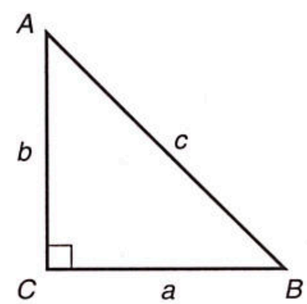
Benchmark Codes: MA.912.T.2.1

Lesson 5: Trigonometry

The word *trigonometry* means "triangle measurement." You can use trigonometric ratios to find missing values of a right triangle.

Ratios of a Right Triangle

The sine, cosine, and tangent ratios show special relationships between the sides and angles of right triangles. To use these ratios properly, you need to be able to tell the difference between the two legs of a right triangle in relation to the acute angles of the right triangle.



Leg a is opposite $\angle A$ and adjacent to $\angle B$.

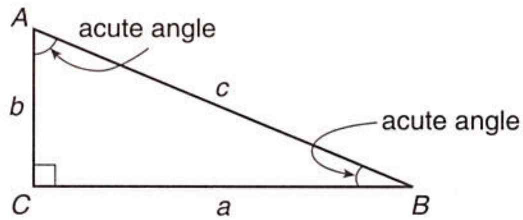
Leg b is opposite $\angle B$ and adjacent to $\angle A$.

Side c is the hypotenuse.

In general, the vertices of a right triangle are labeled with capital letters and the lengths of the sides are labeled with lowercase letters. The right angle is usually indicated with the "square" symbol. If you don't see an indication of a right angle, check the given statements to see whether you can verify that the triangle is a right triangle. Never assume that a triangle is a right triangle just because it looks as if it has a right angle.

Sine Ratio

The **sine (sin)** of an acute angle of a right triangle is the ratio that compares the length of the leg **opposite** the acute angle to the length of the **hypotenuse**.

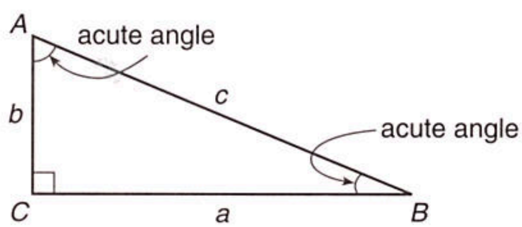


$\sin A = \frac{a}{c}$

$\sin B = \frac{b}{c}$

Cosine Ratio

The **cosine (cos)** of an acute angle of a right triangle is the ratio that compares the length of the leg **adjacent** to the acute angle to the length of the **hypotenuse**.

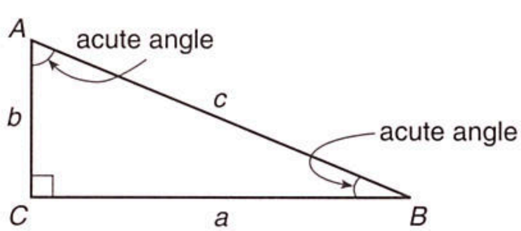


$\cos A = \frac{b}{c}$

$\cos B = \frac{a}{c}$

Tangent Ratio

The **tangent (tan)** of an acute angle of a right triangle is the ratio that compares the length of the leg **opposite** the acute angle to the length of the leg **adjacent** to the acute angle.



$\tan A = \frac{a}{b}$

$\tan B = \frac{b}{a}$

You can remember these three trigonometric ratios by using the mnemonic SOH-CAH-TOA. The Sine of an angle is the Opposite side over the Hypotenuse. The Cosine of an angle is the Adjacent side over the Hypotenuse. The Tangent of an angle is the Opposite side over the Adjacent side.

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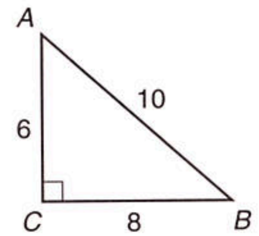
Example

Find the sine, cosine, and tangent ratios for both acute angles of the following triangle.



