GK- Mathematics

Resources for Some Math Questions:

Kaplan et al (2015). Cliff Notes FTCE General Knowledge Test, 3rd Edition

Mander, E. (2015). FTE General Knowledge Test with Online Practice, 3rd Edition

GK- Math Review Overview

Session	Competency/Skill	%	#	Target
1	Pre-Test 15 Questions			
1 & 2	Number Sense	17	8	6
3 & 4	Algebraic Thinking	29	13	9
5 & 6	Geometry	21	9	6
7 & 8	Probability & Statistics	33	15	11
8	Post-Test 15 Questions			
8 Sessions	Total	100	45	32

Algebraic Thinking and the Coordinate Plane

- Determine whether two algebraic expressions are equivalent by applying properties of operations or equality.
- Identify an algebraic expression, equation, or inequality that models a real-world situation.
- Solve equations and inequalities (e.g., linear, quadratic) graphically or algebraically.
- Determine and solve equations or inequalities, graphically or algebraically, in real-world problems.
- Graph and interpret a linear equation in real-world problems (e.g., use data to plot points, explain slope and y-intercept, and determine additional solutions).
- Identify relations that satisfy the definition of a function.
- Compare the slopes of two linear functions represented algebraically and graphically.

29% or Approximately 13 questions

Cliff Notes Text: pages 149-203

Target: 9

Tips for Algebra

- Understand the definition and purpose of the variable.
- Understand the power of substitution.
- Use proportions when comparisons are made.
- Watch out for negative signs.
- Make good use of your calculator.

Expression versus Equation

Expression	Equation
Does not have an equal sign	Has an equal sign
Combine Like Terms	If on the same side of the equal sign, Combine Like Terms. Otherwise, perform inverse operations.
Evaluate	Solve
$4x + 5$	$3x + 6 = 12$

Like Terms

- All numbers without variables are like terms.
- Like terms are terms with the same variable(s) and same exponents.

Examples	Sum	Non-Examples
$4x \ and \ 2x$	$6x$	$6xyz \ and \ -4xy$
$5y^2 and \ -13y^2$	$-8y^2$	$5y \ \ and \ -13y^2$
$3xy^2 \ and \ xy^2$	$4xy^2$	$-34 \ and \ 25x$
$6 \ and \ -12$	-6	$m \ and \ n$

4 Basic Operations & Ways to Undo Them

Operation	How to Undo Operation
Addition	Subtraction
Subtraction	Addition
Multiplication	Division
Division	Multiplication

Other Words Representing Basic Operations

Operation	Other Words Used
addition	sum, plus, increased by
subtraction	difference, minus, decreased by
multiplication	product, times, twice
division	quotient, divided by, ratio

Other Vocabulary

Coefficient: the number in front of a variable. In $4x$, the coefficient is 4.

Constant term: the number without the variable. In $2x - 6$, -6 is the constant.

Inequalities:

- $<$ Less than
- $>$ Greater than
- $\leq$ Less than or equal to
- $\geq$ greater than or equal to
- $\neq$ not equal to

Translations

Translation	Expression
Twice a number	$2x$
The difference between a number and two	$x - 2$
Six more than twice a number	$6 + 2x$
Four times the sum of a number and five	$4(x + 5)$
The square of the sum, x plus 3	$(x + 3)^2$
The ratio of a number and seventeen	$x/17$

Translation-Type Algebraic Question

- A box of pens costs $\$2.49$. What equation would allow you to calculate the cost (C) of multiple boxes (B) of pens?

A) $B = 2.49C$

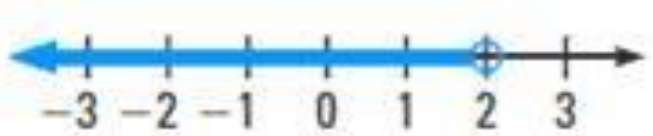
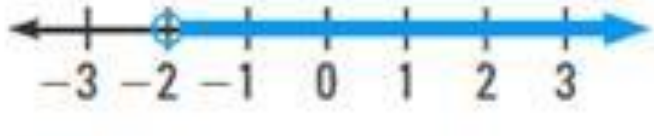
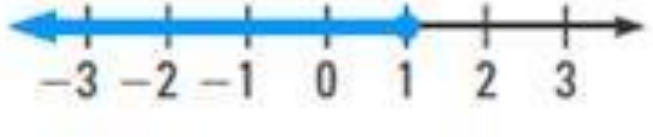
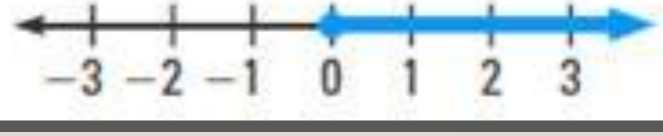
B) $C = 2.49B$

