Lesson 8: Solids

In this lesson, you will review lines and planes in three dimensions. You will examine solid figures and the two-dimensional representations of three-dimensional objects. You will use properties of congruent and similar solids. You will also find the surface area and volume of three-dimensional objects.

Lines and Planes in Three Dimensions

Skew lines: $\overrightarrow{AB}$ and $\overrightarrow{YZ}$ are not parallel but they will never intersect.

Line perpendicular to a plane $(\overrightarrow{AB} \perp \overrightarrow{C})$: A line that forms a right angle to a plane.

Intersecting planes $(\overrightarrow{C} \text{ and } \overrightarrow{D})$: planes that meet at a line

Parallel planes $(\overrightarrow{C} \parallel \overrightarrow{D})$: planes that do not intersect

Perpendicular planes $(\overrightarrow{C} \perp \overrightarrow{D})$: planes that intersect and are at a right angle to each other
Practice

Directions: Use the following figure to answer questions 1 through 8.

1. \( R \) is a ____________________.

2. \( \overrightarrow{JK} \) is a ____________________.

3. \( \overrightarrow{LM} \) and \( \overrightarrow{NO} \) are ____________________ lines.

4. Where does \( \overrightarrow{HK} \) intersect plane \( R \)? ____________________

5. Where do planes \( P \) and \( Q \) intersect? ____________________

6. \( \overrightarrow{HK} \) and \( \overrightarrow{LM} \) are ____________________ lines.

7. \( Q \) and \( R \) are ____________________ planes.

8. \( \overrightarrow{NO} \) is the intersection of planes _______ and ________.

9. Which of the following represents the intersection of two planes?
   
   A. line
   B. point
   C. plane
   D. segment
Solids

A solid is another name for a three-dimensional figure. There are two main categories of simple solids: polyhedrons and non-polyhedrons. A **polyhedron** is a three-dimensional shape with flat faces that meet along straight edges. Some solids feature curved surfaces and are therefore considered **non-polyhedrons**.

**Polyhedrons (Solids with Flat Surfaces)**
The dimensions of a polyhedron are its length, width, and height. The plane figures that make up a polyhedron are called **faces**. Faces intersect to form **edges**. The point of intersection of three or more edges is called a **vertex**. A solid must have all three of these characteristics to be considered a polyhedron.

<table>
<thead>
<tr>
<th>Prisms</th>
<th>Pyramids</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Triangular Prism" /></td>
<td><img src="image2" alt="Triangular Pyramid" /></td>
</tr>
<tr>
<td><img src="image3" alt="Rectangular Prism" /></td>
<td><img src="image4" alt="Rectangular Pyramid" /></td>
</tr>
<tr>
<td><img src="image5" alt="Cube" /></td>
<td></td>
</tr>
</tbody>
</table>

- **Prisms**
  - *5 or more faces*
  - *2 parallel, congruent bases*
- **Pyramids**
  - *1 base*
  - *4 or more faces*

The number of faces, edges, and vertices for any given three-dimensional polyhedron can be found using Euler’s formula: **Faces + Vertices = Edges + 2**.
An **oblique prism** is a prism with bases that are not perpendicular to the prism’s faces. The faces of oblique prisms are parallelograms. An **oblique pyramid** is a pyramid with an apex that is not directly above the center of its base.

<table>
<thead>
<tr>
<th>Oblique Prisms and Pyramids</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Oblique Triangular Prism" /></td>
</tr>
<tr>
<td><img src="image" alt="Oblique Triangular Pyramid" /></td>
</tr>
</tbody>
</table>

A **Platonic solid** is a polyhedron with regular polygons for its bases. Platonic solids are also called regular solids. There are only five platonic solids, as shown.

<table>
<thead>
<tr>
<th>Platonic Solids</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Tetrahedron" /></td>
</tr>
<tr>
<td>4 faces</td>
</tr>
<tr>
<td><img src="image" alt="Dodecahedron" /></td>
</tr>
<tr>
<td>12 faces</td>
</tr>
</tbody>
</table>
Example

Draw the net of the following oblique prism.

The faces of oblique prisms are parallelograms. The bases and two lateral faces are rectangles.

Example

Identify all the vertices and edges of the following polyhedron.

