GK- Mathematics

Resources for Some Math Questions:
Kaplan et al (2015). Cliff Notes FTCE General Knowledge Test, 3rd Edition
Mander, E. (2015). FTE General Knowledge Test with Online Practice, 3rd Edition

Let's Be A.D.U.L.T.S.

- **A**ctive Participants
- **D**o Unto Others
- **U**tilize Respectful Tones
- **L**isten Intently
- **T**ake Breaks as Needed
- **S**tay the Entire Session

Tips for Reducing Test Anxiety

- **Approach the exam with confidence**
  Use whatever strategies you can to personalize success: visualization, logic, talking to your self, practice, team work, journaling, etc. View the exam as an opportunity to show how much you've studied and to receive a reward for the studying you've done
- **Be prepared!**
  Learn your material thoroughly and organize what materials you will need for the test. Use a checklist
- **Choose a comfortable location for taking the test**
  with good lighting and minimal distractions

Reference: http://www.studygs.net/tstprp8.htm

Tips for Reducing Test Anxiety

- **Allow yourself plenty of time**
  Especially to do things you need to do before the test and still get there a little early
- **Avoid thinking you need to cram just before**
- **Strive for a relaxed state of concentration**
  Avoid speaking with any fellow students who have not prepared, who express negativity, who will distract your preparation
- **A program of exercise**
  is said to sharpen the mind

Reference: http://www.studygs.net/tstprp8.htm

Tips for Reducing Test Anxiety

- **Get a good night's sleep**
  the night before the exam
- **Don't go to the exam with an empty stomach**
  fresh fruits and vegetables are often recommended to reduce stress.
  Stressful foods can include processed foods, artificial sweeteners, carbonated soft drinks, chocolate, eggs, fried foods, junk foods, pork, red meat, sugar, white flour products, chips and similar snack foods, foods containing preservatives or heavy spices

Reference: http://www.studygs.net/tstprp8.htm

GK- Math Review Overview

Session	Competency/Skill	%	#	Target
1	Pre-Test 15 Questions			
1 & 2	Number Sense	17	8	6
3 & 4	Geometry	21	9	6
5 & 6	Algebraic Thinking	29	13	9
7 & 8	Probability & Statistics	33	15	11
8	Post-Test 15 Questions			
8 Sessions	Total	100	45	32

GK Mathematics PRE-TEST

Practice Test for the General Knowledge Math Test

Number Sense	Geometry & Measurement	Algebraic Thinking	Probability
1	9	32	38
2	11	33	40
3	13	35	41
	24		44
			45

