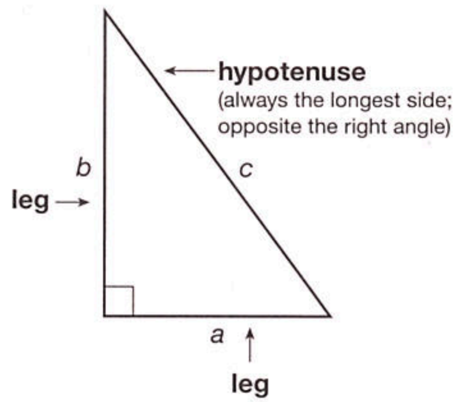
Lesson 4: Right Triangles

Right triangles are triangles with right (90°) angles. These particular types of triangles have several special properties. In this lesson you will learn about the Pythagorean theorem, the HL congruence theorem, 30°–60°–90° triangles, 45°–45°–90° triangles, and the altitudes of right triangles.

Pythagorean Theorem

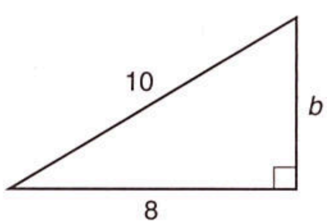
The **Pythagorean theorem** states that for any right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



$$a^2 + b^2 = c^2$$

Example

What is the value of *b* in the following right triangle?



$a^2 + b^2 = c^2$

$8^2 + b^2 = 10^2$

$64 + b^2 = 100$

$b^2 = 36$ Find the square root of both sides.

$b = 6$

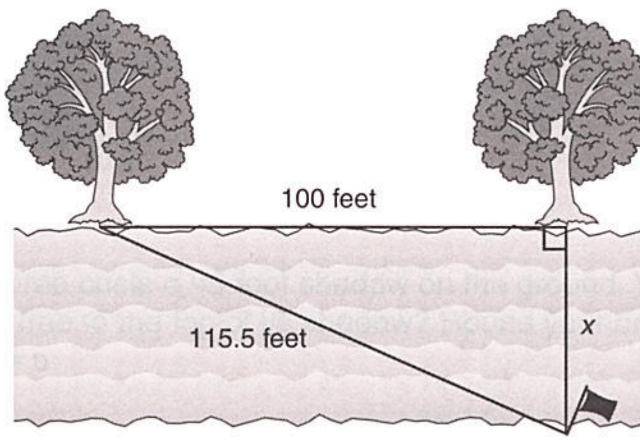
The value of *b* is 6 units.

TIP: For obtuse triangles, $c^2 > a^2 + b^2$. For acute triangles, $c^2 < a^2 + b^2$.

Benchmark Codes: MA.912.G.5.1, MA.912.G.5.4, MA.912.G.8.3

Example

Tommy wants to swim across the Anclote River, but he doesn't know the width of the river. He measures the distance between two trees on one side of the river and finds that they are 100 feet apart. He stakes a point across the river from one of the trees to form a right angle. Tommy knows the distance from the stake to the other tree is 115.5 feet. What is the width of the river?



Looking at the diagram, you have the length of the hypotenuse and one of the legs of the right triangle. Use the Pythagorean theorem to find the length of the other leg.

$$a^2 + b^2 = c^2$$

$$a^2 + 100^2 = 115.5^2$$

$$a^2 + 10{,}000 = 13{,}340.25$$

$$a^2 = 3{,}340.25$$

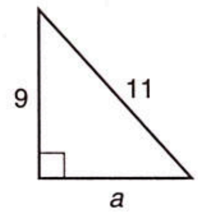
$$a = 57.795\ldots$$

The width of the river is approximately 57.8 feet.

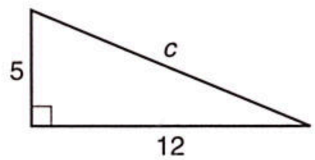
Remember that the longest side in a right triangle will always be the hypotenuse. This is a reasonable answer because it is smaller than the hypotenuse.