C) $2.49 = B + C$

D) $B - 2.49 = C$

Answer: B) $C = 2.49B$

Translations for Inequalities

VERBAL PHRASE	INEQUALITY	GRAPH
All real numbers less than 2	$x < 2$	
All real numbers greater than -2	$x > -2$	
All real numbers less than or equal to 1	$x \leq 1$	
All real numbers greater than or equal to 0	$x \geq 0$	

Note when the circle on the number-line is open and when it is closed.

Distributive Property

- $7(2x + 6)$ $= \mathbf{14x + 42}$
- $-2(5 - 4x)$ $= \mathbf{-10 + 8x}$
- $4 + 5(3x - 1)$ $= \mathbf{4 + 15x - 5 = 15x - 1}$
- $3x - (4x + 8)$ $= \mathbf{3x - 4x - 8 = -x - 8}$

Substitution

- Evaluate: $7y^2 - 8xy + 11, \text{if } x = -1 \text{ and } y = 2$
- Understand the operations 7 times y squared minus 8 times x times y plus 11
- $7(\,)^2 - 8(\,)(\,) + 11$ Use parenthesis to denote where you will need to insert a value.
- $7(2)^2 - 8(-1)(2) + 11$ Next, simplify one part at a time
- $7(4) + 16 + 11$ Take your time
- $28 + 16 + 11$
- **55 Answer**

Solving Equations

- Types of Solutions
  - 1 solution
  - No solution
  - All Real Numbers

No Solution vs All Real Numbers

No Solution	All Real Numbers or Infinitely Many Solutions
$-3p + 2 - 2p = 7 - 5p$ $-5p + 2 = 7 - 5p$ $+5p \quad = \quad +5p$ $2 \neq 7$ When the variables are eliminated and a false statement results, the answer is **No solution**.	$2(x + 6) = 12 + 2x$ $2x + 12 = 12 + 2x$ $-2x \quad = \quad -2x$ $12 = 12$ When the variables are eliminated and a true statement results, the answer is **All Real Numbers.**

Create a graphed solution for each.

Inequalities

- Solve inequalities like equations → isolate the variable.
- **Major Difference**: When multiplying or dividing both sides by a negative value, reverse the inequality symbol.

$2(x+4) \geq 16$	$-5x+1<21$	$-22 < -\frac{2}{3}x+2 \leq 14$
$2x+8 \geq 16$ $-8 \quad -8$ $\frac{2x}{2} \geq \frac{8}{2}$ $x \geq 4$	$-5x+1<21$ $-1 \quad -1$ $* \frac{-5x}{-5} < \frac{20}{-5}$ $x > -4$	$-22 < -\frac{2}{3}x+2 \leq 14$ $-2 \quad -2 \quad -2$ $-24 < -\frac{2}{3}x \leq 12$ $*(-\frac{3}{2})(-24) < (-\frac{3}{2})(-\frac{2}{3}x) \leq (-\frac{3}{2})12$ $36 > x \geq -18$

Real-World Inequality Problem

You Try:

A salesman receives a base salary of $300 a month, plus 10% of his sales. How much do his average weekly sales (s) have to be to make enough money to cover his monthly expenses of $950?

A) $s \leq \$650$

B) $s \leq \$6{,}500$

C) $s \geq \$1{,}625$

D) $s \leq \$1{,}625$

Real-World Inequality Problem Answered

Question: A salesman receives a base salary of $300 a month, plus 10% of his sales. **How much do his average weekly sales (s) have to be** to make enough money to cover his monthly expenses of $950?

