*In Exercises 109–112, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.*

**109.** The equation $y = mx + b$ shows that no line can have a $y$-intercept that is numerically equal to its slope.

**110.** Every line in the rectangular coordinate system has an equation that can be expressed in slope-intercept form.

**111.** The graph of the linear function $5x + 6y - 30 = 0$ is a line passing through the point $(6, 0)$ with slope $-\frac{5}{6}$.

**112.** The graph of $x = 7$ in the rectangular coordinate system is the single point $(7, 0)$.

*In Exercises 113–114, find the coefficients that must be placed in each shaded area so that the function's graph will be a line satisfying the specified conditions.*

**113.** $\square x + \square y - 12 = 0$; $x$-intercept $= -2$; $y$-intercept $= 4$

**114.** $\square x + \square y - 12 = 0$; $y$-intercept $= -6$; slope $= \dfrac{1}{2}$

**115.** Prove that the equation of a line passing through $(a, 0)$ and $(0, b)(a \neq 0, b \neq 0)$ can be written in the form $\dfrac{x}{a} + \dfrac{y}{b} = 1$. Why is this called the *intercept form* of a line?

**116.** Excited about the success of celebrity stamps, post office officials were rumored to have put forth a plan to institute two new types of thermometers. On these new scales, $°E$ represents degrees Elvis and $°M$ represents degrees Madonna. If it is known that $40°E = 25°M$, $280°E = 125°M$, and degrees Elvis is linearly related to degrees Madonna, write an equation expressing $E$ in terms of $M$.

## Group Exercise

**117.** In Exercises 87–88, we used the data in a bar graph to develop linear functions that modeled the percentage of never-married American females and males, ages 25–29. For this group exercise, you might find it helpful to pattern your work after Exercises 87 and 88. Group members should begin by consulting an almanac, newspaper, magazine, or the Internet to find data that appear to lie approximately on or near a line. Working by hand or using a graphing utility, group members should construct scatter plots for the data that were assembled. If working by hand, draw a line that approximately fits the data in each scatter plot and then write its equation as a function in slope-intercept form. If using a graphing utility, obtain the equation of each regression line. Then use each linear function's equation to make predictions about what might occur in the future. Are there circumstances that might affect the accuracy of the prediction? List some of these circumstances.

## Preview Exercises

*Exercises 118–120 will help you prepare for the material covered in the next section.*

**118.** Write the slope-intercept form of the equation of the line passing through $(-3, 1)$ whose slope is the same as the line whose equation is $y = 2x + 1$.

**119.** Write an equation in general form of the line passing through $(3, -5)$ whose slope is the negative reciprocal (the reciprocal with the opposite sign) of $-\frac{1}{4}$.

**120.** If $f(x) = x^2$, find

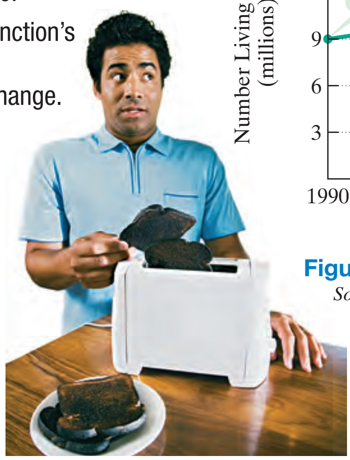
$$\frac{f(x_2) - f(x_1)}{x_2 - x_1},$$

where $x_1 = 1$ and $x_2 = 4$.

# Section 1.5 More on Slope

### Objectives

1. Find slopes and equations of parallel and perpendicular lines.
2. Interpret slope as rate of change.
3. Find a function's average rate of change.



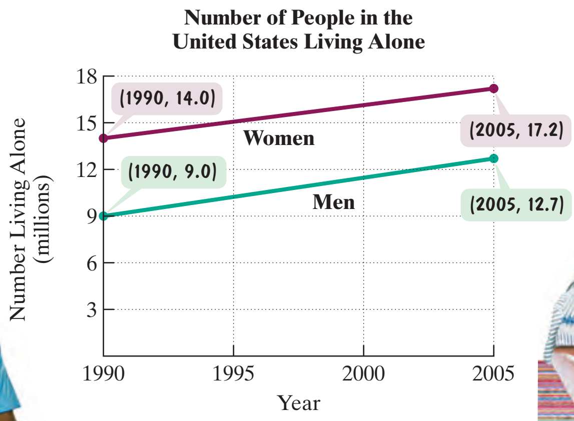



**Figure 1.46**
*Source:* U.S. Census Bureau

A best guess at the future of our nation indicates that the numbers of men and women living alone will increase each year. **Figure 1.46** shows that in 2005, 12.7 million men and 17.2 million women lived alone, an increase over the numbers displayed in the graph for 1990.

Take a second look at **Figure 1.46**. Can you tell that the green graph representing men has a greater slope than the red graph representing women? This indicates a greater rate of change in the number of men living alone than in the number of women living alone over the period from 1990 through 2005. In this section, you will learn to interpret slope as a rate of change. You will also explore the relationships between slopes of parallel and perpendicular lines.

1 Find slopes and equations of parallel and perpendicular lines.

## Parallel and Perpendicular Lines

Two nonintersecting lines that lie in the same plane are **parallel**. If two lines do not intersect, the ratio of the vertical change to the horizontal change is the same for both lines. Because two parallel lines have the same "steepness," they must have the same slope.

### Slope and Parallel Lines

1. If two nonvertical lines are parallel, then they have the same slope.
2. If two distinct nonvertical lines have the same slope, then they are parallel.
3. Two distinct vertical lines, both with undefined slopes, are parallel.