Practice

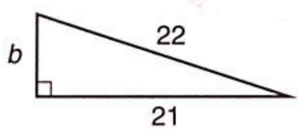
Directions: For questions 1 through 5, use the Pythagorean theorem to find the missing side length. You can leave your answers in radical form or round to the nearest tenth.

1. 

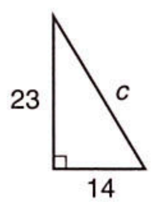
$a =$ ________

2. 

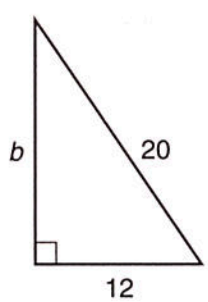
$c =$ ________

3. 

$b =$ ________

4. 

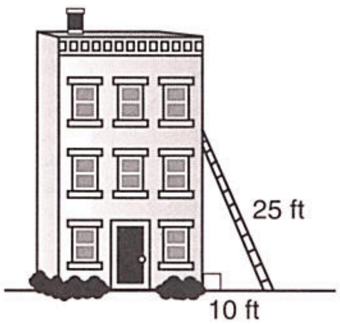
$c =$ ________

5. 

$b =$ ________

Benchmark Codes: MA.912.G.5.1, MA.912.G.5.4, MA.912.G.8.3

6. A 25-foot ladder is positioned 10 feet from the base of a building. What is the distance from the base of the building to the point where the ladder leans up against the building, rounded to the nearest tenth of a foot?



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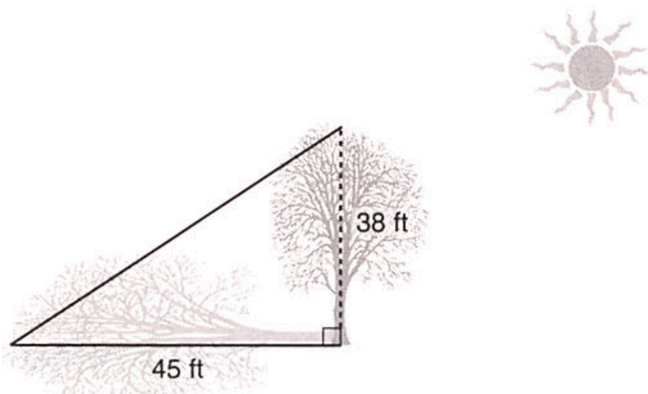
7. A 38-foot tree casts a 45-foot shadow on the ground. What is the distance from the top of the tree to the top of its shadow? Round your answer to the nearest tenth.



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8. If the legs of a right triangle are 17 inches and 36 inches long, what is the length of the hypotenuse? Round your answer to the nearest tenth.

   A. 23.9 inches

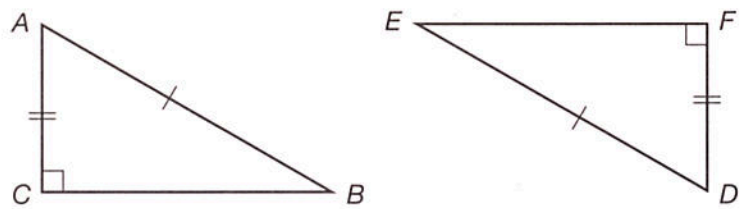
   B. 31.7 inches

   C. 33.4 inches

   D. 39.8 inches

The Hypotenuse-Leg (HL) Congruence Theorem

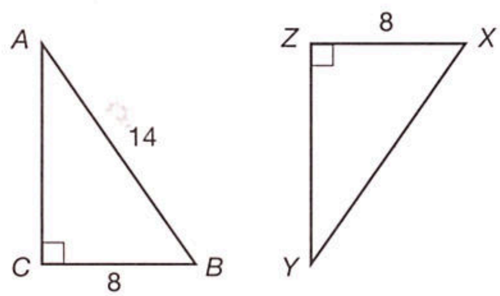
The **HL congruence theorem** states two right triangles are congruent if the hypotenuse and one leg of the corresponding triangles are congruent. The HL congruence theorem is a special case of the SSS postulate (see page 60).