- Note: the information within the question is in monthly terms, but the question is asking for the answer to be in **weekly terms**.
- $Equation$: $300 + 0.1s \geq 950$ Subtract 300 from both sides
- $0.1s \geq 650$ Divide both sides by 0.1
- $s \geq 6500$ Monthly amount.
- $s \geq (6500 \div 4)$ Adjusted for weekly expenses.
- $\boldsymbol{s \geq 1625}$ **The answer is C**.

Consecutive Integers

- When you are confronted with a consecutive integer word problem, you must create a set of variables that represent the numbers.
- It might be helpful to give yourself an example of three numbers that are consecutive; example 3, 4, and 5.
- If the first number 3 is represented by the variable x, how would you represent 4 or 5?
- Using this logic, three consecutive numbers would be represented by x, x+1, and x +2.

Consecutive Integers

The sum of three consecutive integers is 45. What is 5 more than twice the second integer?

- First identify variable representations for the numbers.
- $n, n+1, n+2.$
- Write an equation representing the first part of the question.
- $n+n+1+n+2=45 \rightarrow 3n+3=45 \rightarrow 3n=42 \rightarrow n=14$
- The 3 numbers are 14, 15, and 16.
- 5 more than twice the second integer = **5+2(15)=5+30=35**.

Systems of Linear Equations

- A linear system is two or more equations solved simultaneously.
- Three types of solutions: no solution, 1 solution, many solutions.
- No Solution means the lines are parallel so they don't touch.
- One Solution means the lines touch in exactly one point; meaning the solution is an ordered pair in the form (x, y).
- Many solutions means the lines are on top of one another; meaning the equations are equal. Also called coinciding lines.

Systems of Linear Equations

- There are at least three ways to solve systems: Graphing, Substitution, Elimination.
- Substitution and Elimination are good methods. Another method of solving systems is to use the choices provided and plug the values in. This may not always be possible.
- Let's look at Substitution.

Systems (Substitution)

$x = 5$

$2x - y = 12$

$$\begin{aligned} 2x - y &= 12 \\ 2(5) - y &= 12 \\ 10 - y &= 12 \\ -10 \quad &= -10 \\ \hline -y &= 2 \\ y &= -2 \end{aligned}$$

Use information in the first equation to substitute into the second equation.

Solution (5,-2)

Systems
Combination or Elimination

$$\begin{aligned} 4x + 2y &= -24 \\ 2x - 2y &= 12 \end{aligned}$$

$$\begin{aligned} 4x + 2y &= -24 \\ \underline{2x - 2y} &= \underline{12} \\ 6x &= -12 \\ x &= -2 \end{aligned}$$

$$\begin{aligned} 4x + 2y &= -24 \\ 4(-2) + 2y &= -24 \\ -8 + 2y &= -24 \\ 2y &= -16 \\ y &= -8 \end{aligned}$$

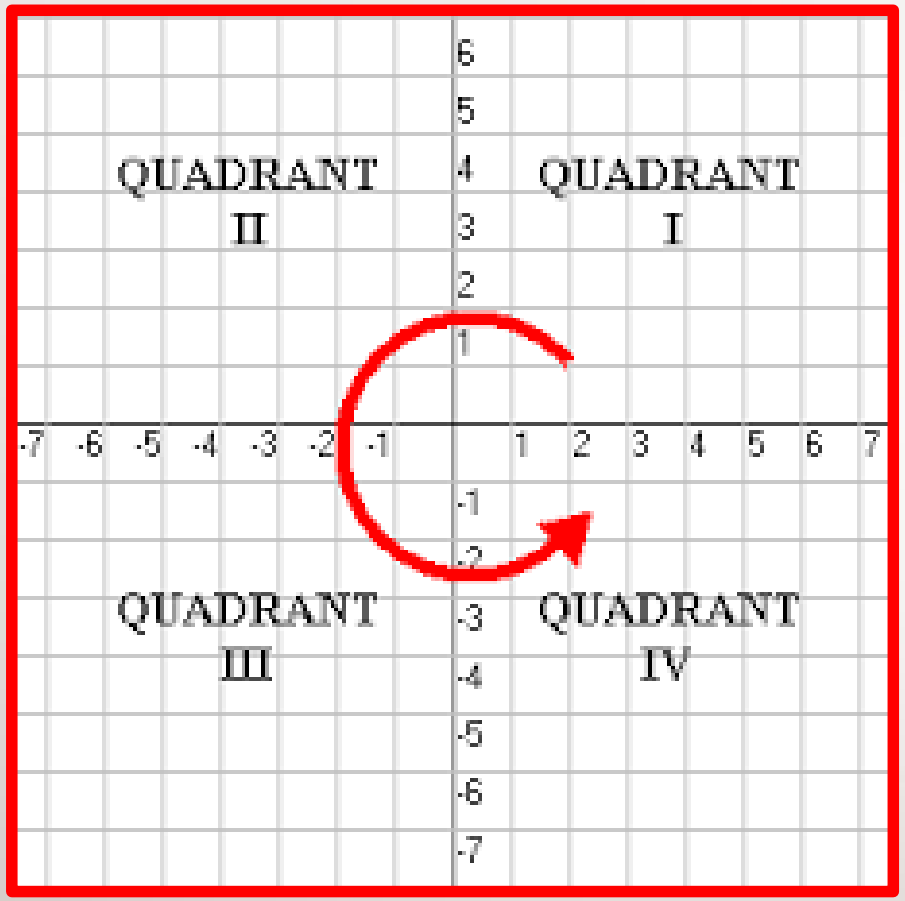
Solution: (-2,-8)