### EXAMPLE 1 Writing Equations of a Line Parallel to a Given Line

Write an equation of the line passing through $(-3, 1)$ and parallel to the line whose equation is $y = 2x + 1$. Express the equation in point-slope form and slope-intercept form.

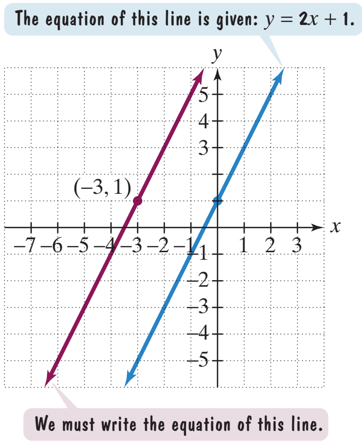



**Figure 1.47**

**Solution** The situation is illustrated in **Figure 1.47**. We are looking for the equation of the red line shown on the left. How do we obtain this equation? Notice that the line passes through the point $(-3, 1)$. Using the point-slope form of the line's equation, we have $x_1 = -3$ and $y_1 = 1$.

$$y - y_1 = m(x - x_1)$$

$y_1 = 1$ $x_1 = -3$

With $(x_1, y_1) = (-3, 1)$, the only thing missing from the equation of the red line is $m$, the slope. Do we know anything about the slope of either line in **Figure 1.47**? The answer is yes; we know the slope of the blue line on the right, whose equation is given.

$$y = 2x + 1$$

The slope of the blue line on the right in **Figure 1.47** is 2.

Parallel lines have the same slope. Because the slope of the blue line is 2, the slope of the red line, the line whose equation we must write, is also 2: $m = 2$. We now have values for $x_1$, $y_1$, and $m$ for the red line.

$$y - y_1 = m(x - x_1)$$

$y_1 = 1$ $m = 2$ $x_1 = -3$

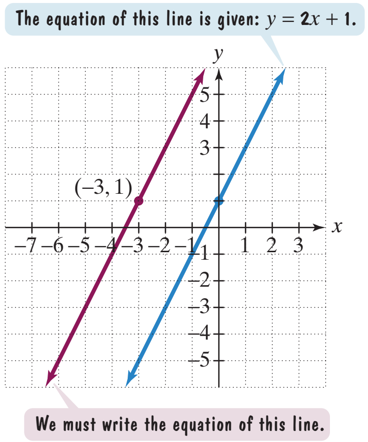



**Figure 1.47** (repeated)

We use $y_1 = 1$, $m = 2$, and $x_1 = -3$. Substitute these values into $y - y_1 = m(x - x_1)$. The point-slope form of the red line's equation is

$$y - 1 = 2[x - (-3)] \text{ or}$$
$$y - 1 = 2(x + 3).$$

Solving for $y$, we obtain the slope-intercept form of the equation.

$$y - 1 = 2x + 6 \quad \text{Apply the distributive property.}$$
$$y = 2x + 7 \quad \text{Add 1 to both sides. This is the slope-intercept form, } y = mx + b, \text{ of the equation. Using function notation, the equation is } f(x) = 2x + 7.$$

**Check Point 1** Write an equation of the line passing through $(-2, 5)$ and parallel to the line whose equation is $y = 3x + 1$. Express the equation in point-slope form and slope-intercept form.

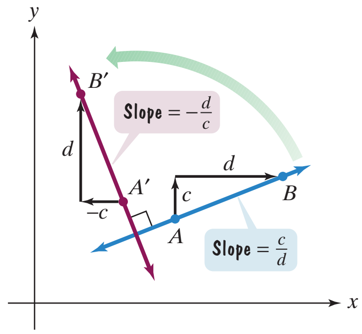



**Figure 1.48** Slopes of perpendicular lines

Two lines that intersect at a right angle (90°) are said to be **perpendicular**, shown in **Figure 1.48**. The relationship between the slopes of perpendicular lines is not as obvious as the relationship between parallel lines. **Figure 1.48** shows line $AB$, with slope $\frac{c}{d}$. Rotate line $AB$ counterclockwise 90° to the left to obtain line $A'B'$, perpendicular to line $AB$. The figure indicates that the rise and the run of the new line are reversed from the original line, but the former rise, the new run, is now negative. This means that the slope of the new line is $-\frac{d}{c}$. Notice that the product of the slopes of the two perpendicular lines is $-1$:

$$\left(\frac{c}{d}\right)\left(-\frac{d}{c}\right) = -1.$$

This relationship holds for all perpendicular lines and is summarized in the following box:

### Slope and Perpendicular Lines

1. If two nonvertical lines are perpendicular, then the product of their slopes is $-1$.
2. If the product of the slopes of two lines is $-1$, then the lines are perpendicular.
3. A horizontal line having zero slope is perpendicular to a vertical line having undefined slope.

An equivalent way of stating this relationship is to say that **one line is perpendicular to another line if its slope is the *negative reciprocal* of the slope of the other line**. For example, if a line has slope 5, any line having slope $-\frac{1}{5}$ is perpendicular to it. Similarly, if a line has slope $-\frac{3}{4}$, any line having slope $\frac{4}{3}$ is perpendicular to it.

## EXAMPLE 2 Writing Equations of a Line Perpendicular to a Given Line

**a.** Find the slope of any line that is perpendicular to the line whose equation is $x + 4y - 8 = 0$.

**b.** Write the equation of the line passing through $(3, -5)$ and perpendicular to the line whose equation is $x + 4y - 8 = 0$. Express the equation in general form.