Vertices: $J, K, L, M, N, P$ (6 total vertices)

Edges: $JK, LK, JL, JM, KN, LP, MN, PN, MP$ (9 total edges)

How many faces are there and what are their shapes?
There are five total faces: 3 rectangular faces and 2 triangular faces.

Notice that Euler's formula holds true in the example above:

$F + V = E + 2$, or $5 + 6 = 9 + 2$
Practice

1. Fill in the table with the shape of the base(s) and the number of edges for each solid.

<table>
<thead>
<tr>
<th>Solid</th>
<th>Base(s)</th>
<th>Number of Edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rectangular Prism</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangular Prism</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rectangular Pyramid</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangular Pyramid</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. What is the name of the solid that has six square faces? ________________

3. Which solid has exactly four vertices? ________________

4. What is the name of a solid with regular polygon faces? ________________

5. Which solid has exactly five faces? ________________

Directions: Use the following solids to answer questions 6 through 8.

6. What is the name of solid III? ________________

7. What solid has a triangular base?
   A. I
   B. II
   C. III
   D. IV

8. What two solids have the same number of faces, edges, and vertices?
   A. I and III
   B. II and III
   C. II and IV
   D. III and IV
Directions: For questions 9 through 12, draw the net of each given solid.

9. oblique rectangular pyramid

10. triangular prism

11. cube

12. rectangular prism

Directions: For questions 13 and 14, draw the solid represented by each net.

13.

14.
Non-Regular Polyhedra

Non-regular polyhedra are polyhedra with faces that are not regular polygons. The faces can include different polygons. The following figures are non-regular polyhedra.

The first solid is composed of two pentagonal pyramids and has 10 triangular faces. The second solid is composed of a cube and a square pyramid, and it has 5 square faces and 4 triangular faces. The third solid is composed of two triangular prisms and it has 4 rectangular faces and 4 triangular faces.

**Example**

What are the faces of the following non-regular polyhedron?

The non-regular polyhedron is composed of three kinds of regular polygons on the top: equilateral triangles, squares, and a regular pentagon. The bottom of it is a ten-sided polygon, with equal sides, so it must be a regular decagon.
Practice

Directions: For questions 1 and 2, draw a net of the non-regular polyhedron shown.

1. 

2. 

Directions: For questions 3 and 4, draw the non-regular polyhedron given its net.

3. 

4.
Spheres

A sphere is a figure where all its points are the same distance from its center. A radius of a sphere is a segment that connects the center to any point on the sphere. A diameter of a sphere is a segment that connects two points on the sphere and passes through the center. A chord of a sphere is a line segment that connects any two points on the sphere. A tangent is a line or plane that intersects exactly one point on a sphere. If a radius and tangent meet at a point on the sphere, the radius and tangent are perpendicular.

If a plane intersects a sphere at its center, the intersection is called a great circle. In the following figure, the shaded area represents a great circle.
Practice

Directions: Use the figure to answer questions 1 through 8. The sphere’s center is $A$, and it has a radius of 8 in.

1. What is a diameter on the sphere? 
2. Which plane creates a great circle? 
3. What are two chords of the sphere? 
4. What are two radii of the sphere? 
5. Which line is a tangent of the sphere? 
6. What is the length of $AH$? 
7. What is the length of $GK$? 
8. What is the length of $KA$?
Surface Area

Surface area (SA) is the total area of the outside of a three-dimensional object. Surface area is measured in square units. The following table gives formulas for the surface areas of various solids.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Formula</th>
<th>where:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right Prism</td>
<td>(SA = Ph + 2B)</td>
<td>(P = \text{perimeter of the base}) (h = \text{height}) (B = \text{area of the base})</td>
</tr>
<tr>
<td>Right Cylinder</td>
<td>(SA = 2\pi rh + 2\pi r^2)</td>
<td>(r = \text{radius of the base}) (h = \text{height}) (\pi \approx 3.14)</td>
</tr>
<tr>
<td>Right Cone</td>
<td>(SA = \pi r \ell + \pi r^2)</td>
<td>(r = \text{radius of the base}) (\ell = \text{slant height}) (\pi \approx 3.14)</td>
</tr>
<tr>
<td>Right Pyramid</td>
<td>(SA = \frac{1}{2}P\ell + B)</td>
<td>(P = \text{perimeter of the base}) (\ell = \text{slant height}) (B = \text{area of the base})</td>
</tr>
</tbody>
</table>

The lateral area is the sum of the areas of the lateral faces of a solid, not including the base(s).