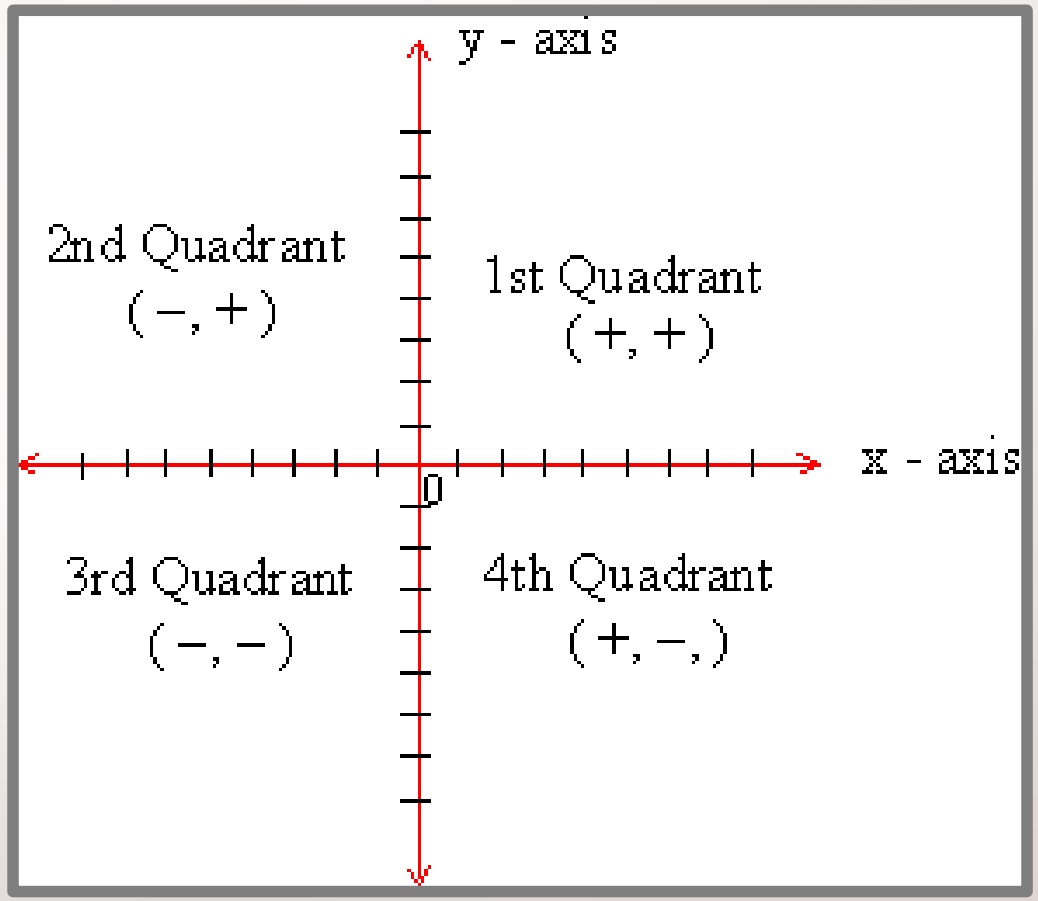
Add the two equations together in an attempt to eliminate the y variable. Solve for x, then use substitution to find the value of y.