$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{10} = \frac{4}{5}$

$\sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{6}{10} = \frac{3}{5}$

$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{6}{10} = \frac{3}{5}$

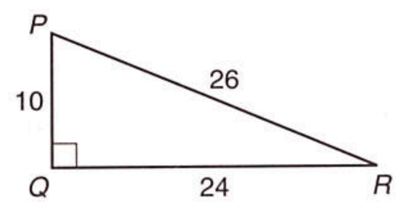
$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{8}{10} = \frac{4}{5}$

$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{8}{6} = \frac{4}{3}$

$\tan B = \frac{\text{opposite}}{\text{adjacent}} = \frac{6}{8} = \frac{3}{4}$

Practice

Directions: For questions 1 through 6, fill in the blanks with the missing values for the following right triangle. Write your answers as fractions in lowest terms.



1. $\sin P =$ ______________
2. $\cos P =$ ______________
3. $\tan P =$ ______________
4. $\sin R =$ ______________
5. $\cos R =$ ______________
6. $\tan R =$ ______________

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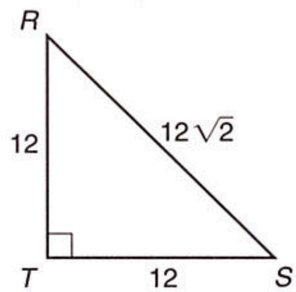
Directions: For questions 7 through 12, fill in the blanks with the missing values for the following right triangle. Write your answers as fractions in simplest form.



7. sin R = ______________

8. cos R = ______________

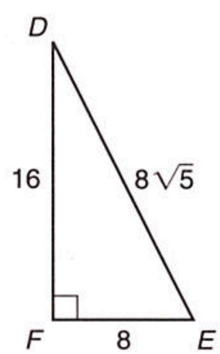
9. tan R = ______________

10. sin S = ______________

11. cos S = ______________

12. tan S = ______________

Directions: For questions 13 through 18, fill in the blanks with the missing values for the following right triangle. Write your answers as fractions in simplest form.



13. sin D = ______________

14. cos D = ______________

15. tan D = ______________

16. sin E = ______________

17. cos E = ______________

18. tan E = ______________

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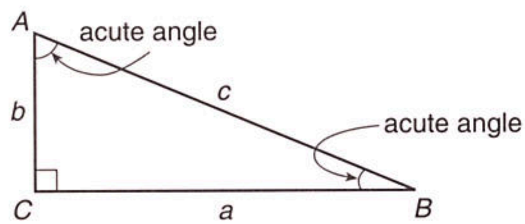
Other Ratios of Right Triangles

There are three other ratios of right triangles: cosecant, secant and cotangent.

Cosecant Ratio

The **cosecant (csc)** of an acute angle of a right triangle is the reciprocal of the sine of the angle. It is the ratio of the length of the hypotenuse to the length of the leg opposite the acute angle.

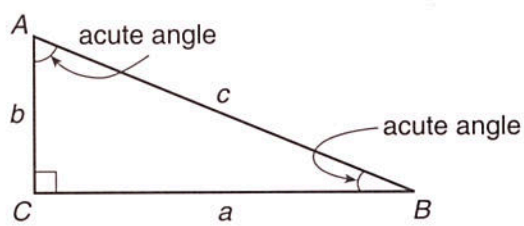


$$\csc A = \frac{1}{\sin A} = \frac{c}{a}$$

$$\csc B = \frac{1}{\sin B} = \frac{c}{b}$$

Secant Ratio

The **secant (sec)** of an acute angle of a right triangle is the reciprocal of the cosine of the angle. It is the ratio of the length of the hypotenuse to the length of the leg adjacent the acute angle.

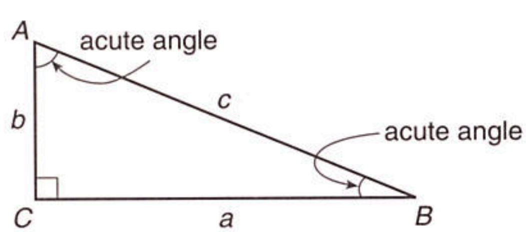


$$\sec A = \frac{1}{\cos A} = \frac{c}{b}$$

$$\sec B = \frac{1}{\cos B} = \frac{c}{a}$$

Cotangent Ratio

The **cotangent (cot)** of an acute angle of a right triangle is the reciprocal of the tangent of the angle. It is the ratio of the length of the leg adjacent the acute angle to the length of the leg opposite the acute angle.