### Solution

**a.** We begin by writing the equation of the given line, $x + 4y - 8 = 0$, in slope-intercept form. Solve for $y$.

$$x + 4y - 8 = 0 \quad \text{This is the given equation.}$$

$$4y = -x + 8 \quad \text{To isolate the y-term, subtract x and add 8 on both sides.}$$

$$y = -\frac{1}{4}x + 2 \quad \text{Divide both sides by 4.}$$

Slope is $-\frac{1}{4}$.

The given line has slope $-\frac{1}{4}$. Any line perpendicular to this line has a slope that is the negative reciprocal of $-\frac{1}{4}$. Thus, the slope of any perpendicular line is 4.

**b.** Let's begin by writing the point-slope form of the perpendicular line's equation. Because the line passes through the point $(3, -5)$, we have $x_1 = 3$ and $y_1 = -5$. In part (a), we determined that the slope of any line perpendicular to $x + 4y - 8 = 0$ is 4, so the slope of this particular perpendicular line must also be 4: $m = 4$.

$$y - y_1 = m(x - x_1)$$

$y_1 = -5$ $m = 4$ $x_1 = 3$

The point-slope form of the perpendicular line's equation is

$$y - (-5) = 4(x - 3) \text{ or}$$

$$y + 5 = 4(x - 3).$$

How can we express this equation in general form $(Ax + By + C = 0)$? We need to obtain zero on one side of the equation. Let's do this and keep $A$, the coefficient of $x$, positive.

$$y + 5 = 4(x - 3) \quad \text{This is the point-slope form of the line's equation.}$$

$$y + 5 = 4x - 12 \quad \text{Apply the distributive property.}$$

$$y - y + 5 - 5 = 4x - y - 12 - 5 \quad \text{To obtain 0 on the left, subtract y and subtract 5 on both sides.}$$

$$0 = 4x - y - 17 \quad \text{Simplify.}$$

In general form, the equation of the perpendicular line is $4x - y - 17 = 0$.

## Check Point 2

**a.** Find the slope of any line that is perpendicular to the line whose equation is $x + 3y - 12 = 0$.

**b.** Write the equation of the line passing through $(-2, -6)$ and perpendicular to the line whose equation is $x + 3y - 12 = 0$. Express the equation in general form.

2 Interpret slope as rate of change.

## Slope as Rate of Change

Slope is defined as the ratio of a change in $y$ to a corresponding change in $x$. It describes how fast $y$ is changing with respect to $x$. For a linear function, slope may be interpreted as the rate of change of the dependent variable per unit change in the independent variable.

Our next example shows how slope can be interpreted as a rate of change in an applied situation. When calculating slope in applied problems, keep track of the units in the numerator and the denominator.

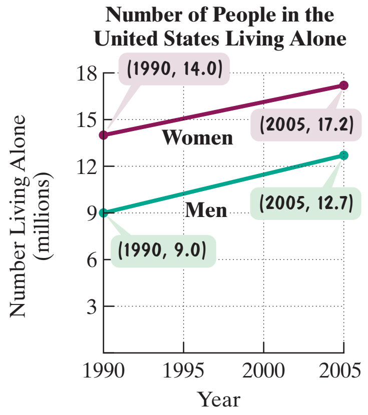



**Figure 1.49**
*Source:* U.S. Census Bureau

## EXAMPLE 3 Slope as a Rate of Change

The line graphs for the number of women and men living alone are shown again in **Figure 1.49**. Find the slope of the line segment for the women. Describe what this slope represents.

**Solution** We let $x$ represent a year and $y$ the number of women living alone in that year. The two points shown on the line segment for women have the following coordinates:

(1990, 14.0) — In 1990, 14 million U.S. women lived alone.

(2005, 17.2) — In 2005, 17.2 million U.S. women lived alone.

Now we compute the slope:

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{17.2 - 14.0}{2005 - 1990}$$

$$= \frac{3.2}{15} \approx \frac{0.21 \text{ million women}}{\text{year}}.$$

The unit in the numerator is *million women*.

The unit in the denominator is *year*.

The slope indicates that the number of American women living alone increased at a rate of approximately 0.21 million each year for the period from 1990 through 2005. The rate of change is 0.21 million women per year.

**Check Point 3** Use the ordered pairs in **Figure 1.49** to find the slope of the green line segment for the men. Express the slope correct to two decimal places and describe what it represents.

In Check Point 3, did you find that the slope of the line segment for the men is different from that of the women? The rate of change for the number of men living alone is greater than the rate of change for the number of women living alone. The green line segment representing men in **Figure 1.49** is steeper than the red line segment representing women. If you extend the line segments far enough, the resulting lines will intersect. They are not parallel.

**3** Find a function's average rate of change.

## The Average Rate of Change of a Function

If the graph of a function is not a straight line, the **average rate of change** between any two points is the slope of the line containing the two points. This line is called a **secant line**. For example, **Figure 1.50** shows the graph of a particular man's height, in inches, as a function of his age, in years. Two points on the graph are labeled: (13, 57) and (18, 76). At age 13, this man was 57 inches tall and at age 18, he was 76 inches tall.