Points are Important in Algebra

- **Ordered Pairs** are written in the form **(x, y)** also called a point.
- With any two points, the **distance (length), midpoint**, and **slope** can be determined.
- In all three formulas, the subscripts focus on one coordinate at a time.
- If it is helpful, indicate on scratch paper which point is first and which is second.

Coordinate System Images





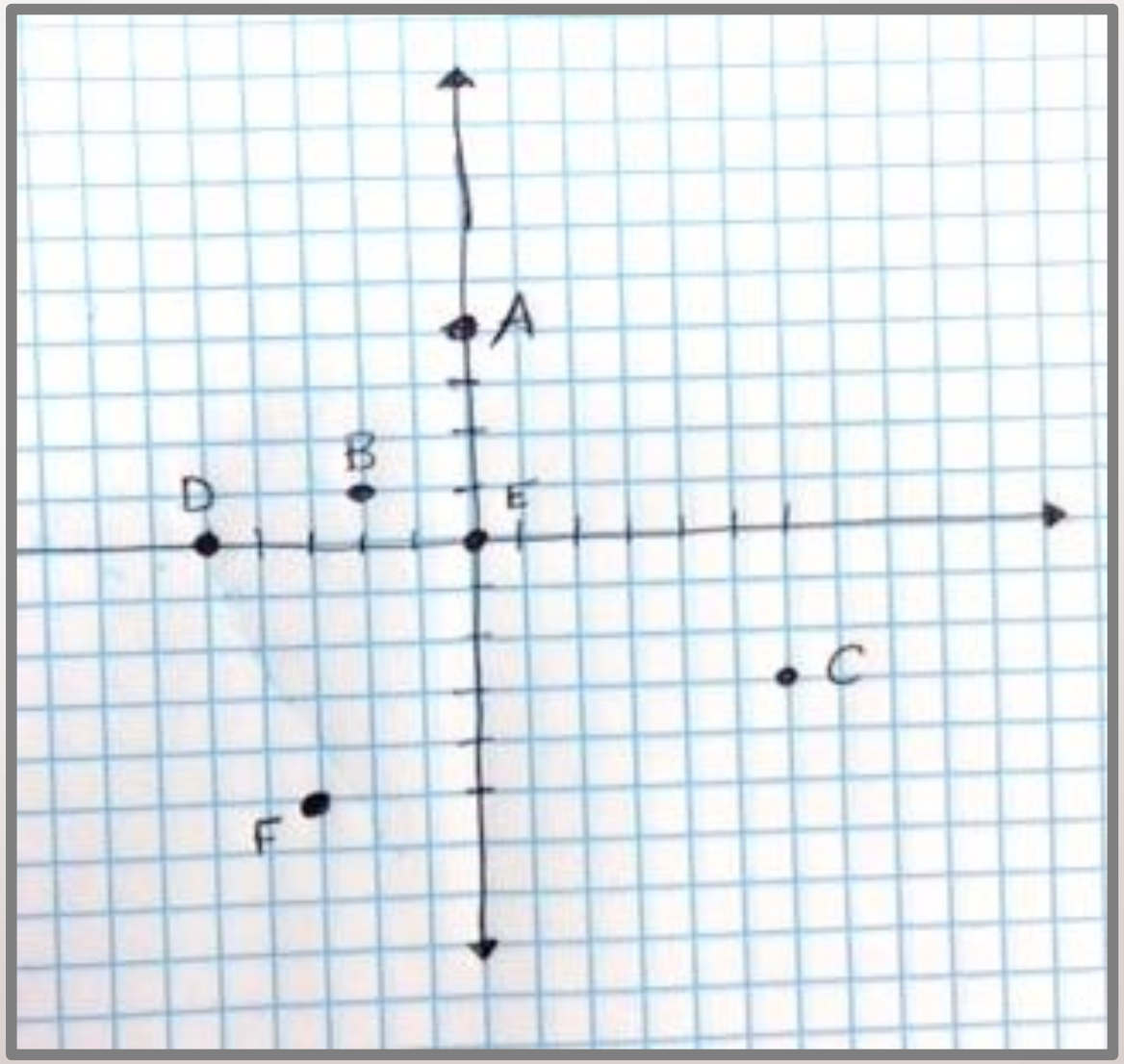
Four quadrants that are counted in a counterclockwise fashion.

