AP Statistics Review I– Probability

First, get out your AP formula sheet and check out the formulas in the probability section.

**The basics**  Suppose \( P(A) = .35, P(B) = .6 \) and \( P(A \cap B) = .27 \). Determine

a. \( 0.65 \)  \( P(A^c) \) (\( A^c \) represents the complement of \( A \))

b. \( 0.68 \)  \( P(A \cup B) \)

c. \( 0.45 \)  \( P(A | B) \)

d. No  are \( A \) and \( B \) independent events? (yes or no)

\( P(A) \cdot P(B) \neq P(A \cap B) \)

**Example 1:** Suppose the probability that a construction company will be awarded a certain contract is .25, the probability that it will be awarded second contract is .21, and the probability that it will get both contracts is .13. What is the probability that the company will win at least one of the two contracts?

\[
P(A \cup B) = .25 + .21 - .13 = .33
\]

**Example 2:** A researcher interested in eye color versus success in a math program collected the following data from a random sample of 2000 high school students.

<table>
<thead>
<tr>
<th></th>
<th>brown</th>
<th>blue</th>
</tr>
</thead>
<tbody>
<tr>
<td>fail</td>
<td>190</td>
<td>10</td>
</tr>
<tr>
<td>pass</td>
<td>1710</td>
<td>90</td>
</tr>
<tr>
<td>1900</td>
<td>100</td>
<td>2000</td>
</tr>
</tbody>
</table>

a) What is the probability that a student from this group fails the math program?

\[
P(\text{fail}) = \frac{1900}{2000} = .95
\]

b) What is the probability that a student from this group fails the math program given that he/she has blue eyes?

\[
P(\text{fail} | \text{blue}) = \frac{10}{100} = .10
\]

c) Are blue eyes and failing the math program independent or dependent?

\[
\text{Yes, } P(F) = P(F|B)
\]

**Example 3:** Of the 60 obese teenagers in a recent study, 15 had type II diabetes, 20 had high blood pressure, and 10 had both high blood pressure and type II diabetes. For the following questions, suppose one of these 60 obese teenagers is randomly selected:

a) Given that the teenager has type II diabetes, what is the probability that he or she also has high blood pressure?

\[
P(H | III) = \frac{10}{15} = \frac{2}{3} = .6667
\]

b) If the obese teenager does NOT have high blood pressure, what is the probability that he or she also does not have type II diabetes?

\[
P(H^c | III^c) = \frac{35}{40} = \frac{7}{8}
\]
Example 4: The probability that Michael misses a free throw shot is .1. If he goes to the line to shoot three free throws:

a) What is the probability that Michael misses all three shots? What assumptions did you make in order to calculate this probability? 
\[ 0.1 \times 0.1 = 0.01 \]

b) What is the probability that Michael makes exactly one of the three shots? 
\[ 3 \times 0.1 \times 0.1 \times 0.9 = 0.027 \]

c) What is the probability that Michael makes at least one of the three shots? 
\[ 1 - 0.01 = 0.99 \]

d) What is the probability that Michael makes only the last of the three shots? 
\[ 0.1 \times 0.9 = 0.09 \]

Example 5: Of the 10,000 freshman at the University of Texas, 7000 must take English, 6000 must take History, and 5000 must take both. Suppose that a student is randomly selected:

a) What is the probability that the selected student must take English? 
\[ \frac{7000}{10000} = 0.7 \]

b) What is the probability that the selected student must take both English and History? 
\[ \frac{5000}{10000} = 0.5 \]

c) Suppose you learn that the selected student must take English, what is the probability that this student must take both English and History? 
\[ P(H|E) = \frac{5000}{7000} = 0.7143 \]

d) Are the outcomes "must take English" and "must take History" independent? Explain. 
\[ \text{No} \quad P(H|E) \neq P(H) \]

e) Answer the question posed in part d if only 4200 of the students must take both English and History.
\[ \frac{4200}{10000} = 0.42 \]

Example 6: Two office aids at Lake Norman High School are responsible for getting the daily tardy list to the appropriate principals by 3:00pm each day. Livi works on the lists 30% of the days and Caitlyn works on the tardy lists 70% of the days. Livi gets the lists to the correct principals in time 90% of the time. Caitlyn gets the tardy lists to the correct principals 92% of the time. If Mr. Gentle sees that the tardy list is on time, what is the probability that today Livi is responsible for the list?

\[ P(T) = 0.914 \]
\[ P(L \cap T) = 0.27 \]
\[ P(L|T) = \frac{0.27}{0.914} = 0.293 \]
Probability Distributions – Discrete Random Variables

Example 7: Let \( y \) denote the number of broken eggs in a randomly selected carton of one dozen “store brand” eggs at a certain market. Suppose that the probability distribution of \( y \) is as follows.