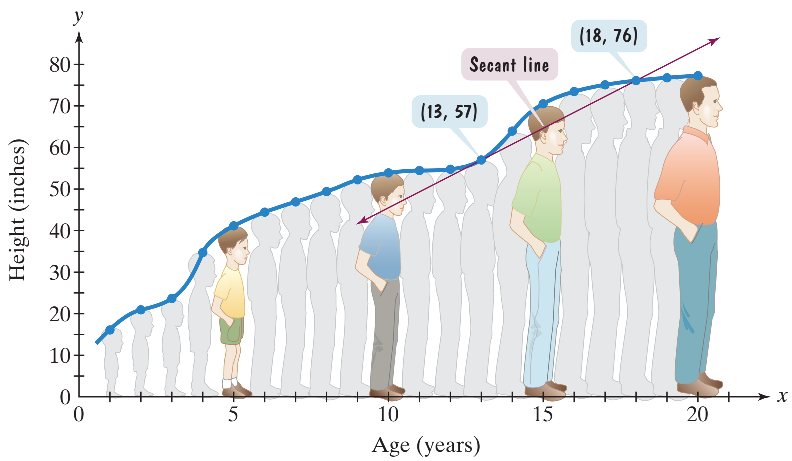



**Figure 1.50** Height as a function of age

The man's average growth rate between ages 13 and 18 is the slope of the secant line containing (13, 57) and (18, 76):

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{76 - 57}{18 - 13} = \frac{19}{5} = 3\frac{4}{5}.$$

This man's average rate of change, or average growth rate, from age 13 to age 18 was $3\frac{4}{5}$, or 3.8, inches per year.

### The Average Rate of Change of a Function

Let $(x_1, f(x_1))$ and $(x_2, f(x_2))$ be distinct points on the graph of a function $f$. (See **Figure 1.51**.) The **average rate of change of $f$** from $x_1$ to $x_2$, denoted by $\frac{\Delta y}{\Delta x}$ (read "delta $y$ divided by delta $x$" or "change in $y$ divided by change in $x$"), is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

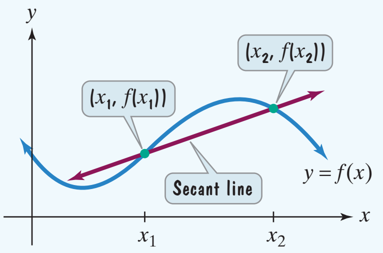



Figure 1.51

## EXAMPLE 4 Finding the Average Rate of Change

Find the average rate of change of $f(x) = x^2$ from

**a.** $x_1 = 0$ to $x_2 = 1$ **b.** $x_1 = 1$ to $x_2 = 2$ **c.** $x_1 = -2$ to $x_2 = 0$.

### Solution

**a.** The average rate of change of $f(x) = x^2$ from $x_1 = 0$ to $x_2 = 1$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1) - f(0)}{1 - 0} = \frac{1^2 - 0^2}{1} = 1.$$

**Figure 1.52(a)** shows the secant line of $f(x) = x^2$ from $x_1 = 0$ to $x_2 = 1$. The average rate of change is positive and the function is increasing on the interval (0, 1).

**b.** The average rate of change of $f(x) = x^2$ from $x_1 = 1$ to $x_2 = 2$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(2) - f(1)}{2 - 1} = \frac{2^2 - 1^2}{1} = 3.$$

**Figure 1.52(b)** shows the secant line of $f(x) = x^2$ from $x_1 = 1$ to $x_2 = 2$. The average rate of change is positive and the function is increasing on the interval (1, 2). Can you see that the graph rises more steeply on the interval (1, 2) than on (0, 1)? This is because the average rate of change from $x_1 = 1$ to $x_2 = 2$ is greater than the average rate of change from $x_1 = 0$ to $x_2 = 1$.

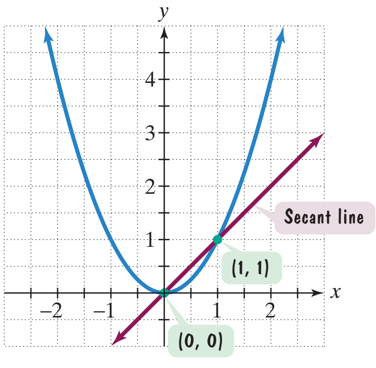



**Figure 1.52(a)** The secant line of $f(x) = x^2$ from $x_1 = 0$ to $x_2 = 1$

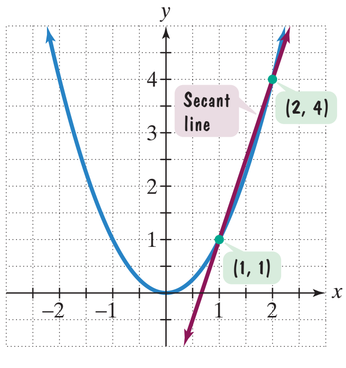



**Figure 1.52(b)** The secant line of $f(x) = x^2$ from $x_1 = 1$ to $x_2 = 2$

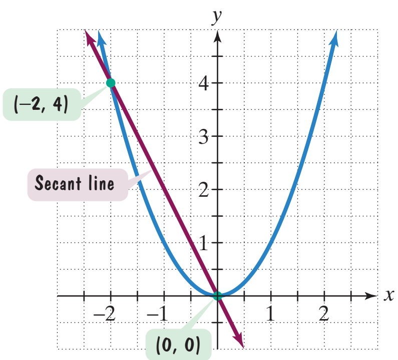



**Figure 1.52(c)** The secant line of $f(x) = x^2$ from $x_1 = -2$ to $x_2 = 0$

**c.** The average rate of change of $f(x) = x^2$ from $x_1 = -2$ to $x_2 = 0$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(0) - f(-2)}{0 - (-2)} = \frac{0^2 - (-2)^2}{2} = \frac{-4}{2} = -2.$$

**Figure 1.52(c)** shows the secant line of $f(x) = x^2$ from $x_1 = -2$ to $x_2 = 0$. The average rate of change is negative and the function is decreasing on the interval $(-2, 0)$.