Points

On the graph paper provided, plot the following points in a coordinate plane. Label each point.

A(0,4); B(-2,1); C(6,-3);

D (-5,0); E(0, 0); F (-3,-5)



Distance

- Distance is also referred to as length.
- The distance between two points is always positive.
- Find the distance between (17, 2) and (14, -8).

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(14 - 17)^2 + (-8 - 2)^2}$$

$$\sqrt{(-3)^2 + (-10)^2} = \sqrt{9 + 100} = \sqrt{109} \approx 10.44.$$

Midpoint

- Find the midpoint of segment AB with endpoints A(-2,5) and B(6, 11).
- The subscripts in the formula means there are two points.
- Ordered Pairs are written in the form (x, y).
- The x-values are -2= x_1 and 6= x_2
- The y-values are 5 =y_1 and 11 = y_2
- Solution: $(\frac{6-2}{2}, \frac{11+5}{2}) = (\frac{4}{2}, \frac{16}{2}) = (2,8)$.

Slope

- 4 types of slope: positive, negative, zero and undefined.
- Find the slope of the line that contains (-2,3) and (-5, 7).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 3}{-5 - (-2)} = \frac{4}{-5 + 2} = \frac{4}{-3}.$$

- Be careful about zeros:
- Find the slope of the line that contains (2,4) and (2,-13).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-13 - 4}{2 - 2} = \frac{-17}{0} = undefined.$$

- Find the slope of the line that contains (11,5) and (-3,5).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 5}{-3 - 11} = \frac{0}{-14} = 0.$$

Parallel vs Perpendicular Lines

- The slopes of parallel lines are equal. (parallel lines never touch)
- The slopes of perpendicular lines are negative reciprocals. (perpendicular lines form 90 degree angles)
- Use y = mx + b to identify the slope (m) and y-intercept (b)
- Are the lines represented by $y = \frac{5}{7}x - 4$ and $y = \frac{7}{5}x - 2$ parallel, perpendicular, or neither?
  - **Answer: Neither because the slopes are reciprocals but they are not negative reciprocals.**

Domain/Range

- **Domain**: the set of x-values.
- **Range**: the set of y-values.
- **Relation**: a set of ordered pairs.
- **Function**: every x must have only one y.

Domain/Range Example 1

Use the relation to answer each question:

$\{(1,2); (-3,1); (5,6)\}$

a) State the domain: $\{-3,1,5\}$

b) State the range: $\{1,2,6\}$

c) Is the relation a function? YES. No values of y repeat for the same value of x.

Domain/Range Example 2

Use the relation to answer each question:

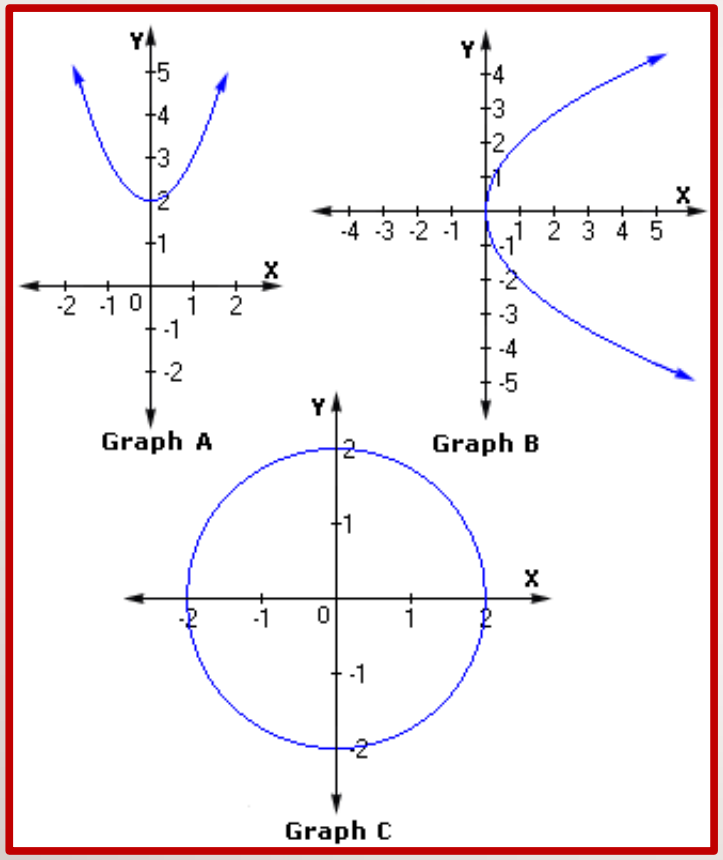
$\{(1,2); (-3,1); (5,6); (-3,5); (0,6); (2,4)\}$