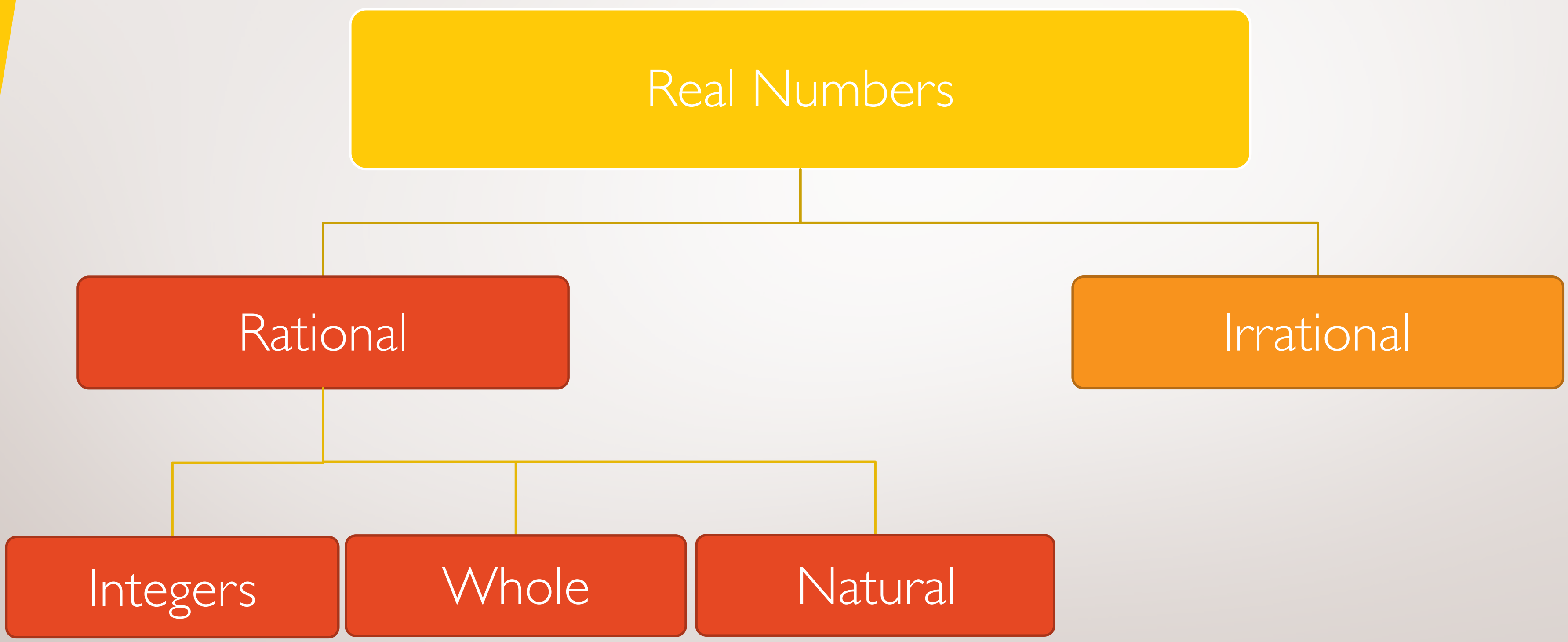
15 Questions 30 Minutes

Number Sense, Concepts, and Operations

- Compare real numbers and identify their location on a number line.
- Solve real-world problems involving the four operations with rational numbers.
- Evaluate expressions involving order of operations.

17% or approximately 8 questions
Cliff Notes Text: pages 59-107
Target: 6

Number Sense



Real Numbers

- **Real Number:** signed numbers, zero, decimals, fractions, prime, composite, even, odd, square roots, cube roots, etc.
- **Rational Numbers:** numbers that are fractions or can be expressed as fractions. Consists of 3 other subsets of numbers: *Integers, Whole, Natural.*
- **Irrational Numbers:** numbers that cannot be written as fractions.

Rational Numbers

- $\frac{1}{3}$ is rational because it is a fraction.
- -5 is rational because it can be written as $\frac{-10}{2}$.
- $\sqrt{25}$ is rational because it is equal to 5, which can be written as a fraction. $(5 \times 5 = 25)$
- $\sqrt[3]{8}$ is rational because it is equal to 2, which can be written as a fraction. $(2 \times 2 \times 2 = 8)$

Rational Numbers

Natural Numbers	Whole Numbers	Integers
Counting Numbers	Zero + Natural #'s	Positive+Negative+Zero
Contain no fractions, decimals, negative numbers, or zero.	Contain no fractions, decimals, or negative numbers.	Contain no fractions or decimals.
{1,2,3,…}	{0,1,2,3,…}	{…-2,-1,0,1,2,…} or {0, ±1, ±2,…}

Irrational Numbers

- All of the radicals that cannot be simplified completely.
- When expressed as decimals, the numbers behind the decimal do not stop or repeat.
- When decimals are used for these numbers we call them approximations. When the radical is used, we say it's the exact value.
- Examples of Irrational Numbers: $\pi, \sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \sqrt{8}, \sqrt{21}$.

Review: Real Numbers

- -4 belongs to the set of Integers, Rational, and Real Numbers.
- 0 is an element of the set of Whole Numbers, Integers, Rational, and Real Numbers.
- $\sqrt{36}$ is an element of the set Natural and Whole Numbers, Integers, Rational and Real Numbers.
- $\sqrt{23}$ belongs to the sets of Irrational and Real Numbers.
- $\frac{2}{3}$ belongs to the set of Rational and Real Numbers.

Complete!

Check all that apply for each value.