<table>
<thead>
<tr>
<th>( y )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p(y) )</td>
<td>.65</td>
<td>.20</td>
<td>.1</td>
<td>.04</td>
<td>?</td>
</tr>
</tbody>
</table>

a) Only values 0, 1, 2, 3, and 4 have probabilities. What is \( p(4) \)?

b) Calculate \( P(y \leq 2) \), the probability that the carton contains at most two broken eggs, and interpret this probability.

c) Calculate \( P(y < 2) \). Why is this smaller than the probability in part b?

d) What is the probability that the carton contains exactly ten unbroken eggs?

e) What is the probability that at least ten eggs are unbroken?

f) What is the expected number of broken eggs per carton?

g) What is the standard deviation of the probability distribution?

Binomial distributions (a special form of a discrete random variable)

Example 8: The AP Statistics exam includes 40 multiple choice questions, each with 5 answer choices. Suppose you believe you have forgotten everything and must guess (randomly choose one of the five answers choices) on every question. Let \( X \) represent the number of correct responses on the test.

a) What makes \( X \) a binomial distribution?

b) What is your expected score on the exam?

\[ \frac{8}{40} \approx 20\% \]

c) Compute the variance and standard deviation of \( X \)?

\[ \sigma \text{Var} = 2.329 \]

d) What is the probability that you will get exactly 25 questions correct?

\[ \binom{40}{25} \times 0.2^{25} \times 0.8^{15} = 4.749 \times 10^{-9} \]

(e) Overall, you believe you can do really well on the free response section. You did not study for the multiple choice section because you figured out that you would probably only need 16 questions correct to earn college credit in the course. What is the probability that you correctly answer at least 16 problems?

\[ P(X \geq 16) = 1 - P(X \leq 15) \]

\[ 1 - 0.978638 = 0.021362 \]

(Bad Idea)
**Geometric Probability Distributions**

Example 9: A new free to play iPhone game allows players to replenish their energy only 10% of the time after successfully completing a level. Consider the random variable \( X \), where \( X \) = number of levels that must be successfully completed until the energy is replenished.

\[
\Pr(X = 1) + \Pr(X = 2) = .19
\]

a) What is the probability that at most 2 levels must be successfully completed?

b) What is the probability that exactly five levels must be successfully completed?

\[
\Pr(\text{5 levels}) = .06561
\]

c) What is the probability that more than three levels must be successfully completed?

\[
1 - \Pr(X \leq 3) = 1 - .271 = .729
\]

d) How could you use simulation to approximate getting energy replenished by the third level?

**Normal Distribution**

Example 10: A machine that puts the center holes in blank CDs operates in such a way that the distribution of the diameter of the holes may be approximated by a normal distribution with a mean of 1.5 cm and a standard deviation of .1 cm. The specifications require the diameters of the holes to be between 1.4 and 1.6 cm. A CD not meeting the specifications is considered defective. (A center hole too small would not fit properly in a CD burner; a hole too large may cause the CD to slide during burning and ruin the quality of the music.) What proportion of CDs produced by this machine are defective due to an improperly sized center hole?

\[
1 - .68 = .32
\]
Sampling Distributions

Example 11: Every Saturday, The Full Deck music store has a draw your card day. A customer may choose to draw a card from a standard deck and buy a second CD for an amount in dollars equal to the value on the card with face cards counting as 10. For example, if a customer draws a 3, his second CD will cost only $3.00. If a customer draws a jack, the CD will cost $10.00. Let X represent the amount paid for a second CD on draw a card day. The expected value of X is $6.50 and the standard deviation of X is $3.10.

a) If a customer draws a card and buys a second CD every Saturday for 10 consecutive weeks, what is the total amount that the customer would expect to pay for these second CDs? 

b) If a customer draws a card and buys a second CD every Saturday for 25 consecutive weeks, what is the approximate probability that the total amount paid for these second CDs will exceed $168.00?

Example 12: A plane used to fly tourists in and out of the rain forest contains seating for 16 passengers. The total weight limit for the passengers is 2500 pounds. Assume the average weight of tourists is 150 pounds, the standard deviation 27 pounds, and that the distribution of tourist weights is approximately normal. If the weight limit is exceeded, the plane has difficulty taking off safely. If a random sample of 16 tourists has booked a flight, what is the chance that the weight limit will be exceeded?

Example 13: A manufacturer of iPhones purchases computer chips from a vendor. When a large shipment is received, a random sample of 200 computer chips is selected, and each is inspected. If the sample proportion of defectives is more than .02, the entire shipment will be returned to the vendor. What is the approximate probability that the shipment will be returned if the true proportion of defectives in the shipment is .05?