a) State the domain: $\{-3,0,1,2,5\}$

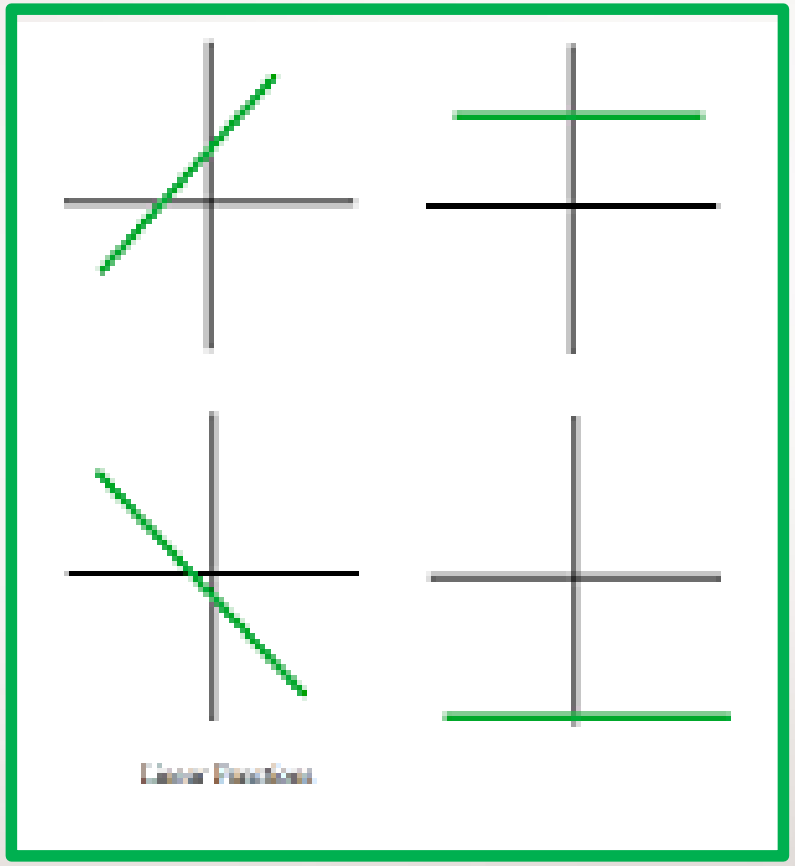
b) State the range: $\{1,2,4,5,6\}$

c) Is the relation a function? NO. -3 corresponds to 1 and 5.

Vertical Line Test to Identify Functions



Only Graph A is a function.



Which of these lines are functions?

- All lines shown in the second image are functions.
- Vertical Lines are absent from the image.
- Vertical lines fail the Vertical Line Test.

Graphing Linear Equations

Use graph paper.

Using $y = mx + b$, identify the y-intercept and slope.

- Identify a partner and practice!
- Graph:

A) $y = 2x + 4$

B) $y = \frac{3}{2}x - 2$

C) $y = 5$

D) $x = -1$

Graphing Inequalities

Use graph paper.

- Graph on a number-line:
- Very similar to graphing equations, except shading is required.
- A) $y > 2x + 4$
- B) $y \leq -2x + 5$
- C) $y \leq 6$
- D) $x > -3$

Complete the two Algebra Worksheets

Use the Cliff Notes text for additional practice.