathematical expression for the conservation of mechanical energy depends on the forms of potential energy in a given problem. For instance, if the only force acting on an object is the force of gravity, as in the egg example, the conservation law can be written as follows:

\[
\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f
\]

If other forces (except friction) are present, simply add the appropriate potential energy terms associated with each force. For instance, if the egg happened to compress or stretch a spring as it fell, the conservation law would also include an elastic potential energy term on each side of the equation.

In situations in which frictional forces are present, the principle of mechanical energy conservation no longer holds because kinetic energy is not simply converted to a form of potential energy. This special situation will be discussed more thoroughly later in this section.

### Table 1 Energy of a Falling 75 g Egg

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Height (m)</th>
<th>Speed (m/s)</th>
<th>(PE_g) (J)</th>
<th>(KE) (J)</th>
<th>(ME) (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.0</td>
<td>0.00</td>
<td>0.74</td>
<td>0.00</td>
<td>0.74</td>
</tr>
<tr>
<td>0.10</td>
<td>0.95</td>
<td>0.98</td>
<td>0.70</td>
<td>0.036</td>
<td>0.74</td>
</tr>
<tr>
<td>0.20</td>
<td>0.80</td>
<td>2.0</td>
<td>0.59</td>
<td>0.15</td>
<td>0.74</td>
</tr>
<tr>
<td>0.30</td>
<td>0.56</td>
<td>2.9</td>
<td>0.41</td>
<td>0.33</td>
<td>0.74</td>
</tr>
<tr>
<td>0.40</td>
<td>0.22</td>
<td>3.9</td>
<td>0.16</td>
<td>0.58</td>
<td>0.74</td>
</tr>
</tbody>
</table>

### Quick Lab

**Materials List**

- medium-sized spring (spring balance)
- assortment of small balls, each having a different mass
- ruler
- tape
- scale or balance

**Safety Caution**

Students should wear goggles to perform this lab.

First, determine the mass of each of the balls. Then, tape the ruler to the side of a tabletop so that the ruler is vertical. Place the spring vertically on the tabletop near the ruler, and compress the spring by pressing down on one of the balls. Release the ball, and measure the maximum height it achieves in the air. Repeat this process five times, and be sure to compress the spring by the same amount each time. Average the results. From the data, can you predict how high each of the other balls will rise? Test your predictions. (Hint: Assume mechanical energy is conserved.)
SAMPLE PROBLEM E

Conservation of Mechanical Energy

PROBLEM
Starting from rest, a child zooms down a frictionless slide from an initial height of 3.00 m. What is her speed at the bottom of the slide? Assume she has a mass of 25.0 kg.

SOLUTION
1. DEFINE
   Given: \( h = h_i = 3.00 \text{ m} \) \( m = 25.0 \text{ kg} \) \( v_i = 0.0 \text{ m/s} \) \( h_f = 0 \text{ m} \)
   Unknown: \( v_f = ? \)

2. PLAN
   Choose an equation or situation:
   The slide is frictionless, so mechanical energy is conserved. Kinetic energy and gravitational potential energy are the only forms of energy present.
   \[ KE = \frac{1}{2}mv^2 \quad PE = mgh \]

   The zero level chosen for gravitational potential energy is the bottom of the slide. Because the child ends at the zero level, the final gravitational potential energy is zero.
   \[ PE_{g,f} = 0 \]

   The initial gravitational potential energy at the top of the slide is
   \[ PE_{g,i} = mgh_i = mgh \]

   Because the child starts at rest, the initial kinetic energy at the top is zero.
   \[ KE_i = 0 \]

   Therefore, the final kinetic energy is as follows:
   \[ KE_f = \frac{1}{2}mv_f^2 \]

3. CALCULATE
   Substitute values into the equations:
   \[ PE_{g,i} = (25.0 \text{ kg})(9.81 \text{ m/s}^2)(3.00 \text{ m}) = 736 \text{ J} \]
   \[ KE_f = \frac{1}{2}(25.0 \text{ kg})v_f^2 \]

   Now use the calculated quantities to evaluate the final velocity.
   \[ ME_i = ME_f \]
   \[ PE_i + KE_i = PE_f + KE_f \]
   \[ 736 \text{ J} + 0 \text{ J} = 0 \text{ J} + (0.500)(25.0 \text{ kg})v_f^2 \]

   \[ v_f = 7.67 \text{ m/s} \]

   CALCULATOR SOLUTION
   Your calculator should give an answer of 7.67333, but because the answer is limited to three significant figures, it should be rounded to 7.67.
The expression for the square of the final speed can be written as follows:

\[ v_f^2 = \frac{2mgh}{m} = 2gh \]

Notice that the masses cancel, so the final speed does not depend on the mass of the child. This result makes sense because the acceleration of an object due to gravity does not depend on the mass of the object.

**Practice E**

**Conservation of Mechanical Energy**

1. A bird is flying with a speed of 18.0 m/s over water when it accidentally drops a 2.00 kg fish. If the altitude of the bird is 5.40 m and friction is disregarded, what is the speed of the fish when it hits the water?

2. A 755 N diver drops from a board 10.0 m above the water’s surface. Find the diver’s speed 5.00 m above the water’s surface. Then find the diver’s speed just before striking the water.

3. If the diver in item 2 leaves the board with an initial upward speed of 2.00 m/s, find the diver’s speed when striking the water.

4. An Olympic runner leaps over a hurdle. If the runner’s initial vertical speed is 2.2 m/s, how much will the runner’s center of mass be raised during the jump?