**TIP:** If you know the height and radius of the base of a cone, you can use the Pythagorean theorem to find its slant height.
Example

What is the lateral area of the following square pyramid?

![Diagram of a square pyramid with labeled sides: \( \ell = 4 \text{ cm} \), \( 3 \text{ cm} \), \( \ell = 3 \text{ cm} \), \( r = 1 \text{ cm} \).]

Substitute the known values into the formula for surface area, without adding the base.

\[
LA = \frac{1}{2} P \ell
= \frac{1}{2} (12 \cdot 4)
= 24
\]

The lateral area is 24 cm².

Example

What is the surface area of the following cone? Use 3.14 for \( \pi \).

\[
SA = \pi r \ell + \pi r^2
= 3.14 \cdot 1 \cdot 3 + 3.14 \cdot 1^2
= 12.56
\]

The surface area is 12.56 cm².
**Practice**

**Directions:** Use the table on page 177 to answer questions 1 through 11. Use 3.14 for π.

1. What is the surface area of the following square prism?

```
8 m
7 m
8 m
```

\[ SA = \]  

2. What is the lateral area of the following rectangular prism?

```
12 in.
8 in.
4 in.
```

\[ LA = \]  

3. What is the surface area of the following cone?

```
h = 8 cm
r = 6 cm
```

\[ SA = \]  

4. What is the surface area of the following cylinder?

```
d = 10 cm
2.2 cm
```

\[ SA = \]  

5. What is the lateral area of the following triangular prism?

```
14 m
5 m
12 m
```

\[ LA = \]
6. What is the surface area of a rectangular prism with a length of 10 cm, a width of 5 cm, and a height of 12 cm?

\[ SA = \] 

7. What is the surface area of a square pyramid with a slant height of 4 in. and a base length of 5 in.?

\[ SA = \] 

8. What is the lateral area of a right regular pentagonal prism with side lengths of 6 m and a height of 10 m?

\[ LA = \] 

9. What is the surface area of a right circular cylinder with a diameter of 3 cm and a height of 7 cm?

\[ SA = \] 

10. What is the surface area of a rectangular box that is open at the top and has a length of 20 inches, a width of 15 inches, and a height of 22 inches?

A. 1,251 in.\(^2\)
B. 1,620 in.\(^2\)
C. 1,840 in.\(^2\)
D. 2,140 in.\(^2\)

11. A square pyramid has a slant height of 30 ft and a base side length of 24 ft. What is its surface area?

A. 1,680 ft\(^2\)
B. 1,784 ft\(^2\)
C. 1,903 ft\(^2\)
D. 2,016 ft\(^2\)
**Volume**

Volume \((V)\) is the measure of the amount of space it takes to fill a three-dimensional object. Volume is measured in **cubic units**. The following table gives formulas for the volumes of various solids.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Formula</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Prism</strong></td>
<td>( V = Bh )</td>
<td>( B = \text{area of the base} ) ( h = \text{height} )</td>
</tr>
<tr>
<td><strong>Cylinder</strong></td>
<td>( V = \pi r^2h )</td>
<td>( r = \text{radius of the base} ) ( h = \text{height} ) ( \pi \approx 3.14 )</td>
</tr>
<tr>
<td><strong>Cone</strong></td>
<td>( V = \frac{1}{3} \pi r^2h )</td>
<td>( r = \text{radius of the base} ) ( h = \text{height} ) ( \pi \approx 3.14 )</td>
</tr>
<tr>
<td><strong>Pyramid</strong></td>
<td>( V = \frac{1}{3} Bh )</td>
<td>( B = \text{area of the base} ) ( h = \text{height} )</td>
</tr>
<tr>
<td><strong>Sphere</strong></td>
<td>( V = \frac{4}{3} \pi r^3 )</td>
<td>( r = \text{radius} ) ( \pi \approx 3.14 )</td>
</tr>
</tbody>
</table>
Example
What is the volume of the following cone? Use 3.14 for $\pi$.

\[ V = \frac{1}{3} \pi r^2 h \]

\[ = \frac{1}{3} \times 3.14 \times 5^2 \times 12 \]

\[ = 314 \]

The volume is 314 cm$^3$.