**Check Point 4** Find the average rate of change of $f(x) = x^3$ from

**a.** $x_1 = 0$ to $x_2 = 1$ **b.** $x_1 = 1$ to $x_2 = 2$ **c.** $x_1 = -2$ to $x_2 = 0$.

Suppose we are interested in the average rate of change of $f$ from $x_1 = x$ to $x_2 = x + h$. In this case, the average rate of change is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x+h) - f(x)}{x + h - x} = \frac{f(x+h) - f(x)}{h}.$$

Do you recognize the last expression? It is the difference quotient that you used in Section 1.3. Thus, the difference quotient gives the average rate of change of a function from $x$ to $x + h$. In the difference quotient, $h$ is thought of as a number very close to 0. In this way, the average rate of change can be found for a very short interval.

## EXAMPLE 5 Finding the Average Rate of Change

When a person receives a drug injected into a muscle, the concentration of the drug in the body, measured in milligrams per 100 milliliters, is a function of the time elapsed after the injection, measured in hours. **Figure 1.53** shows the graph of such a function, where $x$ represents hours after the injection and $f(x)$ is the drug's concentration at time $x$. Find the average rate of change in the drug's concentration between 3 and 7 hours.

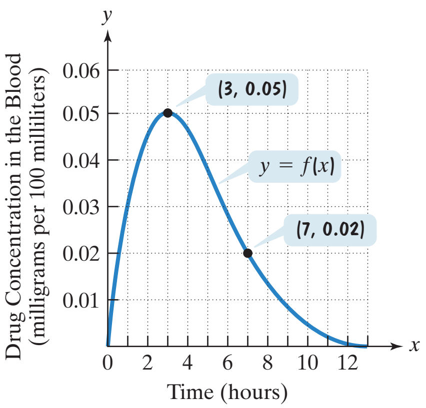



**Figure 1.53** Concentration of a drug as a function of time

**Solution** At 3 hours, the drug's concentration is 0.05 and at 7 hours, the concentration is 0.02. The average rate of change in its concentration between 3 and 7 hours is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(7) - f(3)}{7 - 3} = \frac{0.02 - 0.05}{7 - 3} = \frac{-0.03}{4} = -0.0075.$$

The average rate of change is −0.0075. This means that the drug's concentration is decreasing at an average rate of 0.0075 milligram per 100 milliliters per hour.

### Study Tip

Units used to describe $x$ and $y$ tend to "pile up" when expressing the rate of change of $y$ with respect to $x$. The unit used to express the rate of change of $y$ with respect to $x$ is

the unit used to describe $y$ **per** the unit used to describe $x$.

In **Figure 1.53**, $y$, or drug concentration, is described in milligrams per 100 milliliters.

In **Figure 1.53**, $x$, or time, is described in hours.

In **Figure 1.53**, the rate of change is described in terms of milligrams per 100 milliliters per hour.

**Check Point 5** Use **Figure 1.53** on the previous page to find the average rate of change in the drug's concentration between 1 hour and 3 hours.

## How Calculus Studies Change

Take a rapid sequence of still photographs of a moving scene and project them onto a screen at thirty shots a second or faster. Our eyes see the results as continuous motion. The small difference between one frame and the next cannot be detected by the human visual system. The idea of calculus likewise regards continuous motion as made up of a sequence of still configurations. Calculus masters the mystery of movement by "freezing the frame" of a continuous changing process, instant by instant. For example, **Figure 1.54** shows a male's changing height over intervals of time. Over the period of time from $P$ to $D$, his average rate of growth is his change in height—that is, his height at time $D$ minus his height at time $P$—divided by the change in time from $P$ to $D$. This is the slope of secant line $PD$.

The secant lines $PD$, $PC$, $PB$, and $PA$ shown in **Figure 1.54** have slopes that show average growth rates for successively shorter periods of time. Calculus makes these time frames so small that they approach a single point—that is, a single instant in time. This point is shown as point $P$ in **Figure 1.54**. The slope of the line that touches the graph at $P$ gives the male's growth rate at one instant in time, $P$. In the introduction to calculus in Chapter 11, you'll learn to use a concept called *limits* to determine the slope of this line.

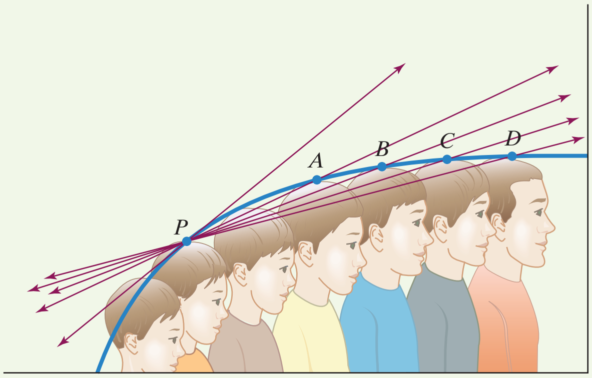



**Figure 1.54** Analyzing continuous growth over intervals of time and at an instant in time

The **average velocity** of an object is its change in position divided by the change in time between the starting and ending positions. If a function expresses an object's position in terms of time, the function's average rate of change describes the object's average velocity.