5. A pendulum bob is released from some initial height such that the speed of the bob at the bottom of the swing is 1.9 m/s. What is the initial height of the bob?

**Energy conservation occurs even when acceleration varies**

If the slope of the slide in Sample Problem E was constant, the acceleration along the slide would also be constant and the one-dimensional kinematic formulas could have been used to solve the problem. However, you do not know the shape of the slide. Thus, the acceleration may not be constant, and the kinematic formulas could not be used.

But now we can apply a new method to solve such a problem. Because the slide is frictionless, mechanical energy is conserved. We simply equate the initial mechanical energy to the final mechanical energy and ignore all the details in the middle. The shape of the slide is not a contributing factor to the system’s mechanical energy as long as friction can be ignored.
Mechanical energy is not conserved in the presence of friction

If you have ever used a sanding block to sand a rough surface, such as in Figure 12, you may have noticed that you had to keep applying a force to keep the block moving. The reason is that kinetic friction between the moving block and the surface causes the kinetic energy of the block to be converted into a nonmechanical form of energy. As you continue to exert a force on the block, you are replacing the kinetic energy that is lost because of kinetic friction. The observable result of this energy dissipation is that the sanding block and the tabletop become warmer.

In the presence of kinetic friction, nonmechanical energy is no longer negligible and mechanical energy is no longer conserved. This does not mean that energy in general is not conserved—total energy is always conserved. However, the mechanical energy is converted into forms of energy that are much more difficult to account for, and the mechanical energy is therefore considered to be “lost.”

Figure 12
(a) As the block slides, its kinetic energy tends to decrease because of friction. The force from the hand keeps it moving.
(b) Kinetic energy is dissipated into the block and surface.

SECTION REVIEW

1. If the spring of a jack-in-the-box is compressed a distance of 8.00 cm from its relaxed length and then released, what is the speed of the toy head when the spring returns to its natural length? Assume the mass of the toy head is 50.0 g, the spring constant is 80.0 N/m, and the toy head moves only in the vertical direction. Also disregard the mass of the spring. (Hint: Remember that there are two forms of potential energy in the problem.)

2. You are designing a roller coaster in which a car will be pulled to the top of a hill of height \( h \) and then, starting from a momentary rest, will be released to roll freely down the hill and toward the peak of the next hill, which is 1.1 times as high. Will your design be successful? Explain your answer.

3. Is conservation of mechanical energy likely to hold in these situations?
   a. a hockey puck sliding on a frictionless surface of ice
   b. a toy car rolling on a carpeted floor
   c. a baseball being thrown into the air

4. Critical Thinking What parts of the kinetic sculpture on the opening pages of this chapter involve the conversion of one form of energy to another? Is mechanical energy conserved in these processes?
SECTION OBJECTIVES

- Relate the concepts of energy, time, and power.
- Calculate power in two different ways.
- Explain the effect of machines on work and power.

SECCTION 4

RATE OF ENERGY TRANSFER

The rate at which work is done is called **power**. More generally, power is the rate of energy transfer by any method. Like the concepts of energy and work, power has a specific meaning in science that differs from its everyday meaning.

Imagine you are producing a play and you need to raise and lower the curtain between scenes in a specific amount of time. You decide to use a motor that will pull on a rope connected to the top of the curtain rod. Your assistant finds three motors but doesn’t know which one to use. One way to decide is to consider the power output of each motor.

If the work done on an object is \( W \) in a time interval \( \Delta t \), then the average power delivered to the object over this time interval is written as follows:

\[
P = \frac{W}{\Delta t}
\]

**Power**

It is sometimes useful to rewrite this equation in an alternative form by substituting the definition of work into the definition of power.

\[
W = Fd
\]

\[
P = \frac{W}{\Delta t} = \frac{Fd}{\Delta t}
\]

The distance moved per unit time is just the speed of the object.

**Conceptual Challenge**

1. **Mountain Roads**
   Many mountain roads are built so that they zigzag up the mountain rather than go straight up toward the peak. Discuss the advantages of such a design from the viewpoint of energy conservation and power.

2. **Light Bulbs**
   A light bulb is described as having 60 watts. What’s wrong with this statement?
The SI unit of power is the **watt**, W, which is defined to be one joule per second. The **horsepower**, hp, is another unit of power that is sometimes used. One horsepower is equal to 746 watts.

The watt is perhaps most familiar to you from your everyday experience with light bulbs (see **Figure 13**). A dim light bulb uses about 40 W of power, while a bright bulb can use up to 500 W. Decorative lights use about 0.7 W each for indoor lights and 7.0 W each for outdoor lights.

In Sample Problem F, the three motors would lift the curtain at different rates because the power output for each motor is different. So each motor would do work on the curtain at different rates and would thus transfer energy to the curtain at different rates.

**POWER (ALTERNATIVE FORM)**

\[
P = Fv
\]

\[
\text{power} = \text{force} \times \text{speed}
\]

The SI unit of power is the **watt**, W, which is defined to be one joule per second. The **horsepower**, hp, is another unit of power that is sometimes used. One horsepower is equal to 746 watts.

The watt is perhaps most familiar to you from your everyday experience with light bulbs (see **Figure 13**). A dim light bulb uses about 40 W of power, while a bright bulb can use up to 500 W. Decorative lights use about 0.7 W each for indoor lights and 7.0 W each for outdoor lights.

In Sample Problem F, the three motors would lift the curtain at different rates because the power output for each motor is different. So each motor would do work on the curtain at different rates and would thus transfer energy to the curtain at different rates.