	Whole	Natural	Integers	Rational	Irrational	Real
	☐	☐	☐	☐	☐	☐
	☐	☐	☐	☐	☐	☐
	☐	☐	☐	☐	☐	☐
	☐	☐	☐	☐	☐	☐

Copy and Complete!(Answers)

<table>
<tr><th></th><th>Whole</th><th>Natural</th><th>Integers</th><th>Rational</th><th>Irrational</th><th>Real</th></tr>
<tr><td><table><tr><th></th><th>Whole</th><th>Natural</th><th>Integers</th><th>Rational</th><th>Irrational</th><th>Real</th></tr><tr><td>-30</td><td>□</td><td>□</td><td></td><td></td><td>□</td><td></td></tr><tr><td>$\sqrt{23}$</td><td>□</td><td>□</td><td>□</td><td>□</td><td></td><td></td></tr><tr><td>5</td><td></td><td></td><td></td><td></td><td>□</td><td></td></tr><tr><td>$-\frac{2}{7}$</td><td>□</td><td>□</td><td>□</td><td></td><td>□</td><td></td></tr></table></td><td>□</td><td>□</td><td>▲</td><td>▲</td><td>□</td><td>▲</td></tr>
<tr><td><table><tr><th></th><th>Whole</th><th>Natural</th><th>Integers</th><th>Rational</th><th>Irrational</th><th>Real</th></tr><tr><td>-30</td><td>□</td><td>□</td><td></td><td></td><td>□</td><td></td></tr><tr><td>$\sqrt{23}$</td><td>□</td><td>□</td><td>□</td><td>□</td><td></td><td></td></tr><tr><td>5</td><td></td><td></td><td></td><td></td><td>□</td><td></td></tr><tr><td>$-\frac{2}{7}$</td><td>□</td><td>□</td><td>□</td><td></td><td>□</td><td></td></tr></table></td><td>□</td><td>□</td><td>□</td><td>□</td><td>▲</td><td>▲</td></tr>
<tr><td><table><tr><th></th><th>Whole</th><th>Natural</th><th>Integers</th><th>Rational</th><th>Irrational</th><th>Real</th></tr><tr><td>-30</td><td>□</td><td>□</td><td></td><td></td><td>□</td><td></td></tr><tr><td>$\sqrt{23}$</td><td>□</td><td>□</td><td>□</td><td>□</td><td></td><td></td></tr><tr><td>5</td><td></td><td></td><td></td><td></td><td>□</td><td></td></tr><tr><td>$-\frac{2}{7}$</td><td>□</td><td>□</td><td>□</td><td></td><td>□</td><td></td></tr></table></td><td>▲</td><td>▲</td><td>▲</td><td>▲</td><td>□</td><td>▲</td></tr>
<tr><td><table><tr><th></th><th>Whole</th><th>Natural</th><th>Integers</th><th>Rational</th><th>Irrational</th><th>Real</th></tr><tr><td>-30</td><td>□</td><td>□</td><td></td><td></td><td>□</td><td></td></tr><tr><td>$\sqrt{23}$</td><td>□</td><td>□</td><td>□</td><td>□</td><td></td><td></td></tr><tr><td>5</td><td></td><td></td><td></td><td></td><td>□</td><td></td></tr><tr><td>$-\frac{2}{7}$</td><td>□</td><td>□</td><td>□</td><td></td><td>□</td><td></td></tr></table></td><td>□</td><td>□</td><td>□</td><td>▲</td><td>□</td><td>▲</td></tr>
</table>

Other Important Numbers

- **Even**: Any number divisible by 2. {…-4,-2,0,2,4,…}
- **Odd**: Numbers not divisible by 2. {…-5,-3,-1,1,3,5,…}
- **Prime**: Any number greater than 1 with exactly 2 factors: one and the number. Examples: 2, 3, 5, 7, 11, 13, 17
- **Composite**: Have 3 or more factors and are all the numbers that are not prime. Examples: 4, 6, 8, 9, 10, 12, 14, 15
- **Note:** 1 is neither prime nor composite.

Exponents

- In the problem 5^2, 5 is the base and 2 is the exponent. If you were being asked to simply the problem, you would multiply $5 \times 5 = 25$.
- When working with negatives, be careful about whether or not the negative is included in the parenthesis.
- -6^2 versus $(-6)^2$ **These are not the same!!!**
- One says repeat only the 6 and use one negative sign.

  $\mathbf{-6^2 = -6 \bullet 6 = -36}$
- The second says repeat everything in parenthesis.

  $\mathbf{(-6)^2 = (-6)(-6) = 36}$

Specific Examples for Exponents

1. Anything to the zero power is 1. That includes numbers or variables.

- $4^0 = 1$
- $4^1 = 4$
- $4^2 = 4 \bullet 4 = 16$
- $4^3 = 4 \bullet 4 \bullet 4 = 64$
- $4^4 = 4 \bullet 4 \bullet 4 \bullet 4 = 256$

2. Notice in the next example how when the negative base is raised by an even exponent, the answer is positive. The opposite is true when the exponent is odd.