$\triangle ABC \cong \triangle DEF$

Example

What must the length of XY be to prove that $\triangle ABC \cong \triangle YXZ$?



Both $\triangle ABC$ and $\triangle YXZ$ are right triangles. To prove that they are congruent, the hypotenuses and one pair of corresponding sides must be congruent.

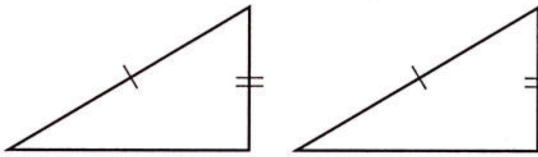
Because the lengths of two corresponding legs are both 8, $CB = XZ$. For the hypotenuses of the triangles to be congruent, AB must be equal to XY. Therefore, the length of XY must be 14 to prove that $\triangle ABC \cong \triangle YXZ$.

Benchmark Codes: MA.912.G.4.6

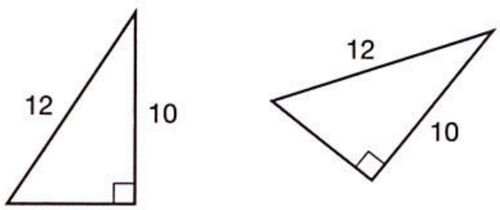
Practice

Directions: For questions 1 through 4, identify whether the pair of triangles can be proven congruent or not.

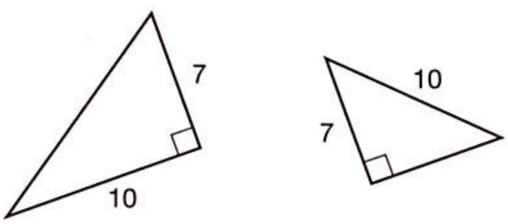
1.



2.



3.



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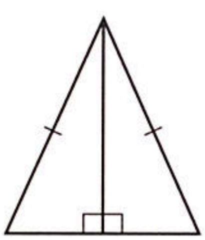
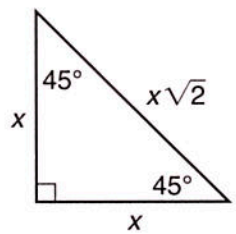


Special Right Triangles

There are two types of right triangles whose side lengths always form specific ratios: an isosceles right triangle and a 30°– 60°– 90° right triangle. With these triangles, you can use ratios to find the missing side length instead of using the Pythagorean theorem.

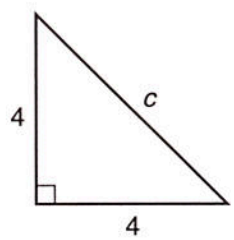
Isosceles Right Triangle (45°– 45°– 90°)

An isosceles right triangle has two 45° angles, one 90° angle, and two congruent sides. The ratio of the side lengths of an isosceles right triangle is $x : x : x\sqrt{2}$.



Example

What is the value of c in the following right triangle?



Use the following ratio to find the missing side length:

$x : x : x\sqrt{2}$

Substitute 4 for x into the ratio.

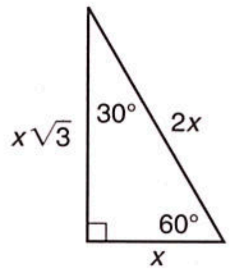
$4 : 4 : 4\sqrt{2}$

The value of c is $4\sqrt{2}$ or approximately 5.7.