**SAMPLE PROBLEM F**

**Power**

**Problem**

A 193 kg curtain needs to be raised 7.5 m, at constant speed, in as close to 5.0 s as possible. The power ratings for three motors are listed as 1.0 kW, 3.5 kW, and 5.5 kW. Which motor is best for the job?

**Solution**

Given: \(m = 193 \text{ kg} \quad \Delta t = 5.0 \text{ s} \quad d = 7.5 \text{ m}\)

Unknown: \(P = ?\)

Use the definition of power. Substitute the equation for work.

\[
P = \frac{W}{\Delta t} = \frac{F \Delta d}{\Delta t} = \frac{mg \Delta d}{\Delta t}
\]

\[
= \frac{(193 \text{ kg})(9.81 \text{ m/s}^2)(7.5 \text{ m})}{5.0 \text{ s}}
\]

\[
P = 2.8 \times 10^3 \text{ W} = 2.8 \text{ kW}
\]

The best motor to use is the 3.5 kW motor. The 1.0 kW motor will not lift the curtain fast enough, and the 5.5 kW motor will lift the curtain too fast.
Power

1. A $1.0 \times 10^3$ kg elevator carries a maximum load of 800.0 kg. A constant frictional force of $4.0 \times 10^3$ N retards the elevator’s motion upward. What minimum power, in kilowatts, must the motor deliver to lift the fully loaded elevator at a constant speed of 3.00 m/s?

2. A car with a mass of $1.50 \times 10^3$ kg starts from rest and accelerates to a speed of 18.0 m/s in 12.0 s. Assume that the force of resistance remains constant at 400.0 N during this time. What is the average power developed by the car’s engine?

3. A rain cloud contains $2.66 \times 10^7$ kg of water vapor. How long would it take for a 2.00 kW pump to raise the same amount of water to the cloud’s altitude, 2.00 km?

4. How long does it take a 19 kW steam engine to do $6.8 \times 10^7$ J of work?

5. A $1.50 \times 10^3$ kg car accelerates uniformly from rest to 10.0 m/s in 3.00 s.
   a. What is the work done on the car in this time interval?
   b. What is the power delivered by the engine in this time interval?

SECTION REVIEW

1. A 50.0 kg student climbs 5.00 m up a rope at a constant speed. If the student’s power output is 200.0 W, how long does it take the student to climb the rope? How much work does the student do?

2. A motor-driven winch pulls the 50.0 kg student in the previous item 5.00 m up the rope at a constant speed of 1.25 m/s. How much power does the motor use in raising the student? How much work does the motor do on the student?

3. **Critical Thinking**  How are energy, time, and power related?

4. **Critical Thinking**  People often use the word *powerful* to describe the engines in some automobiles. In this context, how does the word relate to the definition of *power*? How does this word relate to the alternative definition of *power*?
As the name states, the cars of a roller coaster really do coast along the tracks. A motor pulls the cars up a high hill at the beginning of the ride. After the hill, however, the motion of the car is a result of gravity and inertia. As the cars roll down the hill, they must pick up the speed that they need to whiz through the rest of the curves, loops, twists, and bumps in the track. To learn more about designing roller coasters, read the interview with Steve Okamoto.

How did you become a roller coaster designer?

I have been fascinated with roller coasters ever since my first ride on one. I remember going to Disneyland as a kid. My mother was always upset with me because I kept looking over the sides of the rides, trying to figure out how they worked. My interest in finding out how things worked led me to study mechanical engineering.

What sort of training do you have?

I earned a degree in product design. For this degree, I studied mechanical engineering and studio art. Product designers consider an object's form as well as its function. They also take into account the interests and abilities of the product's consumer. Most rides and parks have some kind of theme, so I must consider marketing goals and concerns in my designs.

What is the nature of your work?

To design a roller coaster, I study site maps of the location. Then, I go to the amusement park to look at the actual site. Because most rides I design are for older parks (few parks are built from scratch), fitting a coaster around, above, and in between existing rides and buildings is one of my biggest challenges. I also have to design how the parts of the ride will work together. The towers and structures that support the ride have to be strong enough to hold up a track and speeding cars that are full of people. The cars themselves need special wheels to keep them locked onto the track and seat belts or bars to keep the passengers safely inside. It's like putting together a puzzle, except the pieces haven't been cut out yet.

What advice do you have for a student who is interested in designing roller coasters?

Studying math and science is very important. To design a successful coaster, I have to understand how energy is converted from one form to another as the cars move along the track. I have to calculate speeds and accelerations of the cars on each part of the track. They have to go fast enough to make it up the next hill! I rely on my knowledge of geometry and physics to create the roller coaster's curves, loops, and dips.
KEY IDEAS

Section 1  Work
• Work is done on an object only when a net force acts on the object to displace it in the direction of a component of the net force.
• The amount of work done on an object by a force is equal to the component of the force along the direction of motion times the distance the object moves.

Section 2  Energy
• Objects in motion have kinetic energy because of their mass and speed.
• The net work done on or by an object is equal to the change in the kinetic energy of the object.
• Potential energy is energy associated with an object’s position. Two forms of potential energy discussed in this chapter are gravitational potential energy and elastic potential energy.

Section 3  Conservation of Energy
• Energy can change form but can never be created or destroyed.
• Mechanical energy is the total kinetic and potential energy present in a given situation.
• In the absence of friction, mechanical energy is conserved, so the amount of mechanical energy remains constant.