Example
What is the volume of the following square pyramid?

\[ V = \frac{1}{3} Bh \]

\[ = \frac{1}{3} \times 8.5 \times 8.5 \times 15.6 \]

\[ = 375.7 \]

The volume is 375.7 m$^3$. 
Practice

Directions: Use the table on page 181 to answer questions 1 through 10. Use 3.14 for π. Round your answers to the nearest tenth.

1. What is the volume of the following rectangular prism?

   \[ V = \]  

2. What is the volume of the following square pyramid?

   \[ V = \]  

3. What is the volume of the following cone?

   \[ V = \]  

4. What is the volume of the following cylinder?

   \[ V = \]
5. What is the volume of a rectangular prism with a length of 8 ft, a width of 10 ft, and a height of 9 ft?

\[ V = \] ____________

6. What is the volume of a square pyramid with a side length of 15 yd and a height of 10 yd?

\[ V = \] ____________

7. What is the volume of a right circular cone with a diameter of 8 mm and a height of 36 mm?

\[ V = \] ____________

8. What is the volume of a right circular cone with a radius of 4.5 ft and a height of 7.5 ft?

\[ V = \] ____________

9. What is the volume of a rectangular swimming pool with a length of 20 ft, width of 15 ft, and a depth of 5 ft?
   A. 1,192 ft\(^3\)
   B. 1,250 ft\(^3\)
   C. 1,500 ft\(^3\)
   D. 2,167 ft\(^3\)

10. What is the volume of a spherical fish bowl with a diameter of 12 inches?
    A. 678.24 in.\(^3\)
    B. 904.32 in.\(^3\)
    C. 1,130.40 in.\(^3\)
    D. 1,356.48 in.\(^3\)
Congruent and Similar Solids

**Congruent solids** are solid figures that have the same shape and size. The corresponding faces, edges, and angles of congruent solids are congruent.

The following two figures are congruent. They have six congruent rectangular faces and congruent interior angles.

![Congruent Solids](image)

**Example**

If prism $A$ is congruent to prism $B$, what is the surface area of prism $B$?

![Prism A and Prism B](image)

Congruent solids have identical side lengths and angle measures. Therefore, you can use the measurements of prism $A$ to determine the surface area of prism $B$.

To find the surface area of a right prism, substitute the known values into the formula for the surface area of a prism. Because the triangular bases of the prisms are $30^\circ-60^\circ-90^\circ$ triangles, the hypotenuse will be equal to twice its shortest leg: $4 \text{ in.} \times 2 = 8 \text{ in.}$

\[
SA = Ph + 2b
\]

\[
SA = (4 + 8 + 4\sqrt{3})(8) + 2\left(\frac{1}{2} \cdot 16\sqrt{3}\right)
\]

\[
SA = (12 + 4\sqrt{3})(8) + 16\sqrt{3}
\]

\[
SA = 96 + 32\sqrt{3} + 16\sqrt{3}
\]

\[
SA = 96 + 48\sqrt{3}
\]

The surface area of prism $B$ is $96 + 48\sqrt{3}$ in.$^2$
Similar solids are solid figures that have the same shape but not necessarily the same size. The dimensions of similar solids are proportional, and the angles are congruent.

To determine if the following solids are similar, you must determine if the dimensions are proportional. Set up ratios comparing the widths, lengths, and heights.

\[
\text{ratio of widths } = \frac{4}{8} = \frac{1}{2} \quad \text{ratio of lengths } = \frac{4}{8} = \frac{1}{2} \quad \text{ratio of heights } = \frac{5}{10} = \frac{1}{2}
\]

Because the ratios are proportional, the prisms must be similar.

**Example**

The following two right cylinders are similar. What is the radius, \( r \), of the larger cylinder?

\[
\text{ratio of heights } = \frac{6}{9} \quad \text{ratio of radii } = \frac{4}{r}
\]

\[
\frac{6}{9} = \frac{4}{r}
\]

\[
6r = 36
\]

\[
r = 6
\]

The radius of the larger cylinder is 6 mm.
Practice

Directions: For questions 1 through 3, use the congruent or similar figures to find the missing lengths or angles.

1. The two cones shown below are congruent.

![Cone A and Cone B diagrams]

What is the height of cone B? 