Benchmark Codes: MA.912.G.5.3

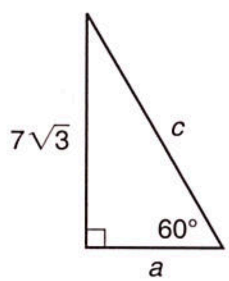
30°– 60°– 90° Triangle

A 30°– 60°– 90° right triangle has one 30° angle, one 60° angle, and one 90° angle. The ratio of the side lengths of a 30°– 60°– 90° triangle is $x : x\sqrt{3} : 2x$.



Example

What are the values of *a* and *c* in the following right triangle?



Because the triangle is a 30°– 60°– 90° triangle, you can use the following ratio to find each missing side length:

$$x : x\sqrt{3} : 2x$$

If you look at the side opposite the 60° angle, you can see that $x = 7$. Substitute 7 for *x* in the ratio.

$$7 : 7\sqrt{3} : 2(7)$$

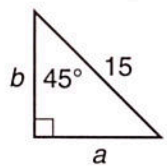
The value of *a* is 7 and the value of *c* is 14.

TIP: You should never have a square root in the denominator of a fraction. To eliminate a square root in the denominator, multiply both the numerator and the denominator by the square root.

Practice

Directions: For questions 1 through 4, use ratios to find the missing side lengths. Write your answers in radical form.

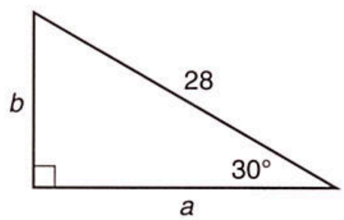
1.



$a =$ ____________

$b =$ ____________

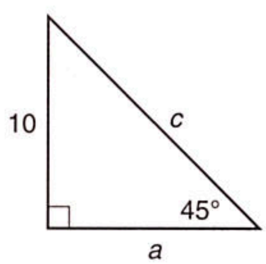
2.



$a =$ ____________

$b =$ ____________

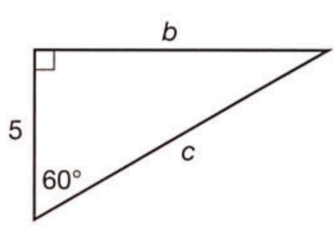
3.



$a =$ ____________

$c =$ ____________

4.



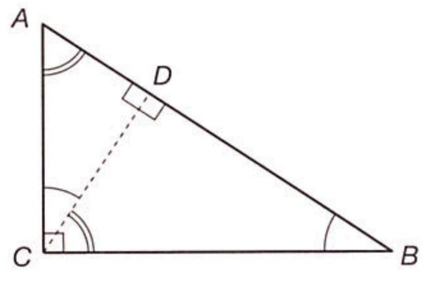
$b =$ ____________

$c =$ ____________

Benchmark Codes: MA.912.G.5.2

Using Altitudes in Right Triangles

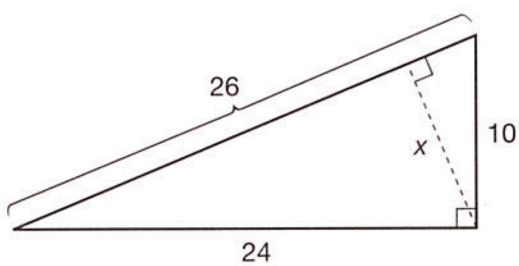
When the altitude of a right triangle is drawn to the hypotenuse, three similar right triangles are formed. The original triangle and the two smaller triangles will all have the same interior angle measures.



$\triangle ABC \sim \triangle ACD \sim \triangle CBD$

Example

What is the length of the altitude, x, rounded to the nearest hundredth?



The ratios of corresponding sides of similar triangles are equal. The ratio of the longer base to the hypotenuse is $\frac{24}{26}$. The altitude forms two smaller similar triangles. The smaller similar triangle has a hypotenuse of 10. The altitude, x, is its longer leg of the triangle on the right. The corresponding ratio is therefore $\frac{x}{10}$. Create a proportion to set the ratios equal to each other.