Section 4  Power
• Power is the rate at which work is done or the rate of energy transfer.
• Machines with different power ratings do the same amount of work in different time intervals.

Variable Symbols

<table>
<thead>
<tr>
<th>Quantities</th>
<th>Units</th>
<th>Conversions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W)  work</td>
<td>J</td>
<td>= N(\cdot)m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= kg(\cdot)m(^2)/s(^2)</td>
</tr>
<tr>
<td>(KE) kinetic energy</td>
<td>J</td>
<td></td>
</tr>
<tr>
<td>(PE_g) gravitational potential energy</td>
<td>J</td>
<td></td>
</tr>
<tr>
<td>(PE_{elastic}) elastic potential energy</td>
<td>J</td>
<td></td>
</tr>
<tr>
<td>(P)  power</td>
<td>W</td>
<td>= J/s</td>
</tr>
</tbody>
</table>

KEY TERMS

work (p. 160)
kinetic energy (p. 164)
work–kinetic energy theorem (p. 166)
potential energy (p. 169)
gravitational potential energy (p. 169)
elastic potential energy (p. 170)
spring constant (p. 170)
mechanical energy (p. 174)
power (p. 179)

PROBLEM SOLVING

See Appendix D: Equations for a summary of the equations introduced in this chapter. If you need more problem-solving practice, see Appendix I: Additional Problems.
**WORK**

**Review Questions**

1. Can the speed of an object change if the net work done on it is zero?

2. Discuss whether any work is being done by each of the following agents and, if so, whether the work is positive or negative.
   - a. a chicken scratching the ground
   - b. a person reading a sign
   - c. a crane lifting a bucket of concrete
   - d. the force of gravity on the bucket in (c)

3. Furniture movers wish to load a truck using a ramp from the ground to the rear of the truck. One of the movers claims that less work would be required if the ramp’s length were increased, reducing its angle with the horizontal. Is this claim valid? Explain.

**Conceptual Questions**

4. A pendulum swings back and forth, as shown at right. Does the tension force in the string do work on the pendulum bob? Does the force of gravity do work on the bob? Explain your answers.

5. The drivers of two identical cars heading toward each other apply the brakes at the same instant. The skid marks of one of the cars are twice as long as the skid marks of the other vehicle. Assuming that the brakes of both cars apply the same force, what conclusions can you draw about the motion of the cars?

6. When a punter kicks a football, is he doing work on the ball while his toe is in contact with it? Is he doing work on the ball after the ball loses contact with his toe? Are any forces doing work on the ball while the ball is in flight?

**Practice Problems**

For problems 7–10, see Sample Problem A.

7. A person lifts a 4.5 kg cement block a vertical distance of 1.2 m and then carries the block horizontally a distance of 7.3 m. Determine the work done by the person and by the force of gravity in this process.

8. A plane designed for vertical takeoff has a mass of $8.0 \times 10^3$ kg. Find the net work done by all forces on the plane as it accelerates upward at $1.0 \text{ m/s}^2$ through a distance of 30.0 m after starting from rest.

9. When catching a baseball, a catcher’s glove moves by 10 cm along the line of motion of the ball. If the baseball exerts a force of 475 N on the glove, how much work is done by the ball?

10. A flight attendant pulls her 70.0 N flight bag a distance of 253 m along a level airport floor at a constant velocity. The force she exerts is 40.0 N at an angle of 52.0° above the horizontal. Find the following:
   - a. the work she does on the flight bag
   - b. the work done by the force of friction on the flight bag
   - c. the coefficient of kinetic friction between the flight bag and the floor

**ENERGY**

**Review Questions**

11. A person drops a ball from the top of a building while another person on the ground observes the ball’s motion. Each observer chooses his or her own location as the level for zero potential energy. Will they calculate the same values for:
   - a. the potential energy associated with the ball?
   - b. the change in potential energy associated with the ball?
   - c. the ball’s kinetic energy?
12. Can the kinetic energy of an object be negative? Explain your answer.

13. Can the gravitational potential energy associated with an object be negative? Explain your answer.

14. Two identical objects move with speeds of 5.0 m/s and 25.0 m/s. What is the ratio of their kinetic energies?

**Conceptual Questions**

15. A satellite is in a circular orbit above Earth’s surface. Why is the work done on the satellite by the gravitational force zero? What does the work–kinetic energy theorem predict about the satellite’s speed?

16. A car traveling at 50.0 km/h skids a distance of 35 m after its brakes lock. Estimate how far it will skid if its brakes lock when its initial speed is 100.0 km/h. What happens to the car’s kinetic energy as it comes to rest?

17. Explain why more energy is needed to walk up stairs than to walk horizontally at the same speed.

18. How can the work–kinetic energy theorem explain why the force of sliding friction reduces the kinetic energy of a particle?

**Practice Problems**

For problems 19–20, see Sample Problem B.

19. What is the kinetic energy of an automobile with a mass of 1250 kg traveling at a speed of 11 m/s?

20. What speed would a fly with a mass of 0.55 g need in order to have the same kinetic energy as the automobile in item 19?

For problems 21–22, see Sample Problem C.

21. A 50.0 kg diver steps off a diving board and drops straight down into the water. The water provides an upward average net force of 1500 N. If the diver comes to rest 5.0 m below the water’s surface, what is the total distance between the diving board and the diver’s stopping point underwater?

22. In a circus performance, a monkey on a sled is given an initial speed of 4.0 m/s up a 25° incline. The combined mass of the monkey and the sled is 20.0 kg, and the coefficient of kinetic friction between the sled and the incline is 0.20. How far up the incline does the sled move?