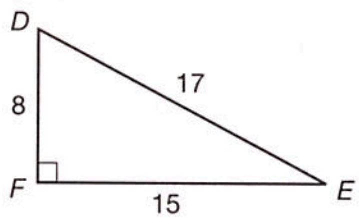


$$\cot A = \frac{1}{\tan A} = \frac{b}{a}$$

$$\cot B = \frac{1}{\tan B} = \frac{a}{b}$$

Example

Find the cosecant, secant, and cotangent ratios for both acute angles of the following triangle.



$\csc D = \frac{1}{\sin D} = \frac{17}{15}$

$\sec D = \frac{1}{\cos D} = \frac{17}{8}$

$\cot D = \frac{1}{\tan D} = \frac{8}{15}$

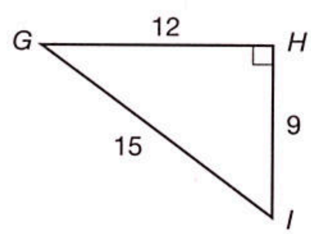
$\csc E = \frac{1}{\sin E} = \frac{17}{8}$

$\sec E = \frac{1}{\cos E} = \frac{17}{15}$

$\cot E = \frac{1}{\tan E} = \frac{15}{8}$

Practice

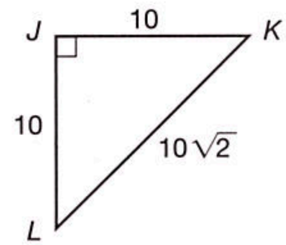
Directions: For questions 1 through 6, fill in the blanks with the missing values for the following right triangle. Write your answers as fractions in lowest terms.



1. $\csc G =$ ______________
2. $\sec G =$ ______________
3. $\cot G =$ ______________
4. $\csc I =$ ______________
5. $\sec I =$ ______________
6. $\cot I =$ ______________

Benchmark Codes: MA.912.T.2.1

Directions: For questions 7 through 12, fill in the blanks with the missing values for the following right triangle. Write your answers as fractions in simplest form.



7. csc $K =$ ______________

8. sec $K =$ ______________

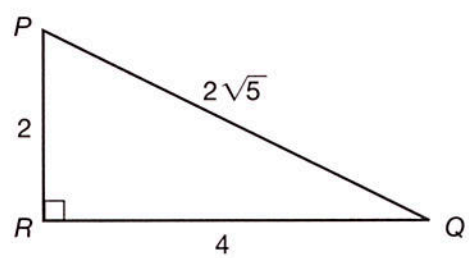
9. cot $K =$ ______________

10. csc $L =$ ______________

11. sec $L =$ ______________

12. cot $L =$ ______________

Directions: For questions 13 through 18, fill in the blanks with the missing values for the following right triangle. Write your answers as fractions in simplest form.



13. csc $P =$ ______________

14. sec $P =$ ______________

15. cot $P =$ ______________

16. csc $Q =$ ______________

17. sec $Q =$ ______________

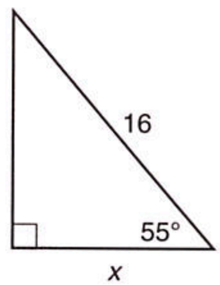
18. cot $Q =$ ______________

Using Trigonometric Ratios

The trigonometric ratios can be used to find missing values in right triangles. You can use a calculator to solve problems involving trigonometric ratios.

Example

What is the value of x in the following right triangle?



You are given an acute angle measure, 55°, and the length of the hypotenuse, 16. You need to find the value of x, the length of the leg adjacent to the acute angle. Therefore, you need to use the cosine ratio to find the value of x.