- $(-3)^0 = 1$
- $(-3)^1 = -3$
- $(-3)^2 = (-3)(-3) = 9$
- $(-3)^3 = (-3)(-3)(-3) = -27$
- $(-3)^4 = (-3)(-3)(-3)(-3) = 81$

Law of exponents

Law	Example
$a^m a^n = a^{m+n}$	$2^3 2^4 = 2^{3+4} = 2^7 = 128$
$(a^m)^n = a^{mn}$	$(2^3)^4 = 2^{3.4} = 2^{12} = 4096$
$(ab)^n = a^n b^n$	$(20)^3 = (2.10)^3 = 2^3.10^3 = 8.1000 = 8000$
$(\frac{a}{b})^n = \frac{a^n}{b^n}$	$(\frac{2}{5})^3 = \frac{2^3}{5^3} = \frac{8}{125}$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{2^5}{2^3} = 2^{5-3} = 2^2 = 4$
$\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$	$\frac{2^3}{2^5} = \frac{1}{2^{5-3}} = \frac{1}{2^2} = \frac{1}{4}$

Scientific Notation

- We use scientific notation to rewrite large numbers or to rewrite small numbers with many values behind the decimal point. Written in the form $a \times 10^b$.
- The goal is to move the decimal until you get a whole value for (a) between 1 and 9.
- The direction that you move the decimal determines if the exponent of ten (b) is positive or negative.

Scientific Notation

(Positive Exponent)

- Let's look at: $2{,}340{,}000.$
- If we rewrite this value using scientific notation, you will need to take the decimal from behind the last zero and move it behind the number two. The number of places you moved the decimal becomes your exponent for 10. Since you are moving the decimal to the left, the exponent is positive.
- Answer: $2{,}340{,}000 \;=\; \mathbf{2.34 \times 10^{6}}$

Scientific Notation

(Negative Exponent)

- Let's look at: 0.0000825
- If we rewrite this value using scientific notation, you will need to take the decimal from its current location and move it to the first whole number between 1 and 9. In this case, we move it till we get behind the 8. The number of places you moved the decimal becomes your exponent for 10. Since you are moving the decimal to the right, the exponent is negative.
- Answer: $0.0000825 = \mathbf{8.25 \times 10^{-5}}$

Scientific Notation

Example

- Simplify: $\frac{28\times10^{12}}{7\times10^{9}}$.
- This problem may look difficult, but think about doing two problems at once. For example, the two values that look alike are 28 and 7. Reduce those first. Keep in mind, this is a division problem. For the second part, there are twelve tens in the numerator and nine in the denominator. If we cancel out an equal number of tens in the numerator and denominator, how many tens would be left?
- **Answer: $\mathbf{4\times10^{3}=4000}$.** (The 3 represents the number of zeros.)

Complete Scientific Notation Worksheet – Number Sense

Factors and GCF

- The factors of 10 are: 1, 2, 5, 10
- The factors of 21 are: 1, 3, 7, 21
- Find the greatest common factor of 10 and 21.
- The factors of 20 are: 1, 2, 4, 5, 10, 20
- Find the GCF of 10 and 20.
- Now, find the GCF of 28 and 36.
- 17 is a number whose factors are 1 and 17. What is this number called?

Factors and GCF Answers

- Find the greatest common factor of 10 and 21. **GCF = 1**
- Find the GCF of 10 and 20. **GCF = 10**
- Find the GCF of 28 and 36. **GCF = 4**
- 17 is a number whose factors are 1 and 17.
  What is this number called? **Prime**

Multiples and LCM

- The multiples of 2 are: 2, 4, 6, 8, 10, 12, 14,...
- The multiples of 3 are: 3, 6, 9, 12, 15, 18, 21,...
- What is the Least Common Multiple of 2 and 3? 6
- What is the Least Common Multiple of 2 and 6?
- What is the LCM for 3 and 5?
- What is the LCM for 15 and 25?
- Add: $\frac{1}{3} + \frac{1}{2}$

Number Sense

Multiples and LCM **Answers**

- What is the Least Common Multiple of 2 and 6?

  When one number is a factor of the other, the LCM is the larger of the two values: LCM= 6.
- What is the LCM for 3 and 5?

  When there are no common factors greater than 1, the LCM is the product of the two values: LCM = 15.
- What is the LCM for 15 and 25?