### Average Velocity of an Object

Suppose that a function expresses an object's position, $s(t)$, in terms of time, $t$. The **average velocity** of the object from $t_1$ to $t_2$ is

$$\frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}.$$

### EXAMPLE 6 Finding Average Velocity

The distance, $s(t)$, in feet, traveled by a ball rolling down a ramp is given by the function

$$s(t) = 5t^2,$$

where $t$ is the time, in seconds, after the ball is released. Find the ball's average velocity from

**a.** $t_1 = 2$ seconds to $t_2 = 3$ seconds.

**b.** $t_1 = 2$ seconds to $t_2 = 2.5$ seconds.

**c.** $t_1 = 2$ seconds to $t_2 = 2.01$ seconds.

**Solution**

**a.** The ball's average velocity between 2 and 3 seconds is

$$\frac{\Delta s}{\Delta t} = \frac{s(3) - s(2)}{3 \text{ sec} - 2 \text{ sec}} = \frac{5 \cdot 3^2 - 5 \cdot 2^2}{1 \text{ sec}} = \frac{45 \text{ ft} - 20 \text{ ft}}{1 \text{ sec}} = 25 \text{ ft/sec}.$$

**b.** The ball's average velocity between 2 and 2.5 seconds is

$$\frac{\Delta s}{\Delta t} = \frac{s(2.5) - s(2)}{2.5 \text{ sec} - 2 \text{ sec}} = \frac{5(2.5)^2 - 5 \cdot 2^2}{0.5 \text{ sec}} = \frac{31.25 \text{ ft} - 20 \text{ ft}}{0.5 \text{ sec}} = 22.5 \text{ ft/sec}.$$

**c.** The ball's average velocity between 2 and 2.01 seconds is

$$\frac{\Delta s}{\Delta t} = \frac{s(2.01) - s(2)}{2.01 \text{ sec} - 2 \text{ sec}} = \frac{5(2.01)^2 - 5 \cdot 2^2}{0.01 \text{ sec}} = \frac{20.2005 \text{ ft} - 20 \text{ ft}}{0.01 \text{ sec}} = 20.05 \text{ ft/sec}.$$

In Example 6, observe that each calculation begins at 2 seconds and involves shorter and shorter time intervals. In calculus, this procedure leads to the concept of *instantaneous*, as opposed to *average*, velocity. Instantaneous velocity is discussed in the introduction to calculus in Chapter 11.

**Check Point 6** The distance, $s(t)$, in feet, traveled by a ball rolling down a ramp is given by the function

$$s(t) = 4t^2,$$

where $t$ is the time, in seconds, after the ball is released. Find the ball's average velocity from

**a.** $t_1 = 1$ second to $t_2 = 2$ seconds.

**b.** $t_1 = 1$ second to $t_2 = 1.5$ seconds.

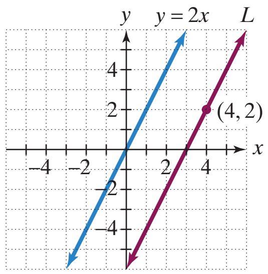
**a.** $t_1 = 1$ second to $t_2 = 1.01$ seconds.

## Exercise Set 1.5

### Practice Exercises

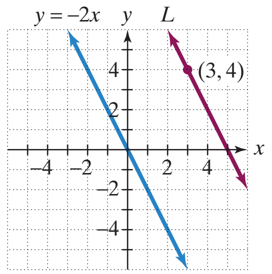
*In Exercises 1–4, write an equation for line L in point-slope form and slope-intercept form.*

**1.**



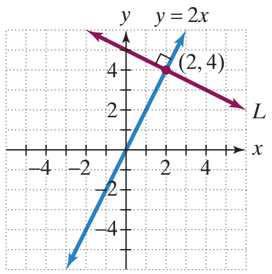

*L* is parallel to $y = 2x$.

**2.**



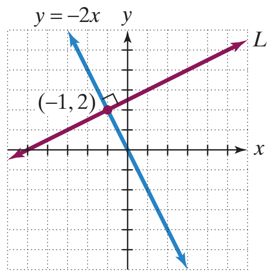

*L* is parallel to $y = -2x$.

**3.**




*L* is perpendicular to $y = 2x$.

**4.**




*L* is perpendicular to $y = -2x$.

*In Exercises 5–8, use the given conditions to write an equation for each line in point-slope form and slope-intercept form.*

**5.** Passing through $(-8, -10)$ and parallel to the line whose equation is $y = -4x + 3$

**6.** Passing through $(-2, -7)$ and parallel to the line whose equation is $y = -5x + 4$

**7.** Passing through $(2, -3)$ and perpendicular to the line whose equation is $y = \frac{1}{5}x + 6$

**8.** Passing through $(-4, 2)$ and perpendicular to the line whose equation is $y = \frac{1}{3}x + 7$

*In Exercises 9–12, use the given conditions to write an equation for each line in point-slope form and general form.*

**9.** Passing through $(-2, 2)$ and parallel to the line whose equation is $2x - 3y - 7 = 0$

**10.** Passing through $(-1, 3)$ and parallel to the line whose equation is $3x - 2y - 5 = 0$

**11.** Passing through $(4, -7)$ and perpendicular to the line whose equation is $x - 2y - 3 = 0$

**12.** Passing through $(5, -9)$ and perpendicular to the line whose equation is $x + 7y - 12 = 0$

*In Exercises 13–18, find the average rate of change of the function from $x_1$ to $x_2$.*