For problems 23–25, see Sample Problem D.

23. A 55 kg skier is at the top of a slope, as shown in the illustration below. At the initial point A, the skier is 10.0 m vertically above the final point B.

   a. Set the zero level for gravitational potential energy at B, and find the gravitational potential energy associated with the skier at A and at B. Then find the difference in potential energy between these two points.
   
   b. Repeat this problem with the zero level at point A.
   
   c. Repeat this problem with the zero level midway down the slope, at a height of 5.0 m.

24. A 2.00 kg ball is attached to a ceiling by a string. The distance from the ceiling to the center of the ball is 1.00 m, and the height of the room is 3.00 m. What is the gravitational potential energy associated with the ball relative to each of the following?

   a. the ceiling
   
   b. the floor
   
   c. a point at the same elevation as the ball

25. A spring has a force constant of 500.0 N/m. Show that the potential energy stored in the spring is as follows:

   a. 0.400 J when the spring is stretched 4.00 cm from equilibrium
   
   b. 0.225 J when the spring is compressed 3.00 cm from equilibrium
   
   c. zero when the spring is unstretched
CONSERVATION OF MECHANICAL ENERGY

**Review Questions**

26. Each of the following objects possesses energy. Which forms of energy are mechanical, which are nonmechanical, and which are a combination?
   - a. glowing embers in a campfire
   - b. a strong wind
   - c. a swinging pendulum
   - d. a person sitting on a mattress
   - e. a rocket being launched into space

27. Discuss the energy transformations that occur during the pole-vault event shown in the photograph below. Disregard rotational motion and air resistance.

28. A strong cord suspends a bowling ball from the center of a lecture hall's ceiling, forming a pendulum. The ball is pulled to the tip of a lecturer's nose at the front of the room and is then released. If the lecturer remains stationary, explain why the lecturer is not struck by the ball on its return swing. Would this person be safe if the ball were given a slight push from its starting position at the person's nose?

**Conceptual Questions**

29. Discuss the work done and change in mechanical energy as an athlete does the following:
   - a. lifts a weight
   - b. holds the weight up in a fixed position
   - c. lowers the weight slowly

30. A ball is thrown straight up. At what position is its kinetic energy at its maximum? At what position is gravitational potential energy at its maximum?

31. Advertisements for a toy ball once stated that it would rebound to a height greater than the height from which it was dropped. Is this possible?

32. A weight is connected to a spring that is suspended vertically from the ceiling. If the weight is displaced downward from its equilibrium position and released, it will oscillate up and down. How many forms of potential energy are involved? If air resistance and friction are disregarded, will the total mechanical energy be conserved? Explain.

**Practice Problems**

For problems 33–34, see Sample Problem E.

33. A child and sled with a combined mass of 50.0 kg slide down a frictionless hill that is 7.34 m high. If the sled starts from rest, what is its speed at the bottom of the hill?

34. Tarzan swings on a 30.0 m long vine initially inclined at an angle of 37.0° with the vertical. What is his speed at the bottom of the swing if he does the following?
   - a. starts from rest
   - b. starts with an initial speed of 4.00 m/s

**POWER**

For problems 35–36, see Sample Problem F.

35. If an automobile engine delivers 50.0 hp of power, how much time will it take for the engine to do $6.40 \times 10^5$ J of work? (Hint: Note that one horsepower, 1 hp, is equal to 746 watts.)

36. Water flows over a section of Niagara Falls at the rate of $1.2 \times 10^6$ kg/s and falls 50.0 m. How much power is generated by the falling water?
37. A 215 g particle is released from rest at point A inside a smooth hemispherical bowl of radius 30.0 cm, as shown at right. Calculate the following:
   a. the gravitational potential energy at A relative to B
   b. the particle’s kinetic energy at B
   c. the particle’s speed at B
   d. the potential energy and kinetic energy at C

38. A person doing a chin-up weighs 700.0 N, disregarding the weight of the arms. During the first 25.0 cm of the lift, each arm exerts an upward force of 355 N on the torso. If the upward movement starts from rest, what is the person’s speed at this point?

39. A 50.0 kg pole vaulter running at 10.0 m/s vaults over the bar. If the vaulter’s horizontal component of velocity over the bar is 1.0 m/s and air resistance is disregarded, how high was the jump?

40. An 80.0 N box of clothes is pulled 20.0 m up a 30.0° ramp by a force of 115 N that points along the ramp. If the coefficient of kinetic friction between the box and ramp is 0.22, calculate the change in the box’s kinetic energy.

41. Tarzan and Jane, whose total mass is 130.0 kg, start their swing on a 5.0 m long vine when the vine is at an angle of 30.0° with the horizontal. At the bottom of the arc, Jane, whose mass is 50.0 kg, releases the vine. What is the maximum height at which Tarzan can land on a branch after his swing continues? (Hint: Treat Tarzan’s and Jane’s energies as separate quantities.)

42. A 0.250 kg block on a vertical spring with a spring constant of 5.00 \( \times \) 10\(^3\) N/m is pushed downward, compressing the spring 0.100 m. When released, the block leaves the spring and travels upward vertically. How high does it rise above the point of release?

43. Three identical balls, all with the same initial speed, are thrown at a juggling clown on a tightrope. The first ball is thrown horizontally, the second is thrown at some angle above the horizontal, and the third is thrown at some angle below the horizontal. Disregarding air resistance, describe the motions of the three balls, and compare the speeds of the balls as they reach the ground.