  Remember: Prime Factorizations: $15 = 3 \cdot 5$ and $25 = 5 \cdot 5$ Use what they have in common once and then use the remaining factors: $LCM = 3 \cdot 5 \cdot 5 = 75$.

Multiples and LCM **Answer**

- Add: $\frac{1}{3} + \frac{1}{2}$
- Step 1: What is the LCM for 3 and 2?
- **When there are no common factors greater than 1, the LCM is the product of the two values: LCM = 6.**
- Step 2: We must multiply the numerator and denominator of each fraction by what is needed to create 6 in the denominator.
- $\frac{2}{2}\left(\frac{1}{3}\right) + \left(\frac{1}{2}\right)\frac{3}{3} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}.$

Fractions

- When multiplying fractions, multiply the 2 numerators, and the two denominators. You may simplify first or at the end. Up to you.
- When adding fractions, make sure the denominators are the same.
- Try the following:
-

$$\frac{5}{6} + \frac{8}{7}$$

- You must find the LCM! Go!
- Now Try:

$$\frac{5}{6} \times \frac{8}{7}$$

Number Sense

Fractions (Add) **Answer**

- $\frac{5}{6} + \frac{8}{7}$ **The LCM or LCD is 6(7) = 42**. We will need to multiply numerator and denominator of the first fraction by 7 and by 6 for the second fraction.
- $\frac{7}{7}\left(\frac{5}{6}\right) + \left(\frac{8}{7}\right)\frac{6}{6}$
- $= \frac{35}{42} + \frac{48}{42}$
- $= \frac{83}{42}$ from here, you'd have to read the question carefully and/or look at the answer choices to know if you should convert this to a mixed number or decimal.

Fractions (Multiply) Answer

- $\frac{5}{6} \times \frac{8}{7}$ **For this question, you just need to multiply the numerators and then the denominator. Be sure to look for ways to simply the answer.**
- $\frac{5}{6} \times \frac{8}{7} = \frac{40}{42}$ Since both 40 and 42 are even, they can be simplified. Let's first try to divide each number by two because they are even.
- $\frac{40}{42} = \frac{2 \cdot 20}{2 \cdot 21} = \frac{20}{21}$ Since there are no other common factors shared by the numerator and denominator, the answer is $\frac{20}{21}$.

Zero in Division Problems

- $\frac{5}{0}$ versus $\frac{0}{5}$
- In order to understand division by zero, we must understand: $\frac{20}{4} = 5$ which works, only because $4(5) = 20$.
- $\frac{18}{6} = 3$ Only because $6(3) = 18$.
- Using this logic, find $\frac{0}{5}$ then try $\frac{5}{0}$.

Zero in Division Problems

- $\frac{5}{0}$ versus $\frac{0}{5}$
- Whenever you see a zero in a division problem, the answer is either zero or undefined.
- $\frac{0}{5} = 0$ because 5(0) = 0.
- $\frac{5}{0} \neq 5$ *nor does it equal* 0, therefore it is undefined.
- **Remember this!**

Proper vs Improper

- **<u>Proper Fractions</u>** have denominators that are greater than numerators.
- $\frac{2}{3}, \frac{5}{6}$, and $\frac{101}{105}$ are examples of proper fractions.
- **<u>Improper fractions</u>** have larger numerators than denominators.
- $\frac{8}{5}, \frac{27}{6}$, and $\frac{111}{102}$ are examples of improper fractions.

Mixed Numbers

- Mixed Numbers are a combination of whole numbers and fractions.
- Examples: $4\frac{2}{5}$, $8\frac{3}{8}$, and $10\frac{1}{2}$.
- When we want to manipulate these numbers, we typically write them as improper fractions but can also write them as decimals.

Mixed Numbers as Improper Fractions

- Improper fractions are created by multiplying the whole number by the denominator, then adding the numerator. Finally placing that sum over the original denominator.
- As *Improper Fractions*: $4\frac{2}{5} = \mathbf{\frac{22}{5}}$, $8\frac{3}{8} = \mathbf{\frac{67}{8}}$, and $10\frac{1}{2} = \mathbf{\frac{21}{2}}$

Mixed Numbers as Decimals

- To write mixed numbers as Decimals, take the fraction part, and turn it into a decimal (using the calculator) and place the whole number in front.
- As Decimals $4\frac{2}{5}$ = $\mathbf{4.4}$, $8\frac{3}{8} = \mathbf{8.375}$, and $10\frac{1}{2}$ = $\mathbf{10.5}$