**13.** $f(x) = 3x$ from $x_1 = 0$ to $x_2 = 5$

**14.** $f(x) = 6x$ from $x_1 = 0$ to $x_2 = 4$

**15.** $f(x) = x^2 + 2x$ from $x_1 = 3$ to $x_2 = 5$

**16.** $f(x) = x^2 - 2x$ from $x_2 = 3$ to $x_2 = 6$

**17.** $f(x) = \sqrt{x}$ from $x_1 = 4$ to $x_2 = 9$

**18.** $f(x) = \sqrt{x}$ from $x_1 = 9$ to $x_2 = 16$

*In Exercises 19–20, suppose that a ball is rolling down a ramp. The distance traveled by the ball is given by the function in each exercise, where t is the time, in seconds, after the ball is released, and s(t) is measured in feet. For each given function, find the ball's average velocity from*

**a.** $t_1 = 3$ to $t_2 = 4$. **b.** $t_1 = 3$ to $t_2 = 3.5$.

**c.** $t_1 = 3$ to $t_2 = 3.01$. **d.** $t_1 = 3$ to $t_2 = 3.001$.

**19.** $s(t) = 10t^2$ **20.** $s(t) = 12t^2$

## Practice Plus

*In Exercises 21–26, write an equation in slope-intercept form of a linear function f whose graph satisfies the given conditions.*

**21.** The graph of $f$ passes through $(-1, 5)$ and is perpendicular to the line whose equation is $x = 6$.

**22.** The graph of $f$ passes through $(-2, 6)$ and is perpendicular to the line whose equation is $x = -4$.

**23.** The graph of $f$ passes through $(-6, 4)$ and is perpendicular to the line that has an $x$-intercept of 2 and a $y$-intercept of $-4$.

**24.** The graph of $f$ passes through $(-5, 6)$ and is perpendicular to the line that has an $x$-intercept of 3 and a $y$-intercept of $-9$.

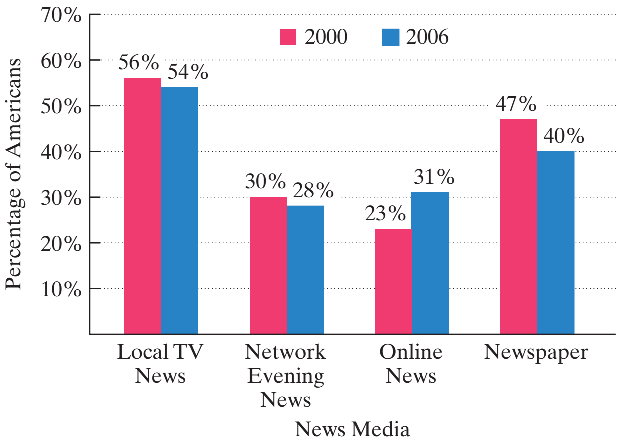
**25.** The graph of $f$ is perpendicular to the line whose equation is $3x - 2y - 4 = 0$ and has the same $y$-intercept as this line.

**26.** The graph of $f$ is perpendicular to the line whose equation is $4x - y - 6 = 0$ and has the same $y$-intercept as this line.

## Application Exercises

*The bar graph shows that as online news has grown, traditional news media have slipped.*

**Percentage of Americans Who Regularly Use Various News Media**




*Source:* Pew Research Center

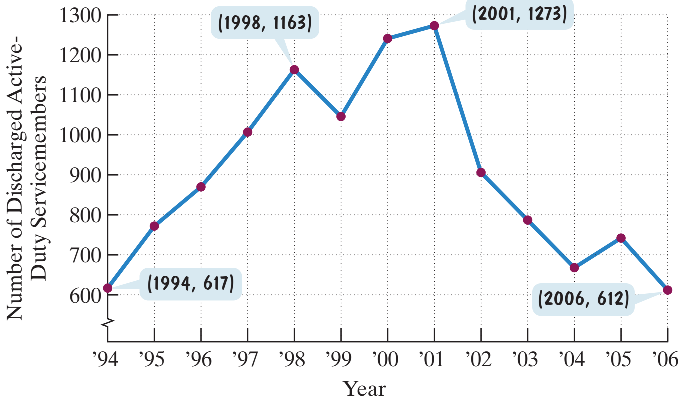
*In Exercises 27–28, find a linear function in slope-intercept form that models the given description. Each function should model the percentage of Americans, P(x), who regularly used the news outlet x years after 2000.*

**27.** In 2000, 47% of Americans regularly used newspapers for getting news and this percentage has decreased at an average rate of approximately 1.2 per year since then.

**28.** In 2000, 23% of Americans regularly used online news for getting news and this percentage has increased at an average rate of approximately 1.3 per year since then.

*The stated intent of the 1994 "don't ask, don't tell" policy was to reduce the number of discharges of gay men and lesbians from the military. Nearly 12,000 active-duty gay servicemembers have been dismissed under the policy. The line graph shows the number of discharges under "don't ask, don't tell" from 1994 through 2006. Use the data displayed by the graph to solve Exercises 29–30.*

**Number of Active-Duty Gay Servicemembers Discharged from the Military for Homosexuality**

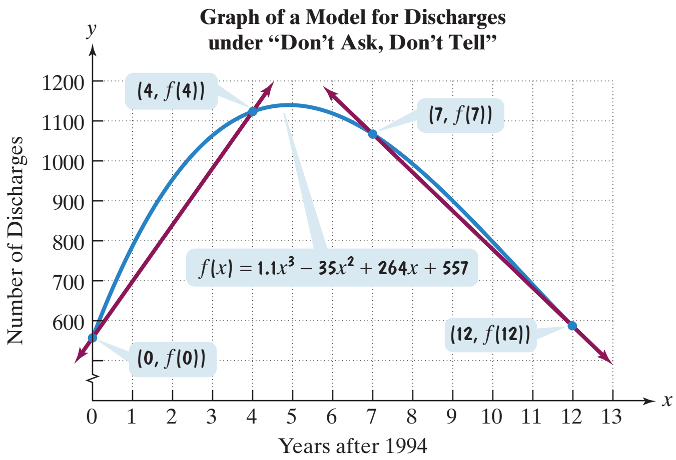



*Source:* General Accountability Office

**29.** Find the average rate of change, rounded to the nearest whole number, from 1994 through 1998. Describe what this means.