44. A 0.60 kg rubber ball has a speed of 2.0 m/s at point A and kinetic energy of 7.5 J at point B. Determine the following:
   a. the ball’s kinetic energy at A
   b. the ball’s speed at B
   c. the total work done on the ball from A to B

45. Starting from rest, a 5.0 kg block slides 2.5 m down a rough 30.0° incline in 2.0 s. Determine the following:
   a. the work done by the force of gravity
   b. the mechanical energy lost due to friction
   c. the work done by the normal force between the block and the incline

46. A skier of mass 70.0 kg is pulled up a slope by a motor-driven cable. How much work is required to pull the skier 60.0 m up a 35° slope (assumed to be frictionless) at a constant speed of 2.0 m/s?

47. An acrobat on skis starts from rest 50.0 m above the ground on a frictionless track and flies off the track at a 45.0° angle above the horizontal and at a height of 10.0 m. Disregard air resistance.
   a. What is the skier’s speed when leaving the track?
   b. What is the maximum height attained?

48. Starting from rest, a 10.0 kg suitcase slides 3.00 m down a frictionless ramp inclined at 30.0° from the floor. The suitcase then slides an additional 5.00 m along the floor before coming to a stop. Determine the following:
   a. the suitcase’s speed at the bottom of the ramp
   b. the coefficient of kinetic friction between the suitcase and the floor
   c. the change in mechanical energy due to friction

49. A light horizontal spring has a spring constant of 105 N/m. A 2.00 kg block is pressed against one end of the spring, compressing the spring 0.100 m. After the block is released, the block moves 0.250 m to the right before coming to rest. What is the coefficient of kinetic friction between the horizontal surface and the block?
50. A 5.0 kg block is pushed 3.0 m at a constant velocity up a vertical wall by a constant force applied at an angle of 30.0° with the horizontal, as shown at right. If the coefficient of kinetic friction between the block and the wall is 0.30, determine the following:

a. the work done by the force on the block
b. the work done by gravity on the block
c. the magnitude of the normal force between the block and the wall

51. A 25 kg child on a 2.0 m long swing is released from rest when the swing supports make an angle of 30.0° with the vertical.

a. What is the maximum potential energy associated with the child?
b. Disregarding friction, find the child’s speed at the lowest position.
c. What is the child’s total mechanical energy?
d. If the speed of the child at the lowest position is 2.00 m/s, what is the change in mechanical energy due to friction?

Graphing Calculator Practice

Work of Displacement

Work done, as you learned earlier in this chapter, is a result of the net applied force, the distance of the displacement, and the angle of the applied force relative to the direction of displacement. Work done is described by the following equation:

\[ W_{\text{net}} = F_{\text{net}}d\cos \theta \]

The equation for work done can be represented on a graphing calculator as follows:

\[ Y_1 = FX\cos(\theta) \]

In this activity, you will use this equation and your graphing calculator to produce a table of results for various values of \( \theta \). Column one of the table will be the displacement (X) in meters, and column two will be the work done (Y1) in joules.

Visit go.hrw.com and enter the keyword HF6WRKX to find this graphing calculator activity. Refer to Appendix B for instructions on downloading the program for this activity.
52. A ball of mass 522 g starts at rest and slides down a frictionless track, as shown at right. It leaves the track horizontally, striking the ground.

a. At what height above the ground does the ball start to move?

b. What is the speed of the ball when it leaves the track?

c. What is the speed of the ball when it hits the ground?

---

**Alternative Assessment**

1. Design experiments for measuring your power output when doing push-ups, running up a flight of stairs, pushing a car, loading boxes onto a truck, throwing a baseball, or performing other energy-transferring activities. What data do you need to measure or calculate? Form groups to present and discuss your plans. If your teacher approves your plans, perform the experiments.

2. Investigate the amount of kinetic energy involved when your car’s speed is 60 km/h, 50 km/h, 40 km/h, 30 km/h, 20 km/h, and 10 km/h. (Hint: Find your car’s mass in the owner’s manual.) How much work does the brake system have to do to stop the car at each speed?

   If the owner’s manual includes a table of braking distances at different speeds, determine the force the braking system must exert. Organize your findings in charts and graphs to study the questions and to present your conclusions.

3. Investigate the energy transformations of your body as you swing on a swing set. Working with a partner, measure the height of the swing at the high and low points of your motion. What points involve a maximum gravitational potential energy? What points involve a maximum kinetic energy? For three other points in the path of the swing, calculate the gravitational potential energy, the kinetic energy, and the velocity. Organize your findings in bar graphs.

4. In order to save fuel, an airline executive recommended the following changes in the airlines’ largest jet flights:

   a. restrict the weight of personal luggage
   b. remove pillows, blankets, and magazines from the cabin
   c. lower flight altitudes by 5 percent
   d. reduce flying speeds by 5 percent

   Research the information necessary to calculate the approximate kinetic and potential energy of a large passenger aircraft. Which of the measures described above would result in significant savings? What might be their other consequences? Summarize your conclusions in a presentation or report.

5. Make a chart of the kinetic energies your body can have. Measure your mass and speed when walking, running, sprinting, riding a bicycle, and driving a car. Make a poster graphically comparing these findings.