Roots

Square Roots		Cube Roots		Fourth Roots	
$\sqrt{1} = 1$	$1 \times 1 = 1$	$\sqrt[3]{1} = 1$	$1 \times 1 \times 1 = 1$	$\sqrt[4]{1} = 1$	$1 \times 1 \times 1 \times 1 = 1$
$\sqrt{4} = 2$	$2 \times 2 = 4$	$\sqrt[3]{8} = 2$	$2 \times 2 \times 2 = 8$	$\sqrt[4]{16} = 2$	$2 \times 2 \times 2 \times 2 = 16$
$\sqrt{9} = 3$	$3 \times 3 = 9$	$\sqrt[3]{27} = 3$	$3 \times 3 \times 3 = 27$	$\sqrt[4]{81} = 3$	$3 \times 3 \times 3 \times 3 = 81$
$\sqrt{16} = 4$	$4 \times 4 = 16$	$\sqrt[3]{64} = 4$	$4 \times 4 \times 4 = 64$	$\sqrt[4]{256} = 4$	$4 \times 4 \times 4 \times 4 = 256$

- Study this table. The first column in each category represents a potential mathematical problem involving roots. The second column in each category represents the justification for the answer. For example, the reason that the cube root of 27 equals 3 is because 3(3)(3) = 27.

Number Sense

Order of Operations

- Most of you are familiar with PEMDAS or Please, Excuse, My, Dear, Aunt, Sally.
- You may continue to use this as long as you **remember that Multiplication and Division must be done from left to right**. So, if division is first in the problem, you must do that first. Multiplication is not more important than division.
- The same is true for Addition and Subtraction. **If subtraction comes first from left to right, do it first.**

Order of Operations Example

- $4 \div 2 \times 4^2 - 2(5-1)$ You Try!
- $4 \div 2 \times 4^2 - 2(4)$ Parenthesis
- $4 \div 2 \times 16 - 2(4)$ Exponents $4^2 = 4 \times 4 = 16$
- $2 \times 16 - 2(4)$ Division (left to right) $4 \div 2$
- $32 - 2(4)$ Multiplication (left to right) 2×16
- $32 - 8$ Multiplication (left to right) $2(4)$
- 24 Subtraction $32 - 8$

Percent Problem

What is 70% of 45?

Two ways to solve this problem. Pick one!	
Step 1: Identify the parts that fit the proportion; do not change the numbers in any way. Step 2: Cross-Multiply and solve for missing value. $\frac{is}{of} = \frac{percent}{100}$ or $\frac{part}{whole} = \frac{percent}{100}$	Step 1: Change the percent to a decimal. (Hint: Move decimal to the left 2 units) Step 2: Multiply the decimal times 45.

Two ways to solve this problem. Pick one!

Step 1: Change the percent to a decimal. (Hint: Move decimal to the left 2 units)
Step 2: Multiply the decimal times 45.

Percent Problem Answer

What is 70% of 45?

$\frac{is}{of} = \frac{percent}{100}$ or $\frac{part}{whole} = \frac{percent}{100}$ $\frac{x}{45} = \frac{70}{100}$ Cross Multiply: $100x = 70(45)$ $100x = 3150$ $x = 31.5$	Step 1: Change the percent to a decimal. Step 2: Multiply the decimal times 45. Or: $0.70(45) = 31.5$

$\frac{is}{of} = \frac{percent}{100}$ or $\frac{part}{whole} = \frac{percent}{100}$ $\frac{x}{45} = \frac{70}{100}$ Cross Multiply: $100x = 70(45)$ $100x = 3150$ $x = 31.5$	Step 1: Change the percent to a decimal. Step 2: Multiply the decimal times 45. Or: $0.70(45) = 31.5$

