Frosty Corn Flake cereal company claims that 20% of all its cereal boxes contain a voucher for a free DVD rental. A group of students believes the company is cheating and the proportion of all boxes with the vouchers is less than 0.20. They decide to collect some data to perform a test of significance with the following hypotheses.

\[ H_0 : p = 0.20 \]
\[ H_A : p < 0.20 \]

where \( p \) is the proportion of all boxes with the voucher

They collect a random sample of 65 boxes and find 11 boxes with the voucher. Using a One Proportion \( z \)-test, the students calculate a \( p \)-value = 0.27 and conclude that they do not have enough evidence to say that the proportion of all boxes is less than 0.20. Although the company may be cheating its customers, the students do not have convincing evidence that this is the case.

**PART I: UNCOVER CORPORATE WRONGDOING?**

If the company is in fact cheating its customers, how likely would it be for a test based on 65 boxes to “catch” the company?

1. The students found 11 out of 65 boxes with vouchers and did not conclude the company was cheating. How many boxes with vouchers out of 65 would they have needed to find in order to conclude that the company is cheating? Use trial and error with One Proportion \( z \)-test on your calculator to find the range of number of voucher boxes that would lead to a conclusion of corporate wrongdoing.

2. The assumption in this handout is the company is cheating, and the question is how likely would it be for the students’ 65 box test with \( \alpha = .05 \) to detect this cheating.

A natural question to ask then is: How badly is the company cheating? Pretend the company’s proportion of all boxes with vouchers is really 0.15 (\( p = 0.15 \)). If a 65 box test using \( \alpha = .05 \) were performed, would the students correctly conclude that the company is cheating (\( p < 0.20 \))? Let’s find out.

You will sample 65 cereal boxes from a population in which 15% of all boxes contain a voucher. The calculator command below, which can be found under Menu, 5: Probability, 4: Random, 3: Binomial, simulates random sampling from this population. Run this command to take a sample of 65.

\[ \text{randBin}(65,.15) \]

In question #1, you should have arrived at the following rule for concluding that the company is cheating. (Recall this rule is based on the significance level of \( \alpha = .05 \).)

In your 65 box trial from question #3, how many boxes with vouchers did you obtain? Based on your result, did you have enough evidence to conclude that the company is cheating?
3. Repeat your simulation from question #2 twenty times and record your results in the table below. (To repeat the command `randBin(65,.15)` simply press ENTER.

<table>
<thead>
<tr>
<th>Trial</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Voucher Boxes</td>
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</tr>
</tbody>
</table>

Remember, the assumption in the simulation is the company is cheating: \( p = 0.15 \). **Out of your 20 trials in question #3, in how many of them did you conclude that the company is cheating?**

**Personal Probability:**

The answer above (to question 3) is the proportion of 20 trials in which you concluded the company is cheating is your estimate of the probability of the 6 box test with \( \alpha = .05 \) detecting the company’s placement of vouchers in only 15% of its boxes.

4. Combine your results as a class in order to make an estimate based on more trials. **What is the class estimate of the probability of the 65 box test with \( \alpha = .05 \) detecting the company’s placement of vouchers in only 15% of its boxes?**

**Class Probability:**

The (class) probability you just calculated is an estimate of the **POWER** of a One Proportion z-test with \( n = 65 \) and \( \alpha = .05 \) against the alternative of \( p = 0.15 \).

5. Using your result in question #4, comment on the students’ ability to detect a company that puts vouchers in only 15% of its boxes by using a 65 box test with \( \alpha = .05 \).

**Final note:** One thousand trials of the simulation from question #2 were conducted using computer software. In 226 of these trials, 7 or fewer boxes with vouchers were found, and thus in 22.6% of the trials it was concluded the company was cheating. So, it is not all that likely for the students’ 65 box test using \( \alpha = .05 \) to detect a company whose proportion of all boxes with vouchers is 0.15!

You have seen what power represents in this scenario. A more general definition of POWER is given below and can be applied to any situation in which a test of significance is performed.

**The POWER of a test of significance against a given alternative is the probability that it rejects the null hypothesis.**