2. Triangular prisms C and D are similar.

![Prism C and Prism D diagrams]

What are the measures of the angles of each triangular base of prism D? 

3. The following two rectangular prisms are similar.

![Rectangular prisms diagrams]

What is the height of the larger prism? 

Benchmark Codes: MA.912.G.7.6
Changes to Surface Area of Solids
When all the dimensions of a solid figure are multiplied by \( n \), the new surface area is \( n^2 \) times the original surface area.

**Example**
A rectangular prism has a surface area of 52 in.\(^2\). What is the surface area of the prism if its length, width, and height are doubled \( (n = 2) \)?

Because all the dimensions used to find the surface area of a prism are doubled, the new surface area will be \( 2^2 \) times the original surface area. The new surface area is 208 in.\(^2\).

**Example**
A right circular cone has a surface area of \( 10\pi \) yd\(^2\). What is the surface area of the cone if its radius and slant height are tripled \( (n = 3) \)?

Because the dimensions of the cone are tripled, the new surface area will be \( 3^2 \) times the original surface area. The new surface area is \( 90\pi \) yd\(^2\).

**Example**
The surface area of a right square pyramid is 56 cm\(^2\). What will its volume be if its slant height and side length are quadrupled \( (n = 4) \)?

Because all the dimensions used to find the surface area of a cone are quadrupled, the new surface area will be \( 4^2 \) times the original surface area. The new surface area is 896 cm\(^2\).
When only some of the dimensions of a solid figure are multiplied by \( n \), use the surface area formulas to identify how the surface area of the figure changes.

**Example**

A rectangular prism has a length of 5 in., a width of 6 in., and a height of 8 in. If its length is tripled \((n = 3)\), what will be the surface area of the new prism?

![Rectangular Prism Diagram]

The formula for the surface area of a rectangular prism is \( SA = Ph + 2B \). If the height is tripled, the formula will be \( SA = P(h \times 3) + 2B \).

\[
\begin{align*}
SA &= P(h \times 3) + 2B \\
SA &= (5 + 6 + 5 + 6)(8 \times 3) + 2(5 \times 6) \\
SA &= 528 + 60 \\
SA &= 588
\end{align*}
\]

The surface area of the new rectangular prism will be 588 in.\(^2\).

**Example**

A right circular cylinder has a diameter of 12 cm and a height of 5 cm. If its radius is multiplied by 6, what will be the surface area of the new cylinder? Use 3.14 for \( \pi \).

![Cylinder Diagram]

The formula for the surface area of a right circular cylinder is \( SA = 2\pi rh + 2\pi r^2 \). If the radius is multiplied by 6, the formula will be \( SA = 2\pi(6r)h + 2\pi(6r)^2 \).

\[
\begin{align*}
SA &= 2\pi(6r)h + 2\pi(6r)^2 \\
SA &= 2\pi(6 \times 6)5 + 2\pi(6 \times 6) \\
SA &= 360\pi + 72\pi = 432\pi
\end{align*}
\]

The surface area of the new cylinder will be \(432\pi \text{ cm}^2\).
Practice

Directions: For questions 1 and 2, find the surface area of each solid given the changes in all of its dimensions. Use 3.14 for π.

1. The height and radius of the cone are both tripled.

New SA = _____________

2. The length, width, and height are all multiplied by 10.

New SA = _____________

Directions: For questions 3 and 4, find the surface area of the given solids given the change in one of its dimensions. Use 3.14 for π.

3. The slant height of the pyramid is doubled.

New SA = _____________

4. The height of the cylinder is multiplied by 8.

New SA = _____________
Changes to Volume of Prisms and Pyramids

Rectangular prisms and square pyramids have three dimensions: length \((l)\), width \((w)\), and height \((h)\). The following rules will help you find the new volume of a prism that changes one, two, or three of its dimensions:

- When **one** dimension of a prism or pyramid is multiplied by \(n\), the new volume is \(n\) times the original volume.
- When **two** dimensions of a prism or pyramid are multiplied by \(n\), the new volume is \(n^2\) times the original volume.
- When **three** dimensions of a prism or pyramid are multiplied by \(n\), the new volume is \(n^3\) times the original volume.