Number Sense

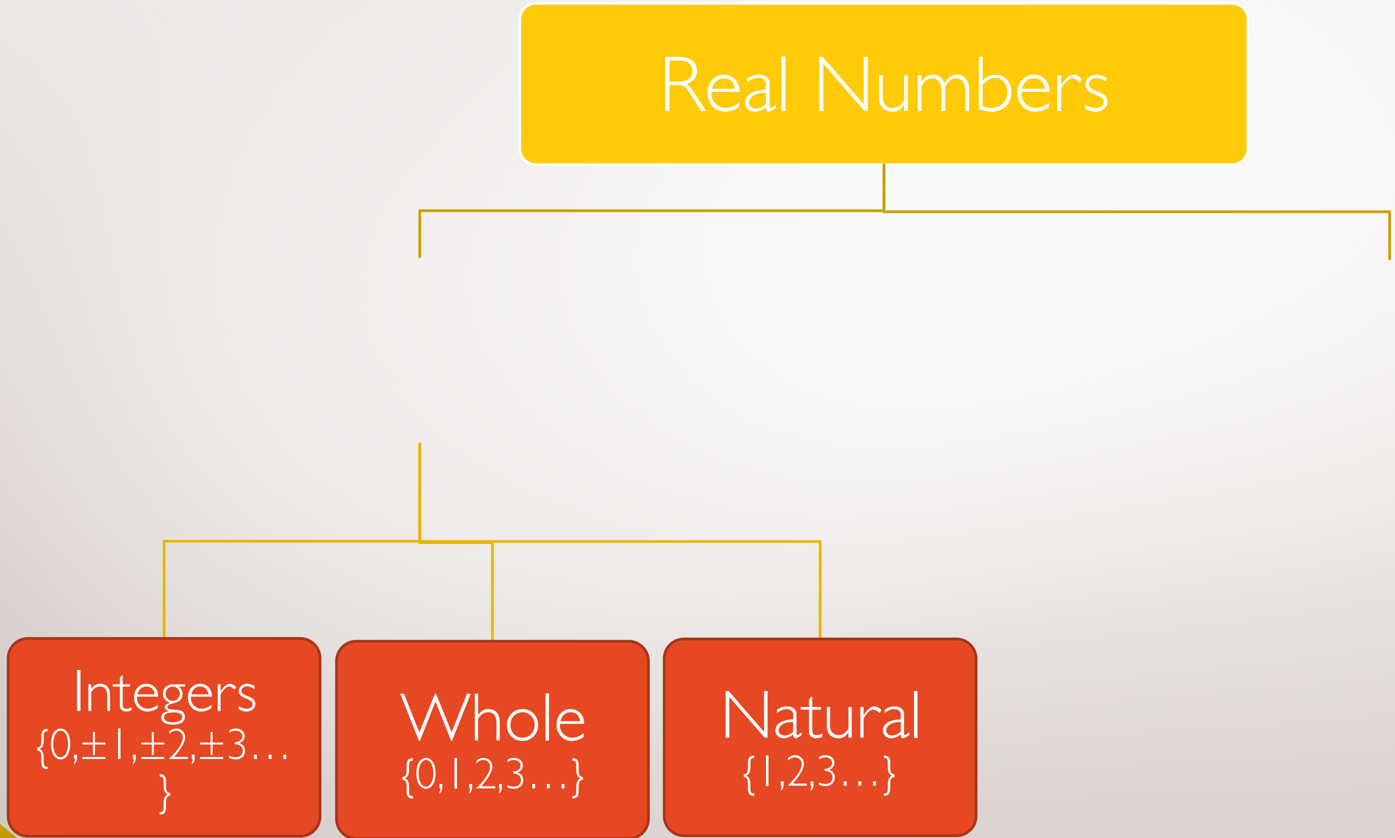
Ratio and Proportion

- You will find several problems that will rely on your knowledge of ratios and proportions.
- A **ratio** is a comparison of two items. We also consider this to be a fraction.
- A **proportion** is when two ratios are set equal to each other.
- Example: Jose' has a bag that contains 4 apples and 6 oranges. Robert has a bag with the same ratio of apples to oranges. If Robert has 12 oranges, how many apples does he have?

Ratio and Proportion Example

- Example: Jose has a bag that contains 4 apples and 6 oranges. Robert has a bag with the same ratio or apples to oranges. If Robert has 12 oranges, how many apples does he have?
- Step 1: Make a ratio (actually make one): $\frac{\boldsymbol{apples}}{\boldsymbol{oranges}}$.
- Step 2: Fill in the information you have using the ratio you created. The first ratio represents Jose's information and the second represents Robert's. $\frac{\boldsymbol{apples}}{\boldsymbol{oranges}} = \frac{\mathbf{4}}{\mathbf{6}} = \frac{\boldsymbol{x}}{\mathbf{12}}$.
- Step 3: Cross Multiply and solve: $\boldsymbol{x} = \mathbf{8}$.

Number Sense



Complete GK-Number Sense Worksheets 1 and 2

Use the Cliff Notes text for additional practice.