1 Interactive Learning

Solve It!

**PURPOSE**
To compare the results of two summations

**PROCESS**
Students may
- substitute the same nonnegative numbers for \(x\) in both expressions and compare results.
- find the length of the diagonal of the rectangular prism using \(d = \sqrt{l^2 + w^2 + h^2}\).

**FACILITATE**
Q For which summation expression does it matter whether \(x_j\) is nonnegative? Explain. [The first expression; negative numbers affect the sum. In the second expression, squaring negates the effects of negatives.]

**ANSWER**
See Solve It in Answers on next page.

**CONNECT THE MATH**
In the Solve It, students compare a sum of data values to the square root of the sum of the squares of those data values. In the lesson, students use the square root of the sum of the differences of data values from the mean to find the standard deviation.

2 Guided Instruction

**Take Note**

Q If all the values in a data set are equal, what is the standard deviation of the set? [0]
Q If you increase each data value in the set by the same number, how does it affect the mean and standard deviation of the set? [The mean increases by that number, but the standard deviation does not change.]

**BIG idea**
Probability

**ESSENTIAL UNDERSTANDING**
Standard deviation is a measure of how far the numbers in a data set deviate from the mean.

**Math Background**
Measures of variation describe how data in a data set are spread out.

Variance is calculated by finding the average squared deviation of each value from the mean of the data. Because of this, it takes all of the data values into account.

Standard deviation is the positive square root of the variance. It is a measure of spread.

Calculating variance and standard deviation is sometimes difficult for students. Suggest that they write a checklist and use it for each problem until they are fully comfortable with the process. For example:
- Find the mean of the data set.
- Find the difference between each data value and the above mean. Write them in a list.
- Find and write down the square of each number on the list.
- Find the mean of the list of squares. This is the variance.
- Take the square root of the variance. This is the standard deviation.

**Mathematical Practices**
Look for and express regularity in repeated reasoning. In calculating for variance and standard deviation, students will look for general methods and shortcuts for the many calculations they will have to perform. Students will also use a calculator to determine variance and standard deviation of a set of data.

**Key Concepts** Finding Variance and Standard Deviation

- Find the mean, \(\bar{x}\), of the \(n\) values in a data set.
- Find the difference, \(x - \bar{x}\), between each value \(x\) and the mean.
- Square each difference, \((x - \bar{x})^2\).
- Find the average (mean) of these squares. This is the variance.
  \[ \sigma^2 = \frac{\sum(x - \bar{x})^2}{n} \]
- Take the square root of the variance. This is the standard deviation.
  \[ \sigma = \sqrt{\frac{\sum(x - \bar{x})^2}{n}} \]
Problem 1  
When calculating variance, squaring each value’s difference from the mean serves two purposes:
• It makes each term positive, so values above the mean do not cancel values below.
• It amplifies larger deviations in the data set.

Q What is the sum of the differences from the mean in the third column of the table? [0]
Q Why is the sum of the differences from the mean of a data set always 0? [The mean lies at the center of a distribution where the sum of data values above it equals the sum of the data values below it.]
Q Can the variance of a data set be negative? Explain. [No; variance is the quotient of two positive numbers—the sum of the squares and the number of data values.]

Finding the square root of the variance serves two purposes:
• It returns the measurement’s unit to the unit of the data values in the set.
• It reduces the size of the measurement so it is easier to use for data analysis.

Got It?
Students can add labels to their tables to help them understand the terms and symbols used: Left to right; “Data Value,” “Mean of Data Set,” “Difference from Mean,” and “Square of Difference.”

Problem 2  
Q Does the VAR STATS screen give the variance of the data set? [No]
Q How can you use the information on the screen to find the variance? [Square the standard deviation.]

Answers

Solve It!
\[ \sum_{i=1}^{3} x_i \] is the sum of 3 values \( x_1, x_2, \) and \( x_3. \)
\[ \sqrt{\sum_{i=1}^{3} (x_i)^2} \] is the square root of the sum of the square of three values i.e. \( \sqrt{(x_1)^2 + (x_2)^2 + (x_3)^2}. \)

Got It?
1. \( \bar{x} = 69.83, \sigma^2 = 115.1389, \sigma = 10.7303 \)
Using a Calculator to Find Standard Deviation

Finding Variance and Standard Deviation

Problem 1

Problem 2

Problem 3

Got It?

Problem 4

Got It?

Got It?

Additional Problems

Answers
3 Lesson Check

Do you know HOW?
• For Exercise 1, suggest using a table to organize the computations as shown in Problem 1.
• For Exercises 1 and 2, students can use a calculator to verify their answers.

Do you UNDERSTAND?
• For Exercise 3, if necessary, remind students that mean, median, and mode are measures of central tendency, and range, interquartile range, variance, and standard deviation are measures of variation.
• For Exercise 4, students can use graphs to help compare and contrast the data sets. The mean is the peak of each graph's curve, which changes to concave-up at approximately 1 standard deviation greater than and less than the mean.

Q What is standard deviation and how is it used? [Answers may vary. Sample: Standard deviation is a measure of the difference of the data from the mean. It is used to analyze data and to make predictions.]

Lesson Check
Do you know HOW?
1. Find the mean, variance, and standard deviation for the data set.
   \[5, 15, 9, 3, 12, 8, 13, 6, 18, 11\]
2. Within how many standard deviations of the mean do all of the data values fall?
   \[12, 17, 15, 13, 9, 10, 12, 10, 15, 17\]

Do you UNDERSTAND?
3. Vocabulary
   Explain the difference between measures of central tendency and measures of variation.