**Example**

A cube has a volume of 64 ft\(^3\). What is the volume of the cube if one dimension is doubled \((n = 2)\)?

One dimension is doubled, so the new volume will be 2 times the original volume. The new volume is 128 ft\(^3\).

**Example**

A rectangular prism has a volume of 350 in\(^3\). What is the volume of the prism if the width and height are tripled \((n = 3)\)?

Two dimensions are tripled, so the new volume will be \(3^2\) times the original volume. The new volume is 3150 in\(^3\).

**Example**

A square pyramid has a volume of 120 cm\(^3\). What is the volume of the pyramid if all three dimensions are quadrupled \((n = 4)\)?

All three dimensions are quadrupled, so the new volume will be \(4^3\) times greater than the original volume. The new volume is 7680 cm\(^3\).

**TIP:** These rules can help you find an answer quickly, but it is always a good idea to check your math by putting the new dimensions into the appropriate volume formula.
Practice

Directions: Use the following rectangular prism to answer questions 1 and 2.

1. If the length, width, and height of the rectangular prism are quadrupled, what will be the new volume?
   
   \[ V = \quad \]

2. If only the height of the rectangular prism is multiplied by 6, what will be the new volume?
   
   \[ V = \quad \]

3. Cierra has a computer on her desk that is shaped like a rectangular prism. The computer is 14 inches long, 6 inches wide, and 15 inches tall. She just bought a new computer that is half as tall. What is the volume of the new computer?
   
   \[ V = \quad \]

4. Jerome's locker has a volume of 54 000 cm³. Next year, Jerome will get a locker with a length and a width 2 times those of his current locker. What is the volume of the locker that Jerome will get next year?
   
   \[ V = \quad \]

5. A square pyramid has a volume of 9 m³. If you multiply its width and height by 9, what is the new volume?
   
   A. 27 m³  
   B. 81 m³  
   C. 324 m³  
   D. 729 m³

6. A square pyramid has a height of 5 m and a volume of 60 m³. If you multiply all its dimensions by 12, what is the new width?
   
   A. 144 m  
   B. 72 m  
   C. 36 m  
   D. 12 m
Changes to Volume of Cylinders and Cones

Cylinders and cones are three-dimensional figures, but their dimensions are not described in the same way as those of prisms or pyramids. A change to a cylinder or cone is described in terms of height \((h)\) and radius \((r)\).

The following rules will help you find the new volume of a cylinder or cone that changes dimensions:

- When only the **height** of a cylinder or cone is multiplied by \(n\), the new volume is \(n\) times the original volume.
- When only the **radius** of a cylinder or cone is multiplied by \(n\), the new volume is \(n^2\) times the original volume.
- When **both** the height and the radius of a cylinder or cone are multiplied by \(n\), the new volume is \(n^3\) times the original volume.

**Example**

A cone has a volume of 803.84 mm\(^3\). What is the volume if the radius is quadrupled \((n = 4)\)?

Only the radius is quadrupled; the height stays the same. The new volume will be \(4^2\) times greater than the original volume. The new volume is 12,861.44 mm\(^3\).

**Example**

A cylinder has a volume of 122.44 in.\(^3\). What is the volume of the cylinder if the height is tripled \((n = 3)\)?

If the height is 3 times larger, the volume of the new sphere will be 3 times the original volume. The new volume is 367.32 in.\(^3\).
Practice

Directions: Use the following cone to answer questions 1 and 2. Use 3.14 for π.

1. What is the volume of the cone if only its height is multiplied by 4?
   \[ V = \text{________} \]

2. What is the volume of the cone if only its radius is multiplied by 4?
   \[ V = \text{________} \]

3. On his ranch, Dwayne has an underground cylindrical fuel tank with a volume of 524 m³. Samuel has an underground cylindrical fuel tank that is the same height but 2 times the diameter. What is the volume of Samuel’s fuel tank?
   \[ V = \text{________} \]

4. If a cylinder has a volume of 225 cm³, what is the volume of a cylinder that has a radius 4 times larger?
   A. 900 cm³
   B. 1800 cm³
   C. 3600 cm³
   D. 14400 cm³

5. If a cone has a volume of 12 ft³, what is the volume of a cone that is 3 times larger in all dimensions?
   A. 972 ft³
   B. 324 ft³
   C. 108 ft³
   D. 36 ft³