**30.** Find the average rate of change, rounded to the nearest whole number, from 2001 through 2006. Describe what this means.

*The function $f(x) = 1.1x^3 - 35x^2 + 264x + 557$ models the number of discharges, f(x), under "don't ask, don't tell" x years after 1994. Use this model and its graph, shown on the domain [0, 12] to solve Exercises 31–32.*




**31. a.** Find the slope of the secant line, rounded to the nearest whole number, from $x_1 = 0$ to $x_2 = 4$.

**b.** Does the slope from part (a) underestimate or overestimate the average yearly increase that you determined in Exercise 29? By how much?

**32. a.** Find the slope of the secant line, rounded to the nearest whole number, from $x_1 = 7$ to $x_2 = 12$.

**b.** Does the slope from part (b) underestimate or overestimate the average yearly decrease that you determined in Exercise 30? By how much?

## Writing in Mathematics

**33.** If two lines are parallel, describe the relationship between their slopes.

**34.** If two lines are perpendicular, describe the relationship between their slopes.

**35.** If you know a point on a line and you know the equation of a line perpendicular to this line, explain how to write the line's equation.

**36.** A formula in the form $y = mx + b$ models the average retail price, $y$, of a new car $x$ years after 2000. Would you expect $m$ to be positive, negative, or zero? Explain your answer.

**37.** What is a secant line?

**38.** What is the average rate of change of a function?

## Technology Exercise

**39.** **a.** Why are the lines whose equations are $y = \frac{1}{3}x + 1$ and $y = -3x - 2$ perpendicular?

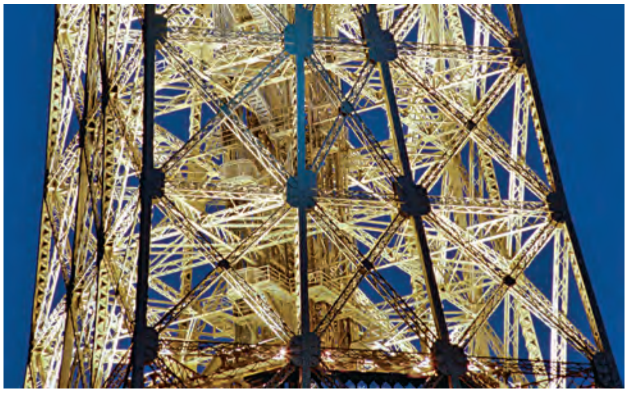
**b.** Use a graphing utility to graph the equations in a $[-10, 10, 1]$ by $[-10, 10, 1]$ viewing rectangle. Do the lines appear to be perpendicular?

**c.** Now use the zoom square feature of your utility. Describe what happens to the graphs. Explain why this is so.

## Critical Thinking Exercises

**Make Sense?** *In Exercises 40–43, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

**40.** Some of the steel girders in this photo of the Eiffel Tower appear to have slopes that are negative reciprocals of each other.



**41.** I have linear functions that model changes for men and women over the same time period. The functions have the same slope, so their graphs are parallel lines, indicating that the rate of change for men is the same as the rate of change for women.

**42.** The graph of my function is not a straight line, so I cannot use slope to analyze its rates of change.

**43.** According to the essay on page 199, calculus studies change by analyzing slopes of secant lines over successively shorter intervals.

**44.** What is the slope of a line that is perpendicular to the line whose equation is $Ax + By + C = 0$, $A \neq 0$ and $B \neq 0$?

**45.** Determine the value of $A$ so that the line whose equation is $Ax + y - 2 = 0$ is perpendicular to the line containing the points $(1, -3)$ and $(-2, 4)$.

## Preview Exercises

*Exercises 46–48 will help you prepare for the material covered in the next section. In each exercise, graph the functions in parts (a) and (b) in the same rectangular coordinate system.*

**46.** **a.** Graph $f(x) = |x|$ using the ordered pairs $(-3, f(-3))$, $(-2, f(-2))$, $(-1, f(-1))$, $(0, f(0))$, $(1, f(1))$, $(2, f(2))$, and $(3, f(3))$.

**b.** Subtract 4 from each $y$-coordinate of the ordered pairs in part (a). Then graph the ordered pairs and connect them with two linear pieces.

**c.** Describe the relationship between the graph in part (b) and the graph in part (a).

**47.** **a.** Graph $f(x) = x^2$ using the ordered pairs $(-3, f(-3))$, $(-2, f(-2))$, $(-1, f(-1))$, $(0, f(0))$, $(1, f(1))$, $(2, f(2))$, and $(3, f(3))$.

**b.** Add 2 to each $x$-coordinate of the ordered pairs in part (a). Then graph the ordered pairs and connect them with a smooth curve.

**c.** Describe the relationship between the graph in part (b) and the graph in part (a).

**48.** **a.** Graph $f(x) = x^3$ using the ordered pairs $(-2, f(-2))$, $(-1, f(-1))$, $(0, f(0))$, $(1, f(1))$, and $(2, f(2))$.

**b.** Replace each $x$-coordinate of the ordered pairs in part (a) with its opposite, or additive inverse. Then graph the ordered pairs and connect them with a smooth curve.

**c.** Describe the relationship between the graph in part (b) and the graph in part (a).