4. Compare and Contrast
   Three data sets each have a mean of 78. Set A has a standard deviation of 10. Set B has a standard deviation of 5. Set C has a standard deviation of 20. Compare and contrast these 3 sets.

5. Reasoning
   What is the effect of an outlier on the standard deviation of a data set?

Practice and Problem-Solving Exercises

Practice
Find the mean, variance, and standard deviation for each data set.
6. 78 90 456 673 111 381 21
7. 13 15 17 18 12 21 10
8. 12 3 2 4 5 7
9. 60 40 35 45 39

Graphing Calculator
Find the mean and the standard deviation.

10. The Dow Jones Industrial average for the first 12 weeks of 1988:

   \[\begin{array}{c}
   1911.31 \\
   1903.51 \\
   1915.82 \\
   1904.98 \\
   1910.48 \\
   1983.26 \\
   \end{array}\]

11. The Dow Jones Industrial average for the first 12 weeks of 2008:

   \[\begin{array}{c}
   12800.18 \\
   12381.02 \\
   12266.39 \\
   11972.25 \\
   \end{array}\]

Determine the whole number of standard deviations from the mean that include all data values.

12. The mean price of the nonfiction books on a best-sellers list is $25.07; the standard deviation is $2.62.
   \[\text{Price range:}\ 22.95, 25.07, 27.95, 24.95, 24.00, 24.95, 24.95, 29.95, 24.95, 24.00, 27.95, 25.00\]

13. The mean length of Beethoven's nine symphonies is 37 minutes; the standard deviation is 12 minutes.
   \[\text{Length range:}\ 27, 30, 47, 35, 30, 40, 35, 22, 65\]

Answers

Lesson Check
1. \(\bar{x} = 10, \quad \sigma^2 = 19.8, \quad \sigma = 4.45\)
2. within 2 standard deviations of the mean
3. Measures of central tendency are specific data pts. which give a summary of the middle of the data set, whereas the measures of variation give a summary of the variation of the data set within the range of distribution.
4. Standard deviation measures how widely spread the data values are. If the data pts. are close to the mean, the standard deviation is small; if the data pts. are far from the mean, the standard deviation is large. The data pts. of Set B are closer to the mean of 70 than the data pts. of Sets A and C; likewise, the data pts. of Set A
5. The effect of an outlier on the standard deviation is to increase the standard deviation.

Practice and Problem-Solving Exercises
6. \(\bar{x} \approx 258.6, \quad \sigma^2 \approx 52,136.81, \quad \sigma \approx 228.3\)
7. \(\bar{x} = 15.1, \quad \sigma^2 = 12.4, \quad \sigma = 3.5\)
8. \(\bar{x} = 5.5, \quad \sigma^2 = 10.9, \quad \sigma = 3.3\)
9. \(\bar{x} = 43.8, \quad \sigma^2 = 75.76, \quad \sigma = 8.7\)
10. \(\bar{x} = 1984.98, \quad \sigma = 57.62\)
11. \(\bar{x} = 12,320.00, \quad \sigma = 273.71\)
12. 2 standard deviations
13. 3 standard deviations
2. Do you know **HOW**?

6. A

12, 17, 15, 13, 9, 10, 15, 17

all of the data values fall?

Within how many standard deviations of the mean do

5, 15, 9, 3, 12, 8, 13, 6, 18, 11

the data set.

Find the mean, variance, and standard deviation for

Lesson Check

include all data values.

Graphing Calculator

27 min, 30 min, 47 min, 35 min, 30 min, 40 min, 35 min, 22 min, 65 min

The mean length of Beethoven’s nine symphonies is 37 minutes; the standard
deviation is 12 minutes.

The mean price of the nonfiction books on a best-sellers list is $25.07; the standard
deviation is $2.62.

The Dow Jones Industrial average for the first 12 weeks of 1988:

12800.18

2014.59

1911.31

12381.02

12606.30

2057.86

11893.69

12207.17

3. Do you **UNDERSTAND**?

**Reasoning**

B has a standard deviation of 5. Set C has a standard

Vocabulary

Compare and Contrast.

of central tendency

PRACTICES

MATHEMATICAL

b. $x = 53.8, \sigma = 3.4; 1\sigma: 7; 2\sigma: 9; 3\sigma: 10$

16. year 2001: $x = 6707$; year 2002: $x = 6738$

17. Overall farm income increased

slightly, but there was less variability

among the states in 2002.

The income in 2001 clustered

more tightly around the mean.

(2001: $\sigma_x = 2679$, 2002: $\sigma_x = 2758$)

18. Iowa, North Dakota, and South Dakota

19. a–b. Check students’ work.

20. a. $\bar{x} = 82.3, \sigma = 4.3$

b. $1\sigma: 7; 2\sigma: 9; 3\sigma: 10$

21. Your first friend; one standard deviation encompasses all values

within one standard deviation above

and below the mean. The graph

shows that all values are within 3

standard deviations of the mean.
22. a. Men: range: 18 yrs, \( \bar{x} \approx 22.44, \sigma \approx 3.58 \);
Women: range: 13 yrs,
\( \bar{x} \approx 25.78, \sigma \approx 4.04 \)
b. No; for the data given, the larger range has the smaller standard deviation.

23. a. no change to \( \sigma \)
b. \( \sigma \) increases by a factor of 10