Substitute the known values into the cosine ratio.

$$\cos 55° = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 55° = \frac{x}{16}$$

Use your calculator to find the decimal value for cos 55° and substitute. The value of cos 55° is about 0.5736. Now, solve for x.

$$0.5736 = \frac{x}{16}$$

$$0.5736 \bullet 16 = x$$

$$x = 9.1776$$

The value of x is approximately 9.2.

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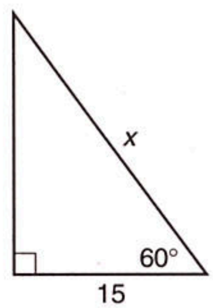
Practice

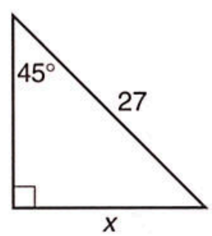
Directions: For questions 1 through 3, find the value of x to the nearest tenth.

1.



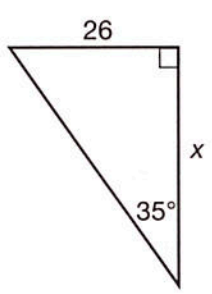
$x =$ ________

2.



$x \approx$ ________

3.



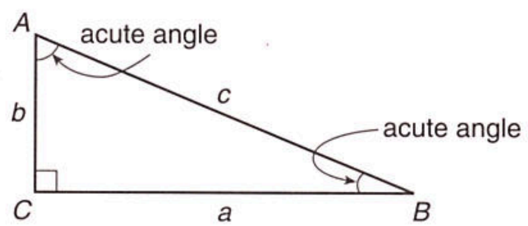
$x \approx$ ________

Inverse Trigonometric Ratios

You can use **inverse trigonometric ratios** to find the measure of an acute angle in a right triangle, given the length of two sides.

Inverse Sine Ratio

The **inverse sine ($\sin^{-1}$ or arcsin)** of x is the measure of the acute angle where $\sin A$ is x.



Given: $\sin A = x$

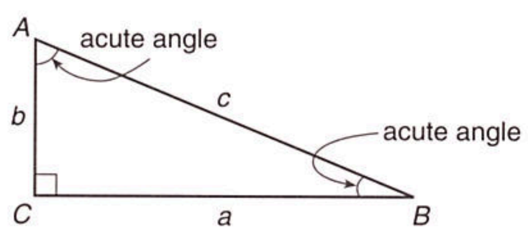
$\sin^{-1} x = m\angle A$

Given: $\sin B = x$

$\sin^{-1} x = m\angle B$

Inverse Cosine Ratio

The **inverse cosine ($\cos^{-1}$ or arccos)** of x is the measure of the acute angle where $\cos A$ is x.



Given: $\cos A = x$

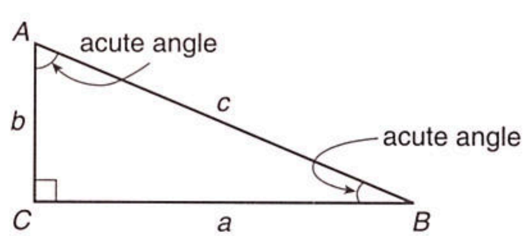
$\cos^{-1} x = m\angle A$

Given: $\cos B = x$

$\cos^{-1} x = m\angle B$

Inverse Tangent Ratio

The **inverse tangent ($\tan^{-1}$ or arctan)** of x is the measure of the acute angle where $\tan A$ is x.



Given: $\tan A = x$

$\tan^{-1} x = m\angle A$

Given: $\tan B = x$

$\tan^{-1} x = m\angle B$

TIP: To find the inverse trigonometric ratio on your calculator, you may have to push the "2nd" button before selecting the usual trigonometry buttons.

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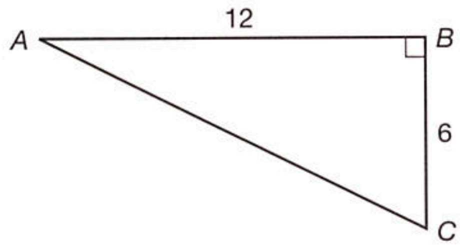
Example

What is the measure of $\angle A$?