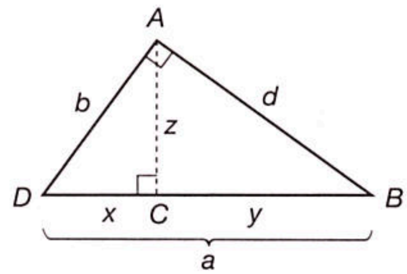
$$\frac{24}{26} = \frac{x}{10}$$

$$26x = 240$$

$$x \approx 9.23$$

TIP: Remember that two of the altitudes of a right triangle are parts of the triangle. The third altitude is inside the triangle.

A **geometric mean** is the positive square root of the product of two numbers. The length of an altitude of a right triangle is equal to the geometric mean of the two segments of the hypotenuse that it creates.



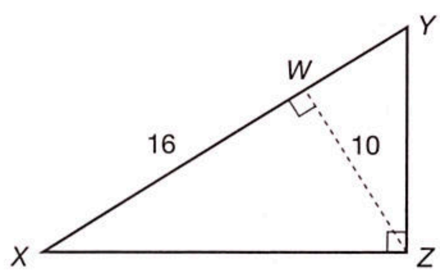
Given that $\overline{AC}$, z, is an altitude of right $\triangle ABD$:

$$\frac{x}{z} = \frac{z}{y}$$

$$z = \sqrt{xy}$$

Example

Given that $\overline{WZ}$ is an altitude of right $\triangle XYZ$, what is the length of WY?



$$WZ = \sqrt{(WX)(WY)}$$

$$10 = \sqrt{(16)(WY)}$$

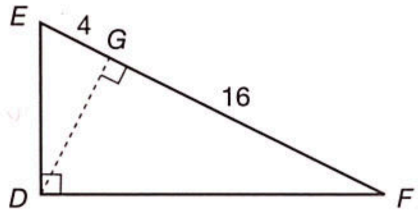
$$100 = (16)(WY)$$

$$6.25 = WY$$

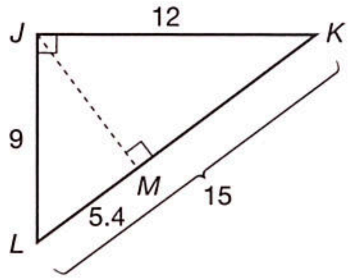
Benchmark Codes: MA.912.G.5.2

Practice

Directions: For questions 1 and 2, find the length of the altitude.

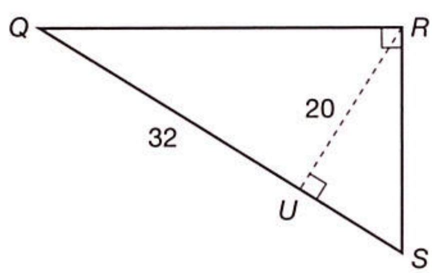
1. 

$GD =$ ____________

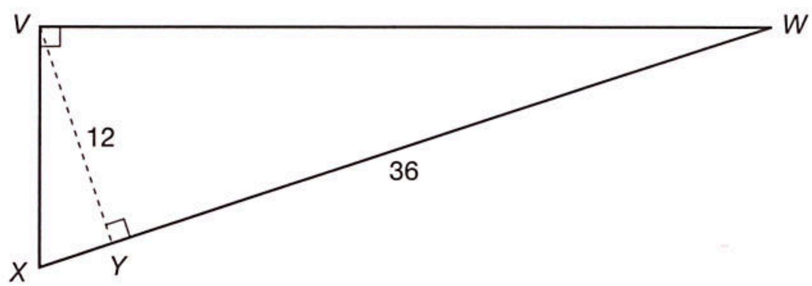
2. 

$JM =$ ____________

Directions: For questions 3 and 4, find the missing length using similar triangles.

3. 

$US =$ ____________

4. 

$XY =$ ____________

Geometry Practice begins on the following page.