6. You are trying to find a way to bring electricity to a remote village in order to run a water-purifying device. A donor is willing to provide battery chargers that connect to bicycles. Assuming the water-purification device requires 18.6 kW•h daily, how many bicycles would a village need if a person can average 100 W while riding a bicycle? Is this a useful way to help the village? Evaluate your findings for strengths and weaknesses. Summarize your comments and suggestions in a letter to the donor.
MULTIPLE CHOICE

1. In which of the following situations is work not being done?
   A. A chair is lifted vertically with respect to the floor.
   B. A bookcase is slid across carpeting.
   C. A table is dropped onto the ground.
   D. A stack of books is carried at waist level across a room.

2. Which of the following equations correctly describes the relation between power, work, and time?
   F. \( W = \frac{P}{t} \)
   G. \( W = \frac{t}{P} \)
   H. \( P = \frac{W}{t} \)
   J. \( P = \frac{t}{W} \)

Use the graph below to answer questions 3–5. The graph shows the energy of a 75 g yo-yo at different times as the yo-yo moves up and down on its string.

3. By what amount does the mechanical energy of the yo-yo change after 6.0 s?
   A. 500 mJ
   B. 0 mJ
   C. −100 mJ
   D. −600 mJ

4. What is the speed of the yo-yo after 4.5 s?
   F. 3.1 m/s
   G. 2.3 m/s
   H. 3.6 m/s
   J. 1.6 m/s

5. What is the maximum height of the yo-yo?
   A. 0.27 m
   B. 0.54 m
   C. 0.75 m
   D. 0.82 m

6. A car with mass \( m \) requires 5.0 kJ of work to move from rest to a final speed \( v \). If this same amount of work is performed during the same amount of time on a car with a mass of \( 2m \), what is the final speed of the second car?
   F. \( 2v \)
   G. \( \sqrt{2}v \)
   H. \( \frac{v}{2} \)
   J. \( \frac{\sqrt{2}v}{2} \)

Use the passage below to answer questions 7–8.

A 70.0 kg base runner moving at a speed of 4.0 m/s begins his slide into second base. The coefficient of friction between his clothes and Earth is 0.70. His slide lowers his speed to zero just as he reaches the base.

7. How much mechanical energy is lost because of friction acting on the runner?
   A. 1100 J
   B. 560 J
   C. 140 J
   D. 0 J

8. How far does the runner slide?
   F. 0.29 m
   G. 0.57 m
   H. 0.86 m
   J. 1.2 m
Use the passage below to answer questions 9–10.
A spring scale has a spring with a force constant of 250 N/m and a weighing pan with a mass of 0.075 kg. During one weighing, the spring is stretched a distance of 12 cm from equilibrium. During a second weighing, the spring is stretched a distance of 18 cm.

9. How much greater is the elastic potential energy of the stretched spring during the second weighing than during the first weighing?

A. \( \frac{9}{4} \)  
B. \( \frac{3}{2} \)  
C. \( \frac{2}{3} \)  
D. \( \frac{4}{9} \)

10. If the spring is suddenly released after each weighing, the weighing pan moves back and forth through the equilibrium position. What is the ratio of the pan’s maximum speed after the second weighing to the pan’s maximum speed after the first weighing? Consider the force of gravity on the pan to be negligible.

F. \( \frac{9}{4} \)  
H. \( \frac{2}{3} \)  
G. \( \frac{3}{2} \)  
J. \( \frac{4}{9} \)

**SHORT RESPONSE**

11. A student with a mass of 66.0 kg climbs a staircase in 44.0 s. If the distance between the base and the top of the staircase is 14.0 m, how much power will the student deliver by climbing the stairs?

**EXTENDED RESPONSE**

Base your answers to questions 14–16 on the information below.

A projectile with a mass of 5.0 kg is shot horizontally from a height of 25.0 m above a flat desert surface. The projectile’s initial speed is 17 m/s. Calculate the following for the instant before the projectile hits the surface:

14. The work done on the projectile by gravity.

15. The change in kinetic energy since the projectile was fired.

16. The final kinetic energy of the projectile.

17. A skier starts from rest at the top of a hill that is inclined at 10.5° with the horizontal. The hillside is 200.0 m long, and the coefficient of friction between the snow and the skis is 0.075. At the bottom of the hill, the snow is level and the coefficient of friction is unchanged. How far does the skier move along the horizontal portion of the snow before coming to rest? Show all of your work.
A mass on a spring will oscillate vertically when it is lifted to the length of the relaxed spring and released. The gravitational potential energy increases from a minimum at the lowest point to a maximum at the highest point. The elastic potential energy in the spring increases from a minimum at the highest point, where the spring is relaxed, to a maximum at the lowest point, where the spring is stretched. Because the mass is temporarily at rest, the kinetic energy of the mass is zero at the highest and lowest points. Thus, the total mechanical energy at those points is the sum of the elastic potential energy and the gravitational potential energy.

A Hooke’s law apparatus combines a stand for mounting a hanging spring and a vertical ruler for measuring the displacement of a mass attached to the spring. In this lab, you will use a Hooke’s law apparatus to determine the spring constant of a spring. You will also collect data during the oscillation of a mass on the spring and use your data to calculate gravitational potential energy and elastic potential energy at different points in the oscillation.

**PREPARATION**

1. Read the entire lab procedure, and plan the steps you will take.

2. If you are not using a datasheet provided by your teacher, prepare a data table in your lab notebook with four columns and seven rows. In the first row, label the first through fourth columns *Trial*, *Mass (kg)*, *Stretched Spring (m)*, and *Force (N)*. In the first column, label the second through seventh rows 1, 2, 3, 4, 5, and 6. Above or below the data table, make a space to enter the value for *Initial Spring (m)*.

3. If you are not using a datasheet provided by your teacher, prepare a second data table in your lab notebook with three columns and seven rows. In the first row, label the first through third columns *Trial*, *Highest Point*. The second through seventh rows will be labeled 1, 2, 3, 4, 5, and 6.
(m), and Lowest Point (m). In the first column, label the second through seventh rows 1, 2, 3, 4, 5, and 6. Above or below the data table, make a space to enter the value for Initial Distance (m).

Spring Constant

4. Set up the Hooke’s law apparatus as shown in Figure 1.

5. Place a rubber band around the scale at the initial resting position of the pointer, or adjust the scale or pan to read 0.0 cm. Record this position of the pointer as Initial Spring (m). If you have set the scale at 0.0 cm, record 0.00 m as the initial spring position.

6. Measure the distance from the floor to the rubber band on the scale. Record this measurement in the second data table under Initial Distance (m). This distance must remain constant throughout the lab.

7. Find a mass that will stretch the spring so that the pointer moves approximately one-quarter of the way down the scale.

8. Record the value of the mass. Also record the position of the pointer under Stretched Spring in the data table.

9. Perform several trials with increasing masses until the spring stretches to the bottom of the scale. Record the mass and the position of the pointer for each trial.

Conservation of Mechanical Energy

10. Find a mass that will stretch the spring to about twice its original length. Record the mass in the second data table. Leave the mass in place on the pan.

Figure 1

Step 5: If the scale is adjusted to read 0.0 cm, record 0.00 m as the initial spring length in your data table.

Step 7: In this part of the lab, you will collect data to find the spring constant of the spring.

Step 10: In this part of the lab, you will oscillate a mass on the spring to find out whether mechanical energy is conserved.
11. Raise the pan until the pointer is at the zero position, the position where you measured the Initial Spring measurement.

12. Gently release the pan to let the pan drop. Watch closely to identify the high and low points of the oscillation.

13. Use a rubber band to mark the lowest position to which the pan falls, as indicated by the pointer. This point is the lowest point of the oscillation. Record the values as Highest Point and Lowest Point in your data table.

14. Perform several more trials, using a different mass for each trial. Record all data in your data table.

15. Clean up your work area. Put equipment away safely so that it is ready to be used again.

**ANALYSIS**

1. **Organizing Data** Use your data from the first data table to calculate the elongation of the spring. Use the equation \( \text{elongation} = \text{initial spring} - \text{stretched spring} \).

2. **Organizing Data** For each trial, convert the masses used to measure the spring constant to their force equivalents. Use the equation \( F_g = ma_g \).

3. **Organizing Data** For each trial, calculate the spring constant using the equation \( k = \frac{\text{force}}{\text{elongation}} \). Take the average of all trials, and use this value as the spring constant.

4. **Organizing Data** Using your data from the second data table, calculate the elongation of the spring at the highest point of each trial. Use the equation \( \text{elongation} = \text{highest point} - \text{initial spring} \). Refer to **Figure 2**.

5. **Organizing Data** Calculate the elongation of the spring at the lowest point of each trial. Use the equation \( \text{elongation} = \text{lowest point} - \text{initial spring} \). Refer to **Figure 2**.

6. **Organizing Data** For each trial, calculate the elastic potential energy, \( P_{Elastic} = \frac{1}{2}kx^2 \), at the highest point of the oscillation.

7. **Organizing Data** For each trial, calculate the elastic potential energy at the lowest point of the oscillation.

8. **Analyzing Results** Based on your calculations in items 6 and 7, where is the elastic potential energy greatest? Where is it the least? Explain these results in terms of the energy stored in the spring.

9. **Organizing Data** Calculate the height of the mass at the highest point of each trial. Use the equation \( \text{highest} = \text{initial distance} - \text{elongation} \).
10. **Organizing Data** Calculate the height of the mass at the lowest point of each trial. Use the equation \( \text{lowest} = \text{initial distance} - \text{elongation} \).

11. **Organizing Data** For each trial, calculate the gravitational potential energy, \( PE_g = mg \text{h} \), at the highest point of the oscillation.

12. **Organizing Data** For each trial, calculate the gravitational potential energy at the lowest point of the oscillation.

13. **Analyzing Results** According to your calculations in items 11 and 12, where is the gravitational potential energy the greatest? Where is it the least? Explain these results in terms of gravity and the height of the mass and the spring.

14. **Organizing Data** Find the total potential energy at the top of the oscillation and at the bottom of the oscillation.

**CONCLUSIONS**

15. **Drawing Conclusions** Based on your data, is mechanical energy conserved in the oscillating mass on the spring? Explain how your data support your answers.

16. **Making Predictions** How would using a stiffer spring affect the value for the spring constant? How would this change affect the values for the elastic and gravitational potential energies?

**EXTENSIONS**

17. **Extending Ideas** Use your data to find the midpoint of the oscillation for each trial. Calculate the gravitational potential energy and the elastic potential energy at the midpoint. Use the principle of the conservation of mechanical energy to find the kinetic energy and the speed of the mass at the midpoint.

18. **Designing Experiments** Based on what you have learned in this lab, design an experiment to measure the spring constants of springs and other elastic materials in common products, such as the springs inside ball point pens, rubber bands, or even elastic waistbands. Include in your plan a way to determine how well each spring or elastic material conserves mechanical energy. If you have time and your teacher approves your plan, carry out the experiment on several items, and make a table comparing your results for the various items.