You are given the adjacent and opposite sides. Therefore, you need to use the inverse tangent ratio to find the measure of $\angle A$.

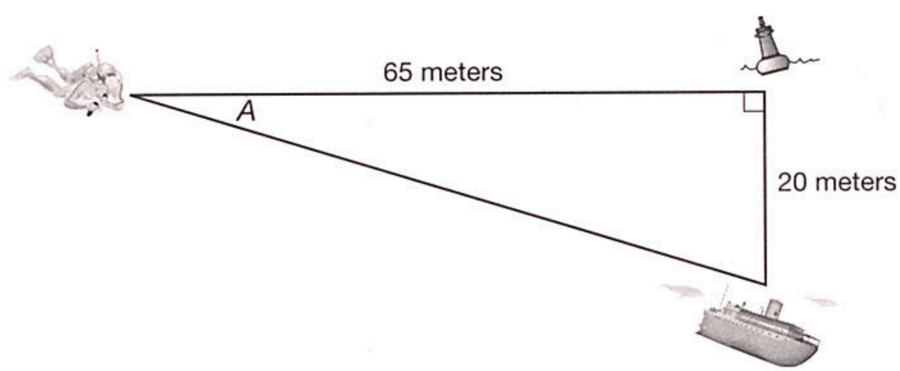
$$\tan A = \frac{6}{12} \text{ or } \frac{1}{2}$$

$$\tan^{-1} \frac{1}{2} = m\angle A$$

$$m\angle A \approx 26.6^\circ$$

Example

A scuba diver is at the surface of the water and is preparing to swim to a shipwreck. The shipwreck is 20 meters underwater, and the diver is 65 meters away from a buoy that shows where the shipwreck lies. At what angle does the diver have to swim to reach the shipwreck, to the nearest tenth of a degree?



You are given the adjacent and opposite sides. Therefore, you need to use the inverse tangent ratio to find the measure of $\angle A$.

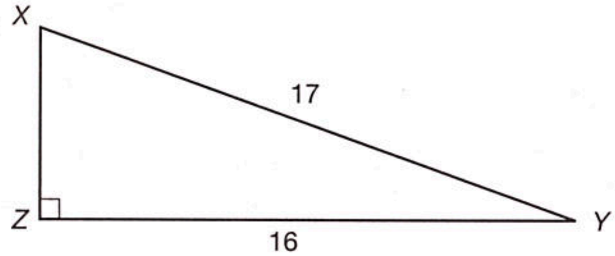
$$\tan A = \frac{20}{65} \text{ or } \frac{4}{13}$$

$$\tan^{-1} \frac{4}{13} = m\angle A$$

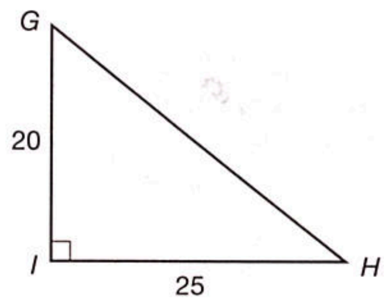
$$m\angle A \approx 17.1^\circ$$

Practice

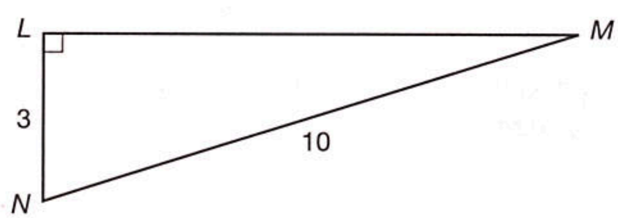
Directions: For questions 1 through 3, use the inverse trigonometric ratios to find the measures of the angles. Round each measure to the nearest tenth of a degree.

1. 

$m\angle Y \approx$ ________

2. 

$m\angle G \approx$ ________

3. 

$m\angle M \approx$ ________

Geometry Practice